ΣΥΝΕΔΡΙΑ ΤΗΣ 8ΗΣ ΙΟΥΝΙΟΥ 1989

ΠΡΟΕΔΡΙΑ ΣΟΛΩΝΟΣ ΚΥΔΩΝΙΑΤΟΥ

ΦΥΣΙΚΗ.— **The Influence of the Penetration of the Radiation in the Focusing Error of the X-Ray Johann Spectrometer**, by *C. N. Koumelis, G. I. Marakis* and *D. A. Vaiopoulos**, διὰ τοῦ Ἀκαδημαϊκοῦ κ. *Καίσαρος Ἀλεξοπούλου*.

ABSTRACT

The focusing error in the Johann X-Ray spectrometer is calculated step by step for an ideal analysing crystal not opaque to the radiation. The effective depth of the penetration which causes the broadening of the lines is depended on either the linear absorption coefficient and the natural width of the used X-Ray line.

INTRODUCTION

The simplest method of selecting a wave length in an X-Ray spectrometer is to use a flat single crystal as a monochromator. Although there are disadvantages for the collected intensity in such a device, this type of monochromators are still used in various spectrometers[1].

To maximise the intensity of the radiation after reflection on the analysing crystal, a bent single crystal is used in the so-called Johann spectrometer[2-3].

The main reasons of the focusing error (broadening of the lines) are generally the following:

---

* ΧΡ. Ν. ΚΟΥΜΕΛΗ, Γ. Ι. ΜΑΡΑΚΗ, Δ. Α. ΒΑΪΟΠΟΥΛΟΥ, **Ἡ ἐπίδρασις τοῦ βάθους διεισδύσεως τῆς ἀκτινοβολίας Roentgen εἰς τὸ σφάλμα ἑστιάσεως τοῦ φασματογράφου Johann.**

1. The natural width of the X-Ray line.
2. The X-Ray source dimensions
3. The mosaic spread of the crystal
4. The depth of the penetrating radiation

In a previous work[4], we have prooved that the first reason is indispensable term for very kinde of broadening and we have examined the focusing error caused by the first two reasons.

In this paper, we are calculating the focusing error because the fourth reason. In the following, we supose that the reflecting crystal planes (hkl) are parallel to the surface of the analysing crystal.

THEORETICAL PROCEDURE

I. BROADENING FROM A «RIGHT» RAY

We define as «central ray» the ray SC from the source S (Fig 1) which meets the sourface of the analysing crystal under the angle θ proper for the wave lenght which corresponds to the peak of the used X-Ray line. Rays from both sides of the central ray SC are defined as «left» and «right» rays respectively.

From the Fig. 1 is evident that the central ray SE from the source S meetting the surface of the analysing crystal at C in the proper Bragg angle θ and penetrating the crystal in a depth (CE)=x, meets the crystal plane $(hkl)_n$ in this depth at an angle:

$$\theta+\omega > \theta$$

So, it will be reflected on it only if it has a natural width containing a wave length suitable for the Bragg angle $\theta+\omega$.

If μ is the linear absorption coefficient of the analysing crystal for the wave lenght corresponding to the Bragg angle θ, the depth of the penetration is practically:

$$x = \frac{3}{\mu}$$

If the X-Ray line has a full width at half maximum (F.W.H.M.) $2.\Delta\lambda_{FWHM}$, the range of the wave lengths between $\lambda-\Delta\lambda$ and $\lambda+\Delta\lambda$ give reflections, where practically:

$$\Delta\lambda = 3\cdot\Delta\lambda_{FWHM}$$

So that, the angle ω and the distance x are defined twice, trom μ and Δλ.

