ΣΥΝΕΔΡΙΑ ΤΗΣ 17ΗΣ ΝΟΕΜΒΡΙΟΥ 1966

ΑΝΑΚΟΙΝΩΣΙΣ ΜΕΛΟΥΣ

ΑΣΤΡΟΝΟΜΙΑ.— **The Probable Mean Values of the Different Indices of Solar Activity During the Sunspot Cycle No. 20 (1964-1975)** *, *by John Xvnthakis***.

Introduction.

I. In a previous paper [Xanthakis, 1966] it has been shown that the currently available mean annual values of the main indices of solar activity can be satisfactorily represented as functions of the time of rise T_R and certain supplementary periodic terms. The basic parameter T_R represents the time of rise corresponding to the observed values of the relative sunspot numbers, i. e. the time interval between the epoch of the minimum of the mean monthly values of the observed relative sunspot numbers and that of the next maximum.

If we represent by

[R_m] the mean annual values of the observed relative sunspot numbers during the years of maximum solar activity, by

[A_m] the mean annual values of the areas of the whole sunspots during the years of maximum solar activity, by

[F_m] the mean annual values of the areas of the prominences during the years of maximum solar activity and by

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[f_m] the mean annual values of the areas of the faculae during the years
of maximum solar activity,

then the relationships giving the values of these quantities as functions
of the time of rise T_R and the supplementary terms Y are of the form :

$$[R_m] = C + 2bT_o(T_o - T_R)^2 + bT_oY \qquad N = 7 \text{ to } 19$$
$$[A_m] = C + 3bT_o(T_o - T_R)^2 + bT_oY \qquad N = 12 \text{ to } 19$$
$$[F_m] = C + bT_o(T_o - T_R)^2 + 3bT_oY_F \qquad N = 15 \text{ to } 19 \tag{1}$$
$$[f_m] = C + bT_o'(T_o' - T_R) + 3bT_o'Y_f \qquad N = 12 \text{ to } 19$$

where $$T_o = 5.76, \qquad T_o' = 7.0$$

and the constants C and b assume the following values :

	$[R_m]$	$[A_m]$	$[F_m]$	$[f_m]$	
C	66.4	1148	0	2070	(2)
b	1	12	114	33	

In the case of the relative sunspot numbers $[R_m]$ and the sunspot
areas $[A_m]$ the term Y is of the form

$$Y = \frac{n}{10 - n} \exp(n - 9), \tag{3}$$

where n assumes for each decade of cycles the values

$$n = 0, 1, 2, \ldots \ldots 9,$$

the value $n = 0$ corresponding to the sunspot cycles $N = -10, 0, 10, 20$ etc.

In the case of the areas of the faculae and the prominences on the
other hand, the terms Y_f and Y_F are of the form :

$$Y_f = \sin N \frac{2\pi}{4}, \qquad Y_F = \sin^2(N + 1)\frac{\pi}{3} \tag{4}$$

i. e. periodic with periods correspondingly equal to 4 and 3 sunspot cycles.

Two additional indices of solar activity i. e. the maxima R_{max} and
the minima R_{min} of the mean monthly relative sunspot numbers observed
during the years of maximum solar activity can also be satisfactorily
represented as functions of the time of rise T_R, the exponential term Y
and another periodic term of the form

$$X_R = 4T_o'' \sin\left(N - \frac{a}{2}\right)\frac{2\pi}{8}, \qquad N = 0 \text{ to } 19$$

with the help of the relations:

$$R_{max} = a^2 + 2T_o^{''}(T_o^{''} - T_R)^2 + aY + 4T_o^{''}\sin\left(N - \frac{a}{2}\right)\frac{2\pi}{8}$$

$$R_{min} = T_o^{''2} + (T_o^{''} + 1)(T_o^{''} - T_R)^2 + \frac{1}{2}T_o^{''}Y$$

$$N = 7 \text{ to } 19$$

(5)

where

$$T_o^{''} = b_R^{'} = 6.3 \text{ and } a = 9.0 .$$

Fig. A represents the values of R_{max}, $[R_m]$ and R_{min} predicted for the sunspot cycle $N = 20$ on the basis of the relations (1) and (5) for the values of the time of rise $T_R = 3.6$ to 5.0 Fig. 8 represents the corresponding values of $[A_m]$, $[f_m]$ and $[F_m]$.

It is worth mentioning that for the sunspot cycle $N = 20$ the term Y in relations (1) assumes the values

$$Y \simeq 0, \qquad Y_F = 0, \qquad Y_f = 0$$

and therefore the values of $[R_m]$, $[A_m]$, $[F_m]$ and $[f_m]$ depend only on the time of rise.

Another interesting point is that the values of $[f_m]$ for the sunspot cycles $N = 12$ to 19 are represented by a linear function of the time of rise T_R while the three remaining relations of (1) are of the second degree.

II. The main part of the variation of the mean annual values of all the above indices of solar activity within each sunspot cycle can be represented by the relation

$$[Z] = [Z_m]\cos^2\frac{\pi}{2\Omega_{1,2}}t \qquad t = 0, 1, 2, \ldots .$$

(6)

where

$\Omega_1 = T_R$ for the years preceding the sunspot maximum (ascending branch)

$\Omega_2 = 11 - T_R$ for the years following the sunspot maximum (descending branch).

Parameter t assumes the value $t = 0$ for the year of maximum solar activity and the values $t = 1, 2, 3, \ldots$ for the first, second, third, ... year preceding or following the year of maximum. In the case of the areas of

the prominences a constant term C_F is added to the second part of rela-
tion (6), due to the fact that during the year of minimum solar activity
the mean annual values of the areas of the prominences do not vanish
but assume for the sunspot cycles $N = 15$ to 19 a minimum value approx-
imatively equal to

$$C_F = 1310 . 10^{-6} \odot .$$

A study of the differences

$G_1(t) = [Z]^o — [Z]^c$ for the years preceding the sunspot maximum
(ascending branch)

$G_2(t) = [Z]^o — [Z]^c$ for the years following the sunspot maximum
(descending branch),

where $[Z]^o$ represents the mean annual values of the above indices given
by the observations and $[Z]^c$ the values of the same quantities computed
with the help of relation (6), shows that the above differences do not
assume random values but vary in a periodic way. We thus conclude
that the mean annual values of the indices of solar activity considered
here show, during the ascending as well as during the descending branch
of solar activity, periodic fluctuations the half-periods of which are found
to be equal to from 2 — 6 years.

In the case of the relative sunspot numbers and the sunspot areas
these fluctuations can be better studied, if we divide the sunspot cycles
in the following groups on the basis of the time of rise T_R :

$$\text{Group (a)} \quad 3.0 < T_R \leqslant 3.5 \quad \text{years}$$
$$\text{Group (b)} \quad 3.5 \leqslant T_R \leqslant 5.0 \quad \text{years}$$
$$\text{Group (c)} \quad 5.0 < T_R \quad \text{years}$$

The values of the functions $G_1(t)$, $G_2(t)$ for each of these groups
are then given by the following relations:

Group (a) $3.0 < T_R \leqslant 3.5$ years,   Cycles No. 8, 11, 18, 19

$$G_1(t) = K_1 Y \sin \frac{\pi}{2} t \sin \frac{\pi}{3} t — L_1 \sin \frac{\pi}{3} t$$

$$G_2(t) = K_2 Y \sin \frac{\pi}{3} t — L_2 \sin \frac{\pi}{6} t$$

(7a)

Group (b) $3.5 \leqslant T_R \leqslant 5.0$ years ,   Cycles No. 8, 9, 10, 13, 15, 16, 17

$$G_1(t) = \left[K_1 Y \sin \frac{\pi}{3} t + L_1 \sin \frac{\pi}{4} t \right] \cos N\pi$$

$$G_2(t) = - K_2 Y \sin \frac{\pi}{3} t - L_2 \sin \frac{\pi}{4} t \tag{7b}$$

Group (c) $5.0 < T_R$ years ,   Cycles No. 7, 12, 14

$$G_1(t) = - K_1 \sin \frac{\pi}{4} t$$

$$G_2(t) = - K_2 \sin \frac{\pi}{4} t \quad \text{for the relative sunspot numbers} \tag{7c}$$

or $G_2(t) = - K_2 \sin \frac{\pi}{5} t$ for the sunspot areas

In the case of the areas of the faculae and the prominences on the contrary, the values of the differences $G_{1,2}(t)$ do not depend on the time of rise but on the polarity of the sunspots i. e. they assume different values for the even and the odd cycles. We thus have

Even Cycles

$$G_1^f(t) = K_1 \cos N \frac{2\pi}{8} \cdot \sin \frac{\pi}{4} (t-1) - L_1 \sin N \frac{2\pi}{8} \sin \frac{\pi}{2} t$$

$$G_1^F(t) = K_1 \cos N \frac{2\pi}{4} \sin \frac{\pi}{2} t$$

$$\tag{8}$$

$$G_2^f(t) = - K_2 \cos^2 N \frac{2\pi}{8} \sin \frac{\pi}{4} (t-1) + L_2 \left[1 - 2 \sin N \frac{2\pi}{8} \right] \sin \frac{\pi}{5} (t-1)$$

$$G_2^F(t) = K_2 \sin \frac{\pi}{3} t$$

Odd Cycles

$$G_1^f(t) = K_1 \left[1 - 2 \cos (N-1) \frac{2\pi}{8} \right] \cos (N-1) \frac{2\pi}{8} \sin \frac{\pi}{3} t$$

$$G_1^F(t) = K_1 \sin (N-1) \frac{2\pi}{8} \sin \frac{\pi}{4} t$$

$$\tag{9}$$

$$G_2^f(t) = - K_2 \cos^2 (N-1) \frac{2\pi}{4} \sin \frac{\pi}{4} t$$

$$G_2^F(t) = - K_2 \sin^2 (N-1) \frac{2\pi}{8} \sin \frac{\pi}{5} t$$

Parameter t assumes again the value $t = 0$ for the year of maximum solar activity and the values $t = 1, 2, 3, \ldots$ for the first, second, third, $\ldots$ year preceding or following the year of maximum. As concerns $K_{1,2}$ and $L_{1,2}$ these coefficients assume specific numerical values in each case.

III. On the basis of relations (1) we find for the sunspot cycle $N = 20$

$$[R_m] = 66.4 + 2T_o(T_o - T_R)^2$$
$$[A_m] = 1148 + 36T_o(T_o - T_R)^2$$
$$[F_m] = 114T_o(T_o - T_R)^2 \tag{10}$$
$$[f_m] = 2070 + 33T_o'(T_o' - T_R)$$

where

$$T_o = 5.76 \quad \text{and} \quad T_o' = 7.0 .$$

Also from relations (6) - (10) we find for the sunspot cycle $N = 20$

$$[R] = [R_m] \cos^2 \frac{\pi}{2\Omega_{1,2}} t + G_{1,2}^R(t)$$

$$G_1^R(t) = 2T_o \sin \frac{\pi}{4} t \tag{11}$$

$$G_2^R(t) = -2T_o \sin \frac{\pi}{4} t$$

$$[A] = [A_m] \cos^2 \frac{\pi}{2\Omega_{1,2}} t + G_{1,2}^A(t)$$

$$G_1^A(t) = 48T_o \sin \frac{\pi}{4} t \tag{12}$$

$$G_2^A(t) = -48T_o \sin \frac{\pi}{5} t$$

$$[F] = C_F + [F_m] \cos^2 \frac{\pi}{2\Omega_{1,2}} t + G_{1,2}^F(t)$$

$$G_1^F(t) = 3b_F \sin \frac{\pi}{2} t \tag{13}$$

$$G_2^F(t) = b_F \sin \frac{\pi}{3} t$$

$$[F] = [f_m] \cos^2 \frac{\pi}{2\Omega_{1,2}} t + G_{1,2}^f (t)$$

$$G_1^f (t) = -3b_f \sin \frac{\pi}{4} (t - 1) \tag{14}$$

$$G_2^f (t) = -3b_f \sin \frac{\pi}{4} (t - 1) + b_f \sin \frac{\pi}{5} (t - 1).$$

Figs. 1 — 8 represent respectively the mean annual values of the relative sunspot numbers [R] and of the areas of the whole sunspots [A], prominences [F] and of faculae [f] computed from the above relations for the successive years of sunspot cycle N = 20 for the values of the time of rise T_R = 3.6 to 5.0 years. The corresponding values are also tabulated in Tables 1 — 4. From Figs. 5, 6 we see that two distinct maxima of the values of [F] during the sunspot cycle N = 20 are to be expected.

Finally if we represent by R_{max}^y , R_{min}^y the highest and lowest values of the mean monthly relative sunspot numbers observed during the successive years of sunspot cycle N = 20 we have the following relations:

$$R_{max}^y = R_{max} \cos^2 \frac{\pi}{2T_R} t + G_1 \quad \text{ascending branch}$$

$$\tag{15}$$

$$R_{min}^y = R_{min} \cos^2 \frac{\pi}{2T_R} t + g_1 \quad \text{ascending branch}$$

$$R_{max}^y = R_{max} \cos^2 \frac{\pi}{2(11 - T_R)} t + G_2 \quad \text{descending branch}$$

$$\tag{16}$$

$$R_{min}^y = R_{min} \cos^2 \frac{\pi}{2(11 - T_R)} t + g_2 \quad \text{descending branch}$$

where R_{max} and R_{min} are given by relations (5) and $G_{1,2}$ and $g_{1,2}$ are periodic terms that, in the case of the sunspot cycle N = 20, are of the form:

$$G_1 = \quad 25.2 \sin \frac{\pi}{4} t$$

$$t = 1, 2, 3, 4 \tag{17}$$

$$g_1 = - \quad 6.3 \sin \frac{\pi}{4} t$$

$$G_2 = -18.9 \sin \frac{\pi}{4} t$$

$$t = 1, 2, 3, 4 . \tag{18}$$

$$g_2 = - \quad 6.6 \sin \frac{\pi}{6} t$$

The values of R^y_{max}, R^y_{min} computed with the help of relations
(15) — (18) for the successive years of sunspot cycle N = 20 are tabulated
in Tables 5 and 6. The same values are also represented in Figs. 9—12.
As it is not possible to predict at present the exact month of each year
during which the values of R^y_{max}, R^y_{min} will occur, the values of
R^y_{max}, R^y_{min} shown in Figs. 9 — 12 have been plotted for the middle of
the corresponding year.

The assistance of Mr. C. Poulakos in performing the computations
and plotting the figures of the present paper is gratefully acknowledged.

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★

Ὁ Ἀκαδημαϊκὸς κ. **Ἰ. Ξανθάκης** παρουσιάζων τὴν ὡς ἄνω ἀνακοίνωσιν
εἶπε τὰ ἑξῆς:

Ἔχω τὴν τιμὴν νὰ ἀνακοινώσω εἰς τὴν Ἀκαδημίαν Ἀθηνῶν τὰ πορίσματα
τῶν ἐρευνῶν μου ἐπὶ τῆς πιθανῆς πορείας τῶν δεικτῶν τῆς ἡλιακῆς δραστηριό-
τητος κατὰ τὸν παρόντα ἡλιακὸν κύκλον, ὅστις ἤρχισε κατὰ μῆνα Ἰούλιον τοῦ
1964 καὶ ἀναμένεται νὰ περαιωθῇ κατὰ τὸ ἔτος 1975.

Πρὶν εἰσέλθω εἰς τὸ κύριον θέμα τῆς ἀνακοινώσεώς μου, εἶναι ἀναγκαῖον
νὰ κάμω μίαν μικρὰν εἰσήγησιν.

Ὡς γνωστόν, ἡ ἡλιακὴ δραστηριότης, ἥτις παρουσιάζει μίαν περιοδικότητα
κατὰ μέσον ὅρον 11ετῆ, γνωστὴν ὑπὸ τὸ ὄνομα «ἑνδεκαετὴς ἡλιακὸς κύκλος»,
ἐκδηλοῦται μὲ διάφορα φαινόμενα, ἐκ τῶν ὁποίων τὰ κυριότερα εἶναι: αἱ ἡλιακαὶ
κηλῖδες, οἱ πυρσοί, αἱ διαφόρου τύπου ἡλιακαὶ προεξοχαί, αἱ ἡλιακαὶ ἐκλάμψεις
καὶ ἡ λαμπρότης ὡρισμένων φασματικῶν γραμμῶν τοῦ ἡλιακοῦ στέμματος. Τὸ
σύνολον τῶν φαινομένων τούτων καθὼς καὶ τινων ἄλλων δευτερευούσης σημασίας
συνιστᾷ τὴν ἡλιακὴν δραστηριότητα, ἡ ὁποία διαδραματίζει σπουδαιότατον ρόλον
εἰς τὴν διαμόρφωσιν τῆς ἑκάστοτε φυσικῆς καταστάσεως τοῦ μεσοπλανητικοῦ δια-
στήματος καὶ εἶναι συνάμα ἡ πρωταρχικὴ πηγὴ τῶν διαφόρων διαταραχῶν τόσον
τῆς ἰονοσφαίρας ὅσον καὶ τῶν ἀπωτάτων ἀτμοσφαιρικῶν στιβάδων τῆς γῆς.

Εἶναι εὐνόητον ἑπομένως τὸ ἐνδιαφέρον ποὺ παρουσιάζουν σήμερον αἱ ἡλια-

καὶ αὗται ἔρευναι ὄχι μόνον διὰ τοὺς ἀστρονόμους, ἀλλὰ καὶ διὰ τοὺς ραδιοηλε-
κτρολόγους, τοὺς μετεωρολόγους, τοὺς γεωφυσικοὺς καὶ δι᾽ ὅλους ἐκείνους ποὺ
ἀσχολοῦνται μὲ τὴν ἐξερεύνησιν τοῦ διαστήματος διὰ τῶν συγχρόνων μέσων ἐρεύ-
νης, τῶν δορυφόρων καὶ τῶν ἐπηνδρωμένων διαστημοπλοίων.