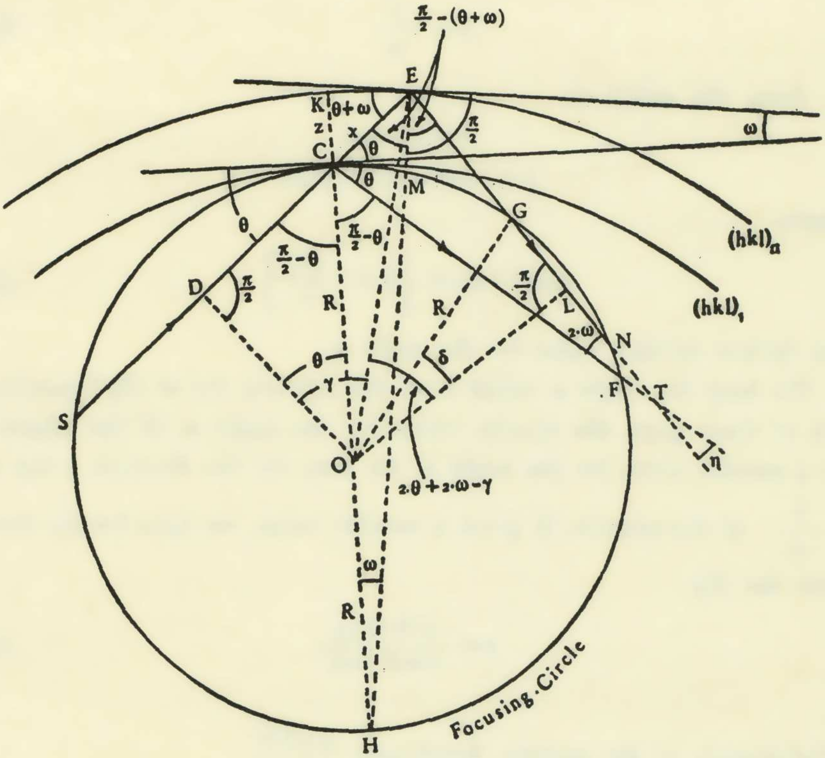

Fig. 1. For the calculation of the broadening from the central ray SC.

i. Calculation of the angle ω

From the triangle HCE (Fig. 1) we have:

$$\frac{\sin\omega}{(CE)} = \frac{\sin\left\{\frac{\pi}{2} - (\theta + \omega)\right\}}{(HC)}$$

or:

$$\frac{\sin\omega}{x} = \frac{\sin(\theta + \omega)}{2 \cdot R}$$

or:

$$\tan\omega = \frac{x\cdot\cos\theta}{2\cdot R + x\cdot\sin\theta} \qquad (1)$$

where:

$$x = \frac{3}{\mu} \qquad (2)$$

Also, from the relations:

$$2\cdot d\cdot\sin\theta = \lambda$$

$$2\cdot d\cdot\sin(\theta+\omega) = \lambda + \Delta\lambda$$

we have:

$$\sin(\theta+\omega) = \left\{1 + \frac{\Delta\lambda}{\lambda}\right\} \qquad (3)$$

which defines another value for the angle ω.

We keep the angle ω either from the relation (1) or (3) depending on which of them gives the smaller value for the angle ω. If the relation (1) gives a smaller value for the angle ω, we keep for the distance x the value $x = \frac{3}{\mu}\cdot$ If the relation (3) gives a smaller value, we have for the distance x from the (1):

$$x = \frac{2\cdot R\cdot\sin\omega}{\cos(\theta+\omega)} \qquad (4)$$

ii. Calculation of the angular broadening $\frac{arc\widehat{FN}}{R}$

We have from Fig. 1:

$$\frac{arc\widehat{HF}}{R} + \frac{arc\widehat{FN}}{R} - \frac{arc\widehat{GM}}{R} = 2\cdot\left\{\frac{\pi}{2} - (\theta+\omega)\right\}$$

or:

$$2\cdot\left\{\frac{\pi}{2} - \theta\right\} + \frac{arc\,\widehat{FN}}{R} - \frac{arc\widehat{GM}}{R} = \pi - 2\cdot\theta - 2\cdot\omega$$

and:

$$\frac{arc\widehat{GM}}{R} = \frac{arc\,\widehat{FN}}{R} + 2\cdot\omega \qquad (5)$$

From the triangle ODE we have:

$$\tan\gamma = \frac{(DE)}{(OD)} = \frac{(DC)+(CE)}{(OD)}$$

or:

$$\tan\gamma = \frac{R\cdot\sin\theta + x}{R\cdot\cos\theta} \tag{6}$$

which defines the angle γ.

Also, from the same triangle ODE we have:

$$(OE) = \frac{(OD)}{\cos\gamma} \quad \text{or} \quad (OE) = \frac{R.\cos\gamma}{\cos\theta}$$

From the triangle OLE we have:

$$(OL) = (OE)\cdot\cos(2\cdot\theta + 2\cdot\omega - \gamma)$$

or:

$$(OL) = \frac{R\cdot\cos\theta\cdot\cos(2\cdot\theta + 2\cdot\omega - \gamma)}{\cos\gamma} \tag{7}$$