Ἀπὸ αἰῶνος καὶ πλέον ἡ ἡλιακὴ δραστηριότης ἐκφράζεται εἰς μίαν αὐθαί-
ρετον κλίμακα εἰσαχθεῖσαν ἐν τῇ ἐπιστήμῃ τὸ 1848 ὑπὸ τοῦ Γερμανοῦ ἀστρονό-
μου Wolf. Εἰς τὴν μέθοδον Wolf λαμβάνεται ὑπ᾽ ὄψιν τὸ πλῆθος τῶν ὁμάδων
τῶν ἡλιακῶν κηλίδων, καθὼς καὶ τὸ πλῆθος τῶν μεμονωμένων τοιούτων ποὺ
ἐμφανίζονται ἑκάστοτε ἐπὶ τῆς ἡλιακῆς ἐπιφανείας. Τὰ στοιχεῖα ταῦτα ἐκφρά-
ζονται δι᾽ ἀριθμῶν, οἱ ὁποῖοι καλοῦνται «Σχετικοὶ Ἀριθμοὶ» ἢ «Ἀριθμοὶ Wolf».

Κατὰ τὰ τέλη τοῦ 19ου αἰῶνος οἱ ἀστρονόμοι ἐπεζήτησαν νὰ ἐκφράσουν
τὴν ἡλιακὴν δραστηριότητα δι᾽ ἄλλων φυσικῶν μεγεθῶν. Οὕτω ἀπὸ τοῦ 1878 παρα-
τηροῦνται καὶ προσδιορίζονται ἀνελλιπῶς εἰς τὸ Ἀστεροσκοπεῖον τοῦ Greenwich
τὰ ἐμβαδὰ τῶν κηλίδων καὶ τῶν πυρσῶν, ἀπὸ δὲ τοῦ 1914 μετροῦνται εἰς τὸ ἡλια-
κὸν κέντρον ἐρευνῶν τῆς Ζυρίχης τὰ ἐμβαδὰ τῶν πάσης φύσεως ἡλικῶν προεξο-
χῶν, ἀπὸ 25ετίας δὲ περίπου παρατηροῦνται ἀνελλιπῶς καὶ προσδιορίζονται ὑπὸ
διαφόρων ἀστεροσκοπείων τῆς Εὐρώπης, τῆς Σοβιετικῆς Ἑνώσεως, τῆς Ἀμερικῆς
καὶ τῆς Ἰαπωνίας τὰ ἐμβαδὰ καὶ ἡ λαμπρότης τῶν ἡλιακῶν ἐκλάμψεων. Πάντα τὰ
στοιχεῖα ταῦτα, ἤτοι, ἀριθμοὶ Wolf, ἐμβαδὰ κηλίδων, πυρσῶν καὶ ἡλιακῶν προεξο-
χῶν ἀποτελοῦν τοὺς λεγομένους **δείκτας** τῆς ἡλιακῆς δραστηριότητος, μὲ τοὺς
ὁποίους ἀσχολούμεθα εἰς τὴν παροῦσαν ἀνακοίνωσιν. Ἀντικείμενον ἰδιαιτέρας
ἐρεύνης μας θὰ ἀποτελέσουν αἱ ἡλιακαὶ ἐκλάμψεις καὶ ἡ ἔντασις τῆς πρασίνης
γραμμῆς τοῦ στέμματος (σλάϊτ).

Λόγῳ τῆς ἐξαιρετικῆς σημασίας, ὅπως εἴπομεν προηγουμένως, τῶν περιοδι-
κῶν τούτων φαινομένων τοῦ ἡλίου, πολλαὶ προσπάθειαι κατεβλήθησαν ὑπὸ δια-
φόρων ἐρευνητῶν διὰ τὴν εὕρεσιν ἐμπειρικῶν στατιστικῶν σχέσεων, ἐπὶ τῇ βάσει
τῶν ὁποίων θὰ ἦτο ἴσως δυνατὸν νὰ προβλεφθῇ τόσον ἡ ἐποχὴ κατὰ τὴν ὁποίαν
ὁ ἥλιος εὑρίσκεται εἰς τὸ μέγιστον τῆς διεγέρσεώς του, ὅσον καὶ τὸ μέγεθος τῆς
διεγέρσεως ταύτης. Ἡ πρώτη ἀξιόλογος στατιστικὴ σχέσις ἐδόθη πρὸ εἰκοσαετίας
ὑπὸ τοῦ Ἑλβετοῦ Ἀστρονόμου Waldmeier, καθηγητοῦ τοῦ Πολυτεχνείου τῆς
Ζυρίχης καὶ Διευθυντοῦ τοῦ ἐκεῖ Κέντρου Ἡλιακῶν Ἐρευνῶν. Ἀλλὰ ἡ σχέσις
Waldmeier, μολονότι παριστᾷ ἱκανοποιητικῶς τὰ μέχρι τοῦ 1944 δεδομένα τῶν
παρατηρήσεων, ἀφίσταται σημαντικῶς τῶν μεταγενεστέρων τοιούτων. Τὸ 1957
εἰς μίαν ἀνακοίνωσίν μου εἰς τὸ Ἀστεροσκοπεῖον τοῦ Pulkovo, γενομένην ἐπ᾽ εὐ-
καιρίᾳ τῆς μεταβάσεώς μου εἰς τὴν Σοβιετικὴν Ἕνωσιν μετ᾽ ἄλλων συναδέλφων
Ἀκαδημαϊκῶν, ὑπεστήριξα ὅτι ἡ βασικὴ παράμετρος ἡ καθορίζουσα τὸ ὕψος τοῦ

μεγίστου τῆς ἡλιακῆς δραστηριότητος, ἐκφραζομένης εἰς ἀριθμοὺς Wolf, εἶναι ὁ χρόνος ἀνόδου, δηλαδὴ τὸ χρονικὸν διάστημα ποὺ μεσολαβεῖ μεταξὺ τῆς ἐποχῆς τοῦ ἐλαχίστου καὶ τοῦ μεγίστου τῶν ἡλιακῶν κηλίδων, ἤτοι μεταξὺ τῆς ἐποχῆς ποὺ ὁ ἥλιος εὑρίσκεται ἐν ἠρεμίᾳ καὶ ἐν πλήρει διεγέρσει ἀπὸ ἀπόψεως κηλίδων. Διετύπωσα δὲ συνάμα τὴν γνώμην ὅτι ἡ σχέσις ἡ συνδέουσα τὴν παράμετρον ταύτην μὲ τὸ μέγιστον τῶν ἡλιακῶν κηλίδων δὲν εἶναι ἀκριβῶς ἐκείνη ποὺ ἔδωσε ὁ Waldmeier, δηλαδὴ λογαριθμικῆς μορφῆς, ἀλλὰ εἶναι σχέσις ἀλγεβρικὴ 2ου βαθμοῦ (παραβολική).

Δύο ἔτη βραδύτερον ὁ Ρῶσος ἀστρονόμος Vitinsky, στηριζόμενος ἐπὶ τῶν πορισμάτων τῆς ἀνακοινώσεώς μου ἐκείνης, διετύπωσε νέαν στατιστικὴν μέθοδον προβλέψεως τῶν τριμηνιαίων τιμῶν τῶν ἀριθμῶν Wolf συναρτήσει τοῦ χρόνου ἀνόδου.

Ἀλλὰ τὸ ὅλον πρόβλημα δὲν περιορίζεται μόνον εἰς τὴν ἔκφρασιν τῶν ἀριθμῶν Wolf συναρτήσει τοῦ χρόνου ἀνόδου, διότι οἱ ἀριθμοὶ Wolf ἀφ' ἑνὸς μὲν εἶναι αὐθαίρετον μαθηματικὸν ἐπινόημα, ἀφ' ἑτέρου δὲ ἀντιπροσωπεύουν ἕνα μόνον ἐκ τῶν δεικτῶν τῆς ἡλιακῆς δραστηριότητος ποὺ ἀναφέρεται εἰς τὸ πλῆθος τῶν ἡλιακῶν κηλίδων. Διὰ τὸν λόγον αὐτὸν ἀπὸ τοῦ ἔτους 1959 ἡ προσοχή μας ἐστράφη καὶ πρὸς τοὺς ἄλλους δείκτας τῆς ἡλιακῆς δραστηριότητος, διὰ τοὺς ὁποίους οὐδεμία συστηματικὴ στατιστικὴ ἔρευνα ἔχει γίνει λόγῳ ἀνεπαρκῶν δεδομένων. (Ὡς γνωστόν, ἓν στατιστικὸν ἐξαγόμενον διὰ νὰ παρέχῃ ἐμπιστοσύνην δέον νὰ στηρίζεται εἰς ὅσον τὸ δυνατὸν μέγα πλῆθος δεδομένων).

Ἀφοῦ πρῶτον συνεκεντρώσαμεν εἰς τὸ Γραφεῖον Ἐρευνῶν καὶ Ὑπολογισμῶν τῆς Ἀκαδημίας πάντα τὰ ὑπὸ τῶν παρατηρήσεων δεδομένα ἐπὶ τῶν διαφόρων δεικτῶν τῆς ἡλιακῆς δραστηριότητος, ἤτοι ὅλα τὰ ἐξαγόμενα τῶν παρατηρήσεων τοῦ Ἀστεροσκοπείου τοῦ Greenwich ἐπὶ τῶν ἐμβαδῶν τῶν ἡλιακῶν κηλίδων καὶ πυρσῶν, ὅλα τὰ ἐξαγόμενα τῶν παρατηρήσεων τοῦ Κέντρου ἡλιακῶν ἐρευνῶν τῆς Ζυρίχης ἐπὶ τῶν ἐμβαδῶν τῶν προεξοχῶν, καθὼς καὶ ὅλον τὸ ἀπὸ τοῦ 1740 μέχρι σήμερον συγκεντρωθὲν ὑλικὸν ἐπὶ τῶν ἀριθμῶν Wolf, ἐπεδόθημεν εἰς τὴν ἀναζήτησιν οὐχὶ ἁπλῶν στατιστικῶν ἀλλ' ἀκριβῶν ἀναλυτικῶν σχέσεων παριστωσῶν πάντα τὰ δεδομένα τῶν παρατηρήσεων συναρτήσει τοῦ χρόνου ἀνόδου.

Τὰ πορίσματα τῶν ἐρευνῶν μας τούτων ἀνεκοινώθησαν σταδιακῶς εἰς τὰς Ἀκαδημίας Ἀθηνῶν καὶ Παρισίων καὶ ἐδημοσιεύθησαν πλέον ἐκτενῶς εἰς εἰδικὰ περιοδικὰ διαφόρων χωρῶν τῆς Εὐρώπης. Αἱ ἐν λόγῳ ἔρευναι, ποὺ ἐξετελέσθησαν ἐξ ὁλοκλήρου εἰς τὸ Γραφεῖον Ἐρευνῶν καὶ Ὑπολογισμῶν τῆς Ἀκαδημίας, προώθησαν σημαντικῶς τὴν λύσιν τοῦ προβλήματος τῆς προγνώσεως τοῦ ὅλου

φαινομένου τῆς ἡλιακῆς δραστηριότητος, διότι ἀπεδείχθη ὅτι οἱ κυριώτεροι δεῖκται αὐτῆς δύνανται νὰ παρασταθῶσιν ἀναλυτικῶς συναρτήσει μιᾶς καὶ μόνης παραμέτρου, τοῦ χρόνου ἀνόδου ἀφ᾽ ἑνὸς καί τινων ἀφ᾽ ἑτέρου συμπληρωματικῶν περιοδικῶν ὅρων ἐκ τῶν προτέρων καθοριζομένων.

Ἀλλὰ διὰ νὰ φθάσωμεν εἰς τὴν τελικὴν λύσιν τοῦ σημαντικοῦ τούτου προβλήματος δέον νὰ γνωρίζωμεν ἐκ τῶν προτέρων τὰς τιμὰς τῆς βασικῆς παραμέτρου, ἐν ἄλλοις λόγοις δέον νὰ γνωρίζωμεν δι᾽ ἕνα ἕκαστον μελλοντικὸν ἡλιακὸν κύκλον ποῖος εἶναι ὁ ἀντίστοιχος χρόνος ἀνόδου. Τοῦτο ὅμως δὲν κατέστη δυνατὸν νὰ ἐπιτευχθῇ μέχρι σήμερον. Κατόπιν τῶν ἀνωτέρω, ὡς εἶναι εὐνόητον, ἡ προσοχὴ τῶν ἀστρονόμων ἐστράφη πρὸς τὴν μυστηριώδη ταύτην παράμετρον, τὸν χρόνον ἀνόδου. Τὸν Μάρτιον τοῦ τρέχοντος ἔτους ὁ Ἄγγλος γεωφυσικὸς King - Helle στηριζόμενος ἐπὶ ἑνὸς φαινομένου ἐπαναληπτικότητος τῶν τιμῶν τοῦ χρόνου ἀνόδου ἀνὰ ἑπτὰ (7) ἡλιακοὺς κύκλους, ὑπεστήριξε διὰ μιᾶς δημοσιεύσεώς του εἰς τὸ γνωστὸν ἀγγλικὸν περιοδικὸν Nature ὅτι ὁ χρόνος ἀνόδου κατὰ τὸν παρόντα ἡλιακὸν κύκλον δέον νὰ εἶναι 3, 4 ἔτη καὶ συνεπῶς τὸ μέγιστον τῆς ἡλιακῆς δραστηριότητος δέον νὰ λάβῃ χώραν τὸν Ἰανουάριον τοῦ 1968. Τὴν ἄποψιν ὅμως ταύτην ἠναγκάσθην νὰ ἀντικρούσω διὰ μιᾶς δημοσιεύσεώς μου εἰς τὸ αὐτὸ περιοδικὸν κατὰ τὸν παρελθόντα μῆνα Ἰούνιον. Πράγματι ἡ ἐπαναληπτικότης τῶν τιμῶν τοῦ χρόνου ἀνόδου ἀνὰ ἑπτὰ ἡλιακοὺς κύκλους, ὡς ὑπεστήριξεν ὁ King - Helle, παρατηρεῖται μόνον ἐὰν ληφθοῦν ὑπ᾽ ὄψιν αἱ ἀπὸ τοῦ 1880 καὶ ἐντεῦθεν παρατηρήσεις, καταστρέφεται δὲ ἐὰν ληφθῶσιν ὑπ᾽ ὄψιν καὶ αἱ προγενέστεραι παρατηρήσεις. Ἐὰν θεωρήσωμεν ὅλον τὸ μέχρι σήμερον συγκεντρωθὲν ὑλικὸν τῶν παρατηρήσεων καὶ λάβωμεν συγχρόνως ὑπ᾽ ὄψιν τὸν ὑπὸ τοῦ Ἀμερικανοῦ ἀστροφυσικοῦ Hall διατυπωθέντα Νόμον τῆς ἀντιστροφῆς τῆς πολικότητος τῶν ἡλιακῶν κηλίδων ἀπὸ κύκλου εἰς κύκλον, τότε ἀνευρίσκομεν σαφῶς δύο περιοδικότητας, ἐκ τῶν ὁποίων ἡ μὲν μία ἔχει περίοδον ὀκτὼ (8) ἡλιακῶν κύκλων, ἡ δὲ ἄλλη δέκα (10). Εἶναι δὲ ἀξιοσημείωτον ὅτι αἱ περιοδικότητες αὗται δὲν παρατηροῦνται μόνον εἰς τὸν χρόνον ἀνόδου, ἀλλὰ καὶ εἰς τὸ χρονικὸν διάστημα τὸ μεσολαβοῦν μεταξὺ τῶν μεγίστων τῆς ἡλιακῆς δραστηριότητος εἰς κύκλους τῆς αὐτῆς πολικότητος, δηλαδὴ εἰς κύκλους μὲ ἄρτιον καὶ περιττὸν ἀριθμὸν εἰς τὴν ἀρίθμησιν Κάρινγκτων. Ἀλλὰ καὶ μὲ τὸ νέον τοῦτο ἐπιπρόσθετον στοιχεῖον ποὺ ἦλθεν εἰς φῶς τὸν παρελθόντα Ἰούνιον, καὶ πάλιν δὲν δυνάμεθα νὰ προσδιορίσωμεν ἐπακριβῶς τὸν χρόνον ἀνόδου, διότι τὸ εὖρος τῶν ἐν λόγῳ περιοδικοτήτων δὲν παραμένει σταθερόν, ἀλλὰ μεταβάλλεται ἀπὸ περιόδου εἰς περίοδον. Οὕτω, ὁ χρόνος ἀνόδου ἐξακολουθεῖ νὰ εἶναι ὁ μέγας ἄγνωστος τοῦ προβλήματος. Τὸ μόνον τὸ ὁποῖον δυνάμεθα νὰ εἴπωμεν ἐπὶ τῇ βάσει τῆς προσ-

φάτου ταύτης ἐρεύνης μας εἶναι ὅτι τὸ προσεχὲς μέγιστον τῆς ἡλιακῆς δρα-
στηριότητος δὲν πρέπει νὰ ἀναμένεται πρὸ τοῦ Ἰουνίου τοῦ 1968.

Ἡ ἀτελὴς γνῶσις τῆς ἀκριβοῦς τιμῆς τοῦ χρόνου ἀνόδου μᾶς ἠνάγκασε νὰ
λάβωμεν ὑπ᾽ ὄψιν εἰς τοὺς ὑπολογισμούς μας διὰ τὸν καθορισμὸν τῆς πορείας
τῶν διαφόρων δεικτῶν τῆς ἡλιακῆς δραστηριότητος κατὰ τὸν παρόντα ἡλιακὸν
κύκλον οὐχὶ μίαν μόνον τιμὴν τῆς παραμέτρου ταύτης, ἀλλὰ μίαν ὁμάδα τιμῶν
κειμένων ἐντὸς τοῦ διαστήματος 3, 6 ἕως 5 ἐτῶν.