From the triangle OLG we have:

$$(OL) = (OG)\cdot\cos\delta = R\cdot\cos\delta$$

and from (7):

$$\cos\delta = \frac{\cos\theta\cdot\cos(2\cdot\theta + 2\cdot\omega - \gamma)}{\cos\gamma} \tag{8}$$

which defines the angle δ. And concequently:

$$\frac{\text{arc}\,\widehat{NG}}{R} = 2\cdot\delta \tag{9}$$

From the Fig. 1 we have:

$$\frac{\text{arc}\,\widehat{FN}}{R} + \frac{\text{arc}\,\widehat{NG}}{R} + \frac{\text{arc}\,\widehat{GM}}{R} + \frac{\text{arc}\,\widehat{MC}}{R} = 2\cdot\theta$$

And according to (5) and (9):

$$\frac{\text{arc}\,\widehat{FN}}{R} + 2\cdot\delta + \frac{\text{arc}\,\widehat{FN}}{R} + 2\cdot\omega + 2\cdot\omega = 2\cdot\theta$$

And finally:

$$\frac{\text{arc}\,\widehat{FN}}{R} = \theta - \delta - 2\cdot\omega$$

Angular broadening from the central ray SE

iii. Perceptive broadening

From the Fig. 1 we have:

$$\eta = 2 \cdot \omega \qquad \text{Perceptive broadening}$$

iv. Maximum angular broadening

The broadening $\frac{\text{arc}\,\widehat{FN}}{R}$ becomes maximum when the ray EGN becomes tangent to the focusing circle (Fig. 2). In this case, the angle δ is zero and G coincides to N. So that:

$$\left\{\frac{\text{arc}\,\widehat{FN}}{R}\right\}_{max} = \theta - 2 \cdot \omega \qquad \text{Maximum angular broadening}$$

From the relation (5) we have:

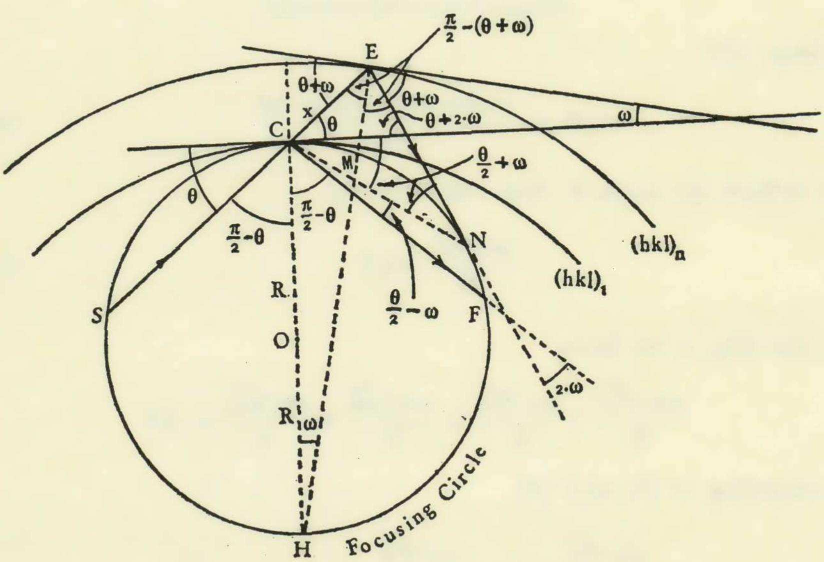

Fig. 2. Maximum broadening [arc $\widehat{FN}$] max.

$$\frac{\text{arc}\widehat{NM}}{R} = \frac{\text{arc }\widehat{FN}}{R} + 2\cdot\omega = \theta - 2\cdot\omega + 2\cdot\omega = \theta$$

We have also from the triangle CON (ON is not plotted) (Fig. 2):

$$(CN) = 2\cdot R\cdot\sin\left\{\frac{\theta}{2} + \omega\right\}$$

and from the triangle CEN:

$$\frac{x}{\sin\left\{\frac{\theta}{2} + \omega\right\}} = \frac{(CN)}{\sin\left\{\pi - 2\cdot(\theta + \omega)\right\}}$$

Or:

$$x = \frac{2\cdot R\cdot\sin^2\left\{\frac{\theta}{2} + \omega\right\}}{\sin\left\{2\cdot(\theta + \omega)\right\}} \tag{10}$$