Οὕτω δίδεται σήμερον, διὰ πρώτην φοράν, πρόβλεψις τῶν κυριωτέρων φαι-
νομένων τῶν συνιστώντων τὴν ἡλιακὴν δραστηριότητα δι᾽ ὅλα τὰ μέχρι τοῦ 1975
ἔτη μὲ πιθανότητας 70 % διὰ τὰς ἡλιακὰς προεξοχάς, 82 % διὰ τὰ ἐμβαδὰ τῶν
κηλίδων καὶ πυρσῶν καὶ 85 % διὰ τοὺς ἀριθμοὺς Wolf.

Αἱ προσεχεῖς παρατηρήσεις καὶ ἰδίᾳ αἱ τοῦ ἔτους 1968 θὰ μᾶς δείξουν
μέχρι ποίου βαθμοῦ αἱ προβλέψεις μας αὗται εἶναι ἐπιτυχεῖς.

Εἶναι γνωστόν, ὅτι ἡ φύσις συχνάκις ἀρέσκεται νὰ μᾶς ἀπατᾷ.

T A B L E 1.

Predicted values of the mean annual relative sunspot numbers [R] for the successive years of the sunspot cycle No. 20 corresponding to the values of the time of rise $3.6 \leqslant T_R \leqslant 5.0$

T_R Years	3.6	3.8	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
1964	3.6	0.8	0.0	0.1	0.6	1.1	1.8	2.5	1.6	0.8	0.3	0.1	0.0
5	16.1	19.7	23.0	24.5	25.8	27.1	28.3	2.5	3.4	4.3	5.2	6.1	6.9
6	61.1	62.3	62.5	62.4	62.2	61.9	61.6	29.3	30.2	31.0	31.9	32.6	33.3
7	106.7	100.9	95.2	92.5	89.9	87.4	85.1	61.2	60.8	60.3	59.9	59.6	59.3
8	120.1	110.6	102.0	98.1	94.4	90.9	87.7	82.9	80.8	78.8	77.2	75.6	74.2
9	106.7	97.3	88.8	85.0	81.4	77.9	74.7	84.7	81.9	79.3	77.0	74.9	73.0
1970	88.2	79.3	71.3	67.7	64.1	60.9	57.8	71.8	69.0	66.4	64.0	61.9	60.0
1	69.5	61.5	54.2	51.0	47.8	44.8	42.0	54.9	52.2	49.6	47.3	45.2	43.3
2	52.4	45.7	39.7	36.9	34.3	31.8	29.5	39.3	36.9	34.5	32.3	30.3	28.4
3	36.7	31.7	27.3	25.4	23.5	21.8	20.2	27.4	25.3	23.4	21.6	19.8	18.2
4	21.8	18.9	16.6	15.5	14.7	14.0	13.3	18.8	17.4	16.1	15.0	13.9	13.0
5	—	—	—	—	—	—	—	12.8	12.3	12.0	11.7	11.6	11.5

T A B L E 2.

Predicted values of the mean annual areas of the whole sunspots [A] for the successive years of the sunspot cycle No. 20 corresponding to the values of the time of rise 3.6 ≤ T_R ≤ 5.0

T_R Years	3.6	3.8	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
1964	63	14	0	2	10	19	31	44	27	14	5	1	0
5	338	400	457	483	507	528	548	44	59	75	90	105	120
6	1150	1169	1171	1168	1164	1158	1151	565	581	595	610	621	633
7	1933	1828	1725	1674	1627	1582	1541	1143	1135	1127	1118	1111	1107
8	2116	1945	1790	1719	1653	1590	1532	1500	1463	1428	1397	1368	1344
9	1860	1691	1537	1468	1404	1341	1283	1477	1427	1381	1339	1301	1268
1970	1493	1334	1190	1124	1061	1003	947	1230	1180	1134	1092	1053	1020
1	1104	960	831	772	716	662	612	895	847	802	760	722	688
2	760	640	533	483	437	393	352	564	520	479	440	404	371
3	504	414	337	303	269	240	211	314	278	244	212	182	154
4	345	293	253	233	219	206	194	186	161	139	121	101	85
5	—	—	—	—	—	—	—	185	177	171	167	164	163

T A B L E 3.

Predicted values of the mean annual areas of the prominences [F] for the successive years of the sunspot cycle No. 20 corresponding to the values of the time of rise $3.6 < T_R \leqslant 5.0$

T_R \ Years	3.6	3.8	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
1964	1402	1328	1310	1312	1320	1327	1334	1341	1327	1317	1312	1310	1310
5	1515	1574	1606	1611	1610	1602	1589	1341	1346	1350	1350	1349	1346
6	2572	2465	2324	2247	2166	2085	2002	1570	1548	1523	1497	1469	1441
7	5784	5387	5008	4827	4655	4492	4339	1920	1841	1763	1690	1621	1558
8	**4366**	**3827**	**3339**	**3115**	**2904**	**2706**	**2522**	**2350**	**2192**	**2046**	**1914**	**1795**	**1689**
9	4799	4276	3805	3590	3388	3198	3021	4193	4058	3982	3817	3712	3618
1970	4413	3943	3525	3334	3154	2988	2834	2858	2707	2568	2443	2330	2231
1	3284	2893	2550	2397	2254	2122	2002	2692	2563	2444	2338	2244	2161
2	2075	1783	1532	1422	1322	1232	1150	1892	1794	1705	1627	1559	1500
3	1470	1279	1124	1061	1003	954	910	1079	1016	960	912	872	838
4	1573	1479	1411	1384	1364	1348	1334	874	843	817	797	781	768
5	—	—	—	—	—	—	—	1326	1319	1314	1312	1310	1310

T A B L E 4.

Predicted values of the mean annual areas of the faculae [f] for the successive years of the sunspot cycle No. 20 corresponding to the values of the time of rise $3.6 \leqslant T_R \leqslant 5.0$

T_R \ Years	3.6	3.8	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
1964	86	20	0	3	16	32	53	79	50	26	10	3	0
5	191	295	403	457	510	562	614	79	108	140	173	207	240
6	690	800	892	933	969	1006	1035	661	708	751	796	835	873
7	2341	2354	2357	2354	2350	2347	2343	1065	1090	1113	1132	1151	1169
8	2852	2806	2760	2737	2714	2691	2668	2336	2328	2318	2311	2300	2290
9	2727	2674	2622	2597	2573	2546	2519	2645	2622	2599	2576	2553	2530
1970	2014	1951	1888	1856	1821	1789	1755	2494	2467	2440	2414	2387	2360
1	1371	1294	1215	1177	1136	1095	1052	1721	1687	1651	1615	1580	1545
2	974	890	805	760	716	673	627	1010	968	925	881	839	794
3	814	733	654	617	577	541	503	585	541	498	452	408	363
4	733	676	626	600	580	561	541	468	431	397	367	334	305
5	—	—	—					528	514	504	496	491	488

T A B L E 5.

Predicted values of the highest values R^y_{max} of the mean monthly relative sunspot numbers for the successive years of the sunspot cycle No. 20 corresponding to the values of the time of rise $3.6 \leqslant T_R \leqslant 5.0$

T_R \ Years	3.6	3.8	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
1964	4.9	1.1	0.0	0.1	0.8	1.5	2.3	3.4	2.0	1.0	0.4	0.1	0.0
5	24.3	29.2	33.5	35.5	37.3	38.9	40.3	33.4	34.4	35.6	36.7	37.8	38.8
6	86.3	87.8	87.9	87.6	87.0	86.5	85.6	41.4	42.5	43.3	44.2	44.8	45.3
7	147.4	139.3	131.3	127.2	123.3	119.6	116.0	84.8	83.8	82.7	81.6	80.6	79.6
8	**163.2**	**150.1**	**138.0**	**132.3**	**126.9**	**121.8**	**116.8**	112.5	109.1	105.8	102.8	100.0	97.2
9	142.6	129.6	117.7	112.2	106.9	101.8	96.9	**112.2**	**107.8**	**103.6**	**99.7**	**96.1**	**92.6**
1970	116.6	104.3	93.2	87.9	82.7	78.1	73.4	92.4	88.0	83.9	80.0	76.5	73.0
1	92.0	81.0	70.9	66.2	61.7	57.5	53.3	69.1	65.0	61.0	57.3	53.8	50.5
2	71.2	62.0	53.7	49.7	46.1	42.6	39.2	49.4	45.8	42.2	38.9	35.9	32.9
3	52.2	45.4	39.3	36.7	34.1	31.8	29.5	36.2	33.3	30.6	27.9	25.5	23.2
4	32.9	29.0	25.8	24.3	23.2	22.2	21.2	27.5	25.6	23.9	22.4	20.9	19.6
5	—	—	—	—	—	—	—	20.6	20.0	19.5	19.2	19.0	18.9

T A B L E 6.

Predicted values of the lowest values R_{min}^y of the mean monthly relative sunspot numbers for the successive years of the sunspot cycle No. 20 corresponding to the values of the time of rise $3.6 \leqslant T_R \leqslant 5.0$

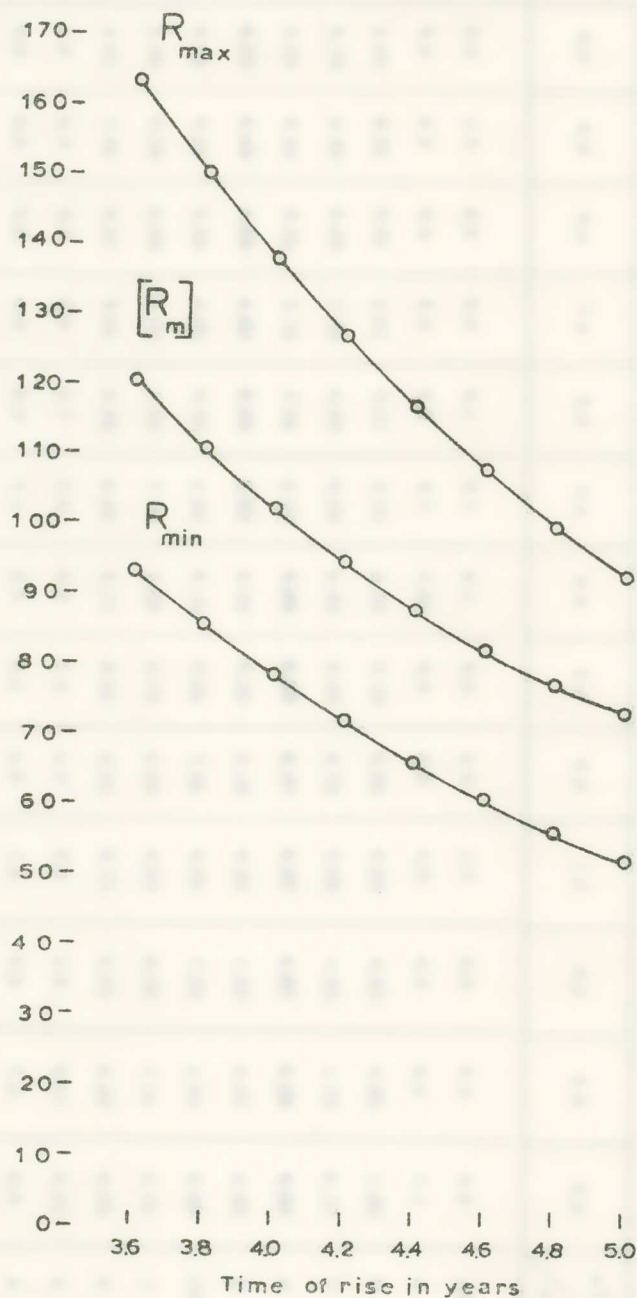| T_R / Years	3.6	3.8	4.0	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
1964	2.8	0.6	0.0	0.1	0.4	0.8	1.3	1.9	1.2	0.6	0.2	0.1	0.0
5	1.7	4.5	6.9	8.0	9.0	9.9	10.7	1.9	2.5	3.2	3.8	4.4	4.9
6	32.1	32.9	32.9	32.6	32.3	31.9	31.4	11.3	11.9	12.4	12.8	13.2	13.4
7	71.8	67.1	62.4	60.0	57.8	55.6	53.4	30.9	30.3	29.7	29.0	28.4	27.8
8	**92.9**	**85.3**	**78.3**	**75.0**	**71.9**	**68.9**	**66.0**	51.4	49.5	47.6	45.8	44.2	42.6
9	82.5	75.0	68.1	64.9	61.9	58.9	56.0	**63.3**	**60.8**	**58.4**	**56.1**	**54.0**	**52.0**
1970	66.2	59.1	52.7	49.6	46.7	43.9	41.2	53.4	50.9	48.5	46.3	44.2	42.2
1	47.4	41.1	35.2	32.6	30.0	27.5	25.1	38.7	36.4	34.1	32.0	30.0	28.1
2	29.6	24.3	19.6	17.3	15.2	13.2	11.3	22.8	20.8	18.8	16.9	15.1	13.4
3	15.8	11.9	8.4	6.9	5.4	4.1	2.8	9.5	7.9	6.3	4.8	3.4	2.1
4	8.0	5.7	3.9	3.1	2.4	1.9	1.3	1.7	0.6	0.0	0.0	0.0	0.0
5	—	—	—	—	—	—	—	0.9	0.6	0.4	0.2	0.1	0.0

Fig. A. Predicted values of : 1) the maximum R_{max} of the mean monthly relative sunspot numbers, 2) the mean annual relative sunspot number $[R_m]$ and 3) the minimum R_{min} of the mean monthly relative sunspot numbers, for the expected year of sunspot maximum (1968 or 1969), corresponding to the values of the time of rise $3.6 \leqslant T_R \leqslant 5.0$.
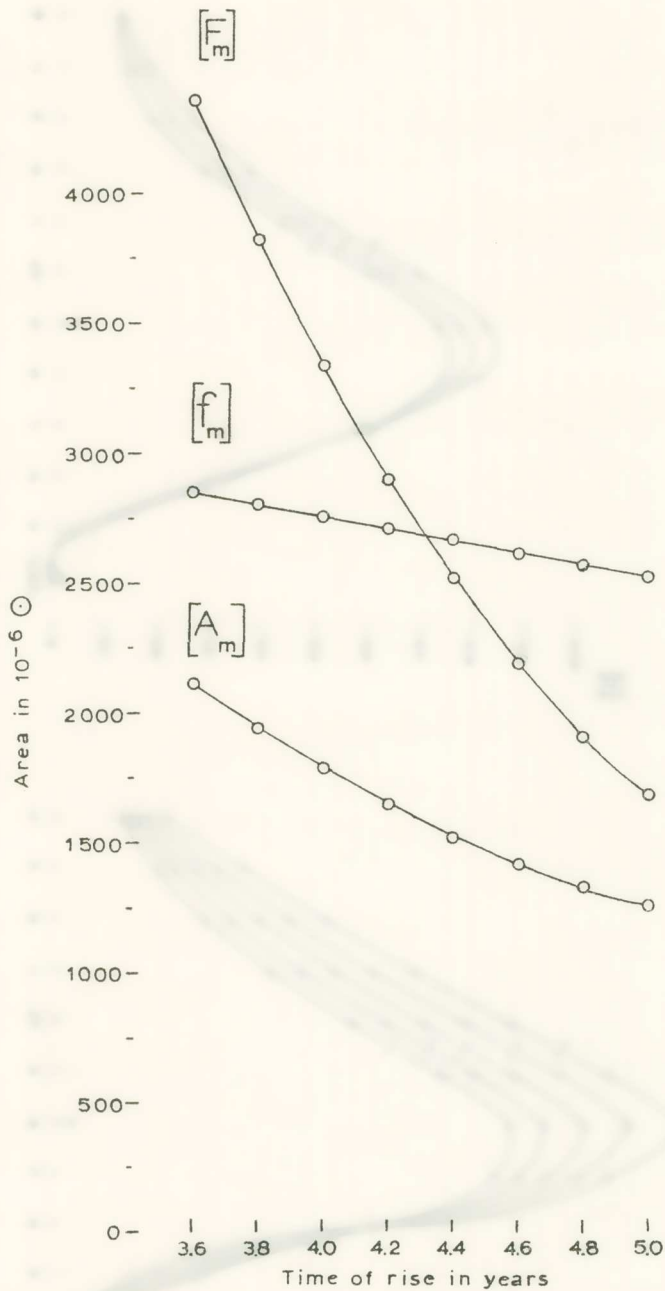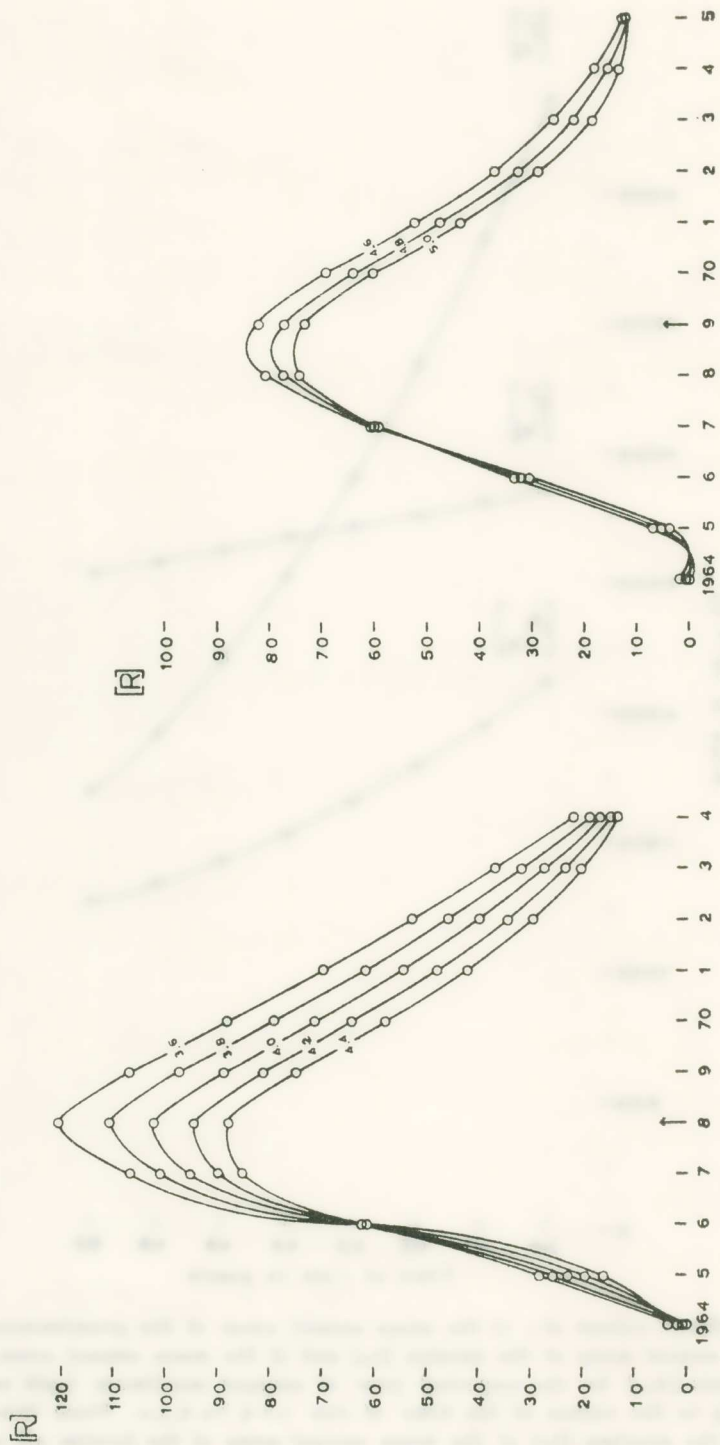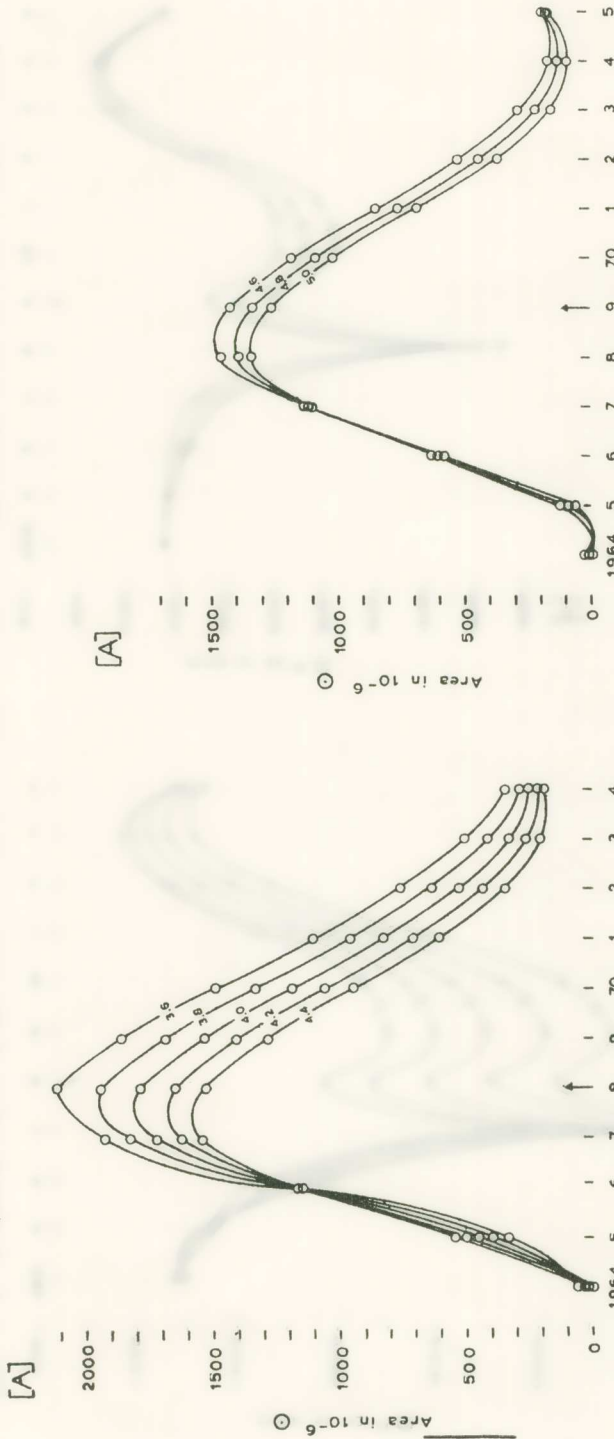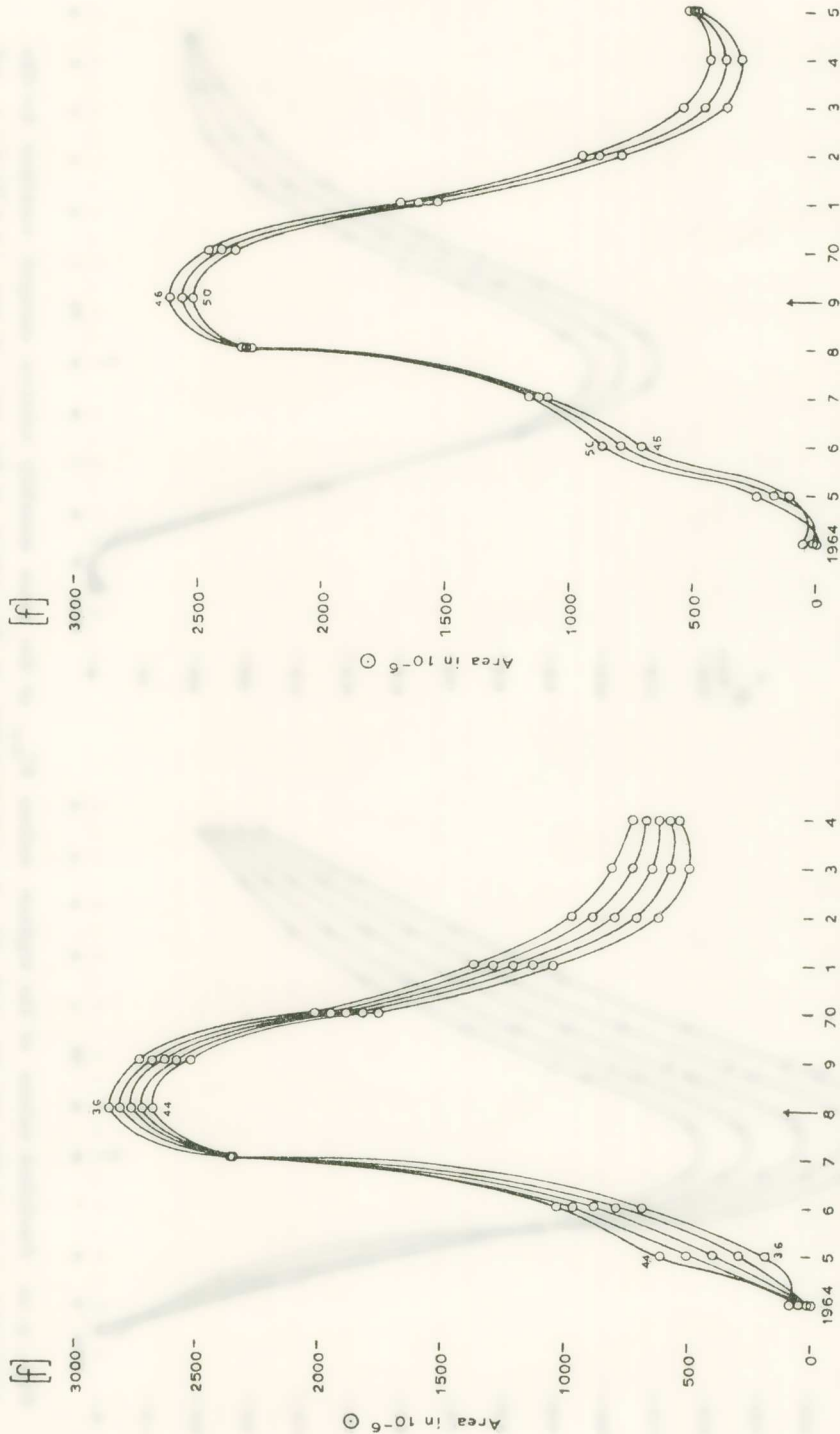
Fig. B. Predicted values of: 1) the mean annual areas of the prominences [F_m], 2) the mean annual areas of the faculae [f_m] and 3) the mean annual areas of the whole sunspots [A_m] for the expected year of sunspot maximum (1968 or 1969), corresponding to the values of the time of rise $3.6 \leqslant T_R \leqslant 5.0$. From this figure we see that the maxima [f_m] of the mean annual areas of the faculae are a linear function of the corresponding time of rise.
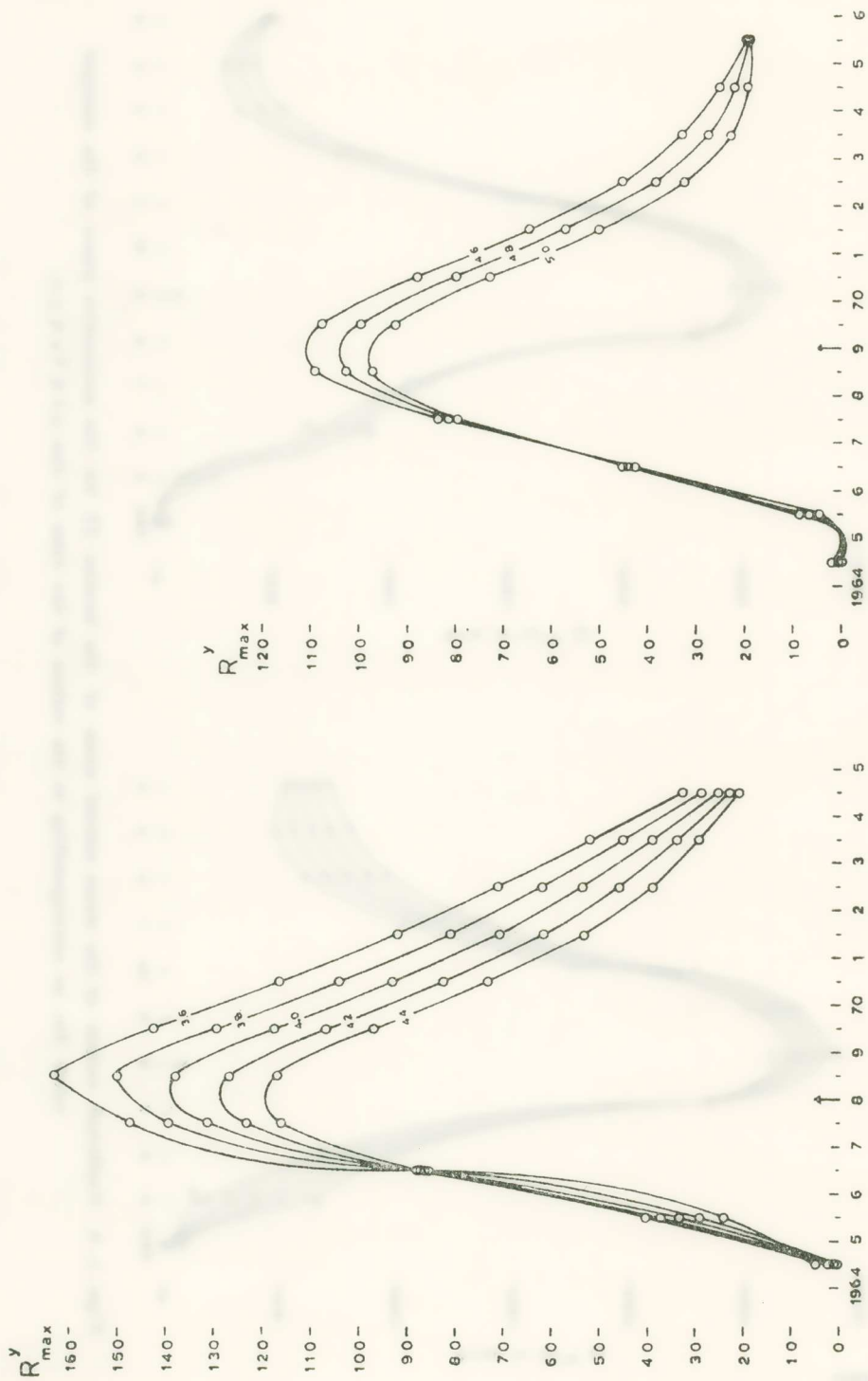
Figs. 1, 2. Predicted values of the mean annual relative sunspot number [R] for the successive years of the sunspot cycle No. 20 corresponding to the values of the time of rise $3.6 \leqslant T_R \leqslant 5.0$.
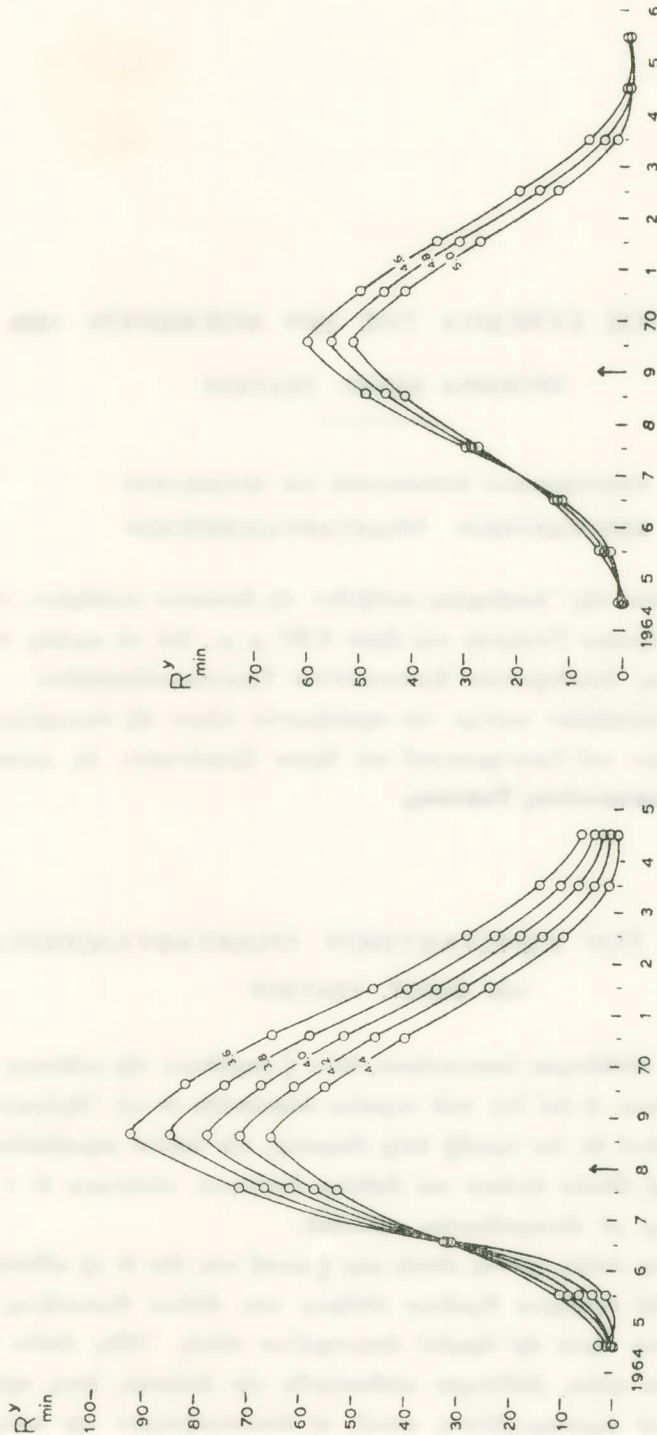
Figs. 3, 4. Predicted values of the mean annual areas of the whole sunspots [A] for the successive years of the sunspot cycle No. 20 corresponding to the values of the time of rise $3.6 \leqslant T_R \leqslant 5.0$.

Figs. 5, 6. Predicted values of the mean annual areas of the prominences [F] for the successive years of the sunspot cycle No. 20 corresponding to the values of the time of rise $3.6 \leqslant T_R \leqslant 5.0$. From these figures we see that two distinct maxima of the areas of the prominences occuring respectively during the years immediately preceding and following the expected year of sunspot maximum (1968 or 1969) are predicted for cycle No. 20.

Figs. 7, 8. Predicted values of the mean annual areas of the faculae [f] for the successive years of the sunspot cycle No. 20 corresponding to the values of the time of rise $3.6 \leqslant T_R \leqslant 5.0$.

Figs. 9, 10. Predicted values of the highest values R_{max}^y of the mean monthly relative sunspot numbers for the successive years of the sunspot cycle No. 20 corresponding to the values of the time of rise $3.6 \leqslant T_R \leqslant 5.0$. All values are plotted for the middle of the corresponding year.

Figs. 11, 12. Predicted values of the lowest values R_{min}^y of the mean monthly relative sunspot numbers for the successive years of the sunspot cycle No. 20 corresponding to the values of the time of rise $3.6 \leqslant T_R \leqslant 5.0$. All values are plotted for the middle of the corresponding year.