So that, when the broadening is maximum, the distance x must fulfil both the relations (4) and (10), i.e.:

$$\frac{2\cdot R\cdot\sin\omega}{\cos(\theta + \omega)} = \frac{2\cdot R\cdot\sin^2\left\{\frac{\theta}{2} + \omega\right\}}{\sin\left[2\cdot(\theta + \omega)\right]}$$

or:

$$2\cdot R\cdot\sin(\theta + \omega)\cdot\sin\omega = \sin^2\left\{\frac{\theta}{2} + \omega\right\}$$

or:

$$\cos(\theta + 2\cdot\omega) - \cos\theta = -\sin^2\left\{\frac{\theta}{2} + \omega\right\}$$

or:

$$\cos^2\left\{\frac{\theta}{2} + \omega\right\} - \sin^2\left\{\frac{\theta}{2} + \omega\right\} - \cos\theta = -\sin^2\left\{\frac{\theta}{2} + \omega\right\}$$

or:

$$\cos^2\left\{\frac{\theta}{2}+\omega\right\}=\cos\theta$$

which defines the relation between the angles ω and θ when the broadening is maximum.

ν. Calculation of the number n of the crystallographic planes (hkl) spacing d, penetrated by the radiation

If:

$$(CK)=n\cdot d=z$$

where d is the spacing of the crystallographic planes (hkl), we have from the triangle HCE (Fig. 1):

$$(HE)^2=(HC)^2+(CE)^2-2\cdot(HC)\cdot(CE)\cdot\cos\left\{\frac{\pi}{2}+\theta\right\}$$

or:

$$2\cdot R+n\cdot d=\sqrt{4\cdot R^2+x^2+4\cdot R\cdot x\cdot\sin\theta} \qquad (11)$$

and:

$$n=\frac{\sqrt{4\cdot R^2+x^2+4\cdot R\cdot x\cdot\sin\theta}}{d}-\frac{2\cdot R}{d}$$

where x is defined either by $x=\frac{3}{\mu}$ or from (3) and (4).

II. BROADENING FROM A "RIGHT" RAY

We supose that the front face of the bent crystal has a given length $2\cdot(CP)=2\cdot y$ (Fig. 3). The angle ε is then:

$$\varepsilon=\frac{y}{2\cdot R}$$

We accept that the limit «right» ray penetrates approximatelly the crystal until the same crystal plane $(hkl)_n$ as the central ray SC. The limit «right» ray is then the SV. We have:

$$(CP)=(CM)=y,\ (CE)=x,\ (CK)=n\cdot d=z$$

Because there are two values for x, there are also two values for the distance $(CK)=z$.

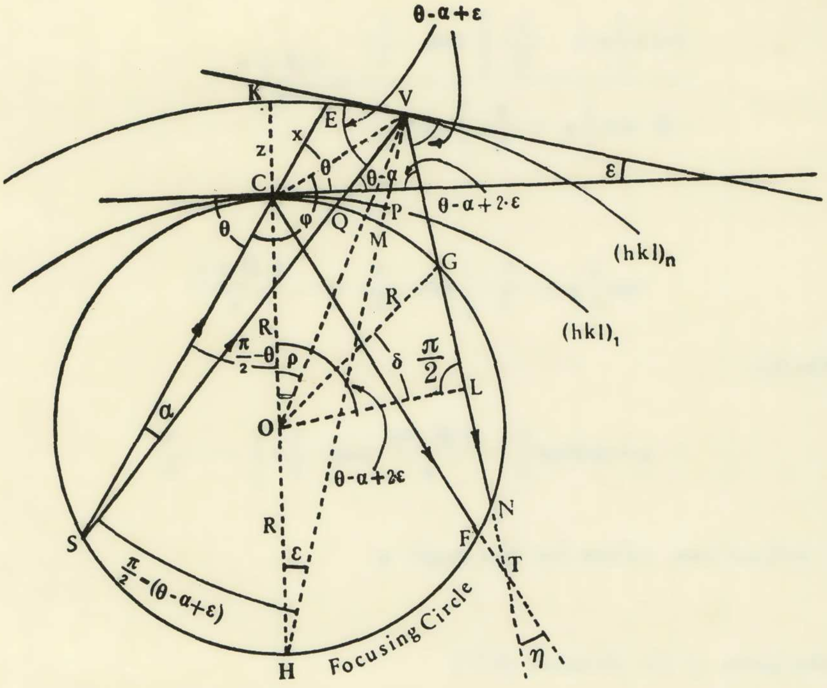

Fig. 3. For the calculation of the broadening from the «right» ray SV.

i. Calculation of the angle φ

From the triangle HCV we have:

$$\frac{\sin\varphi}{(HV)} = \frac{\sin[\pi - (\varphi + \varepsilon)]}{(HC)}$$

or:

$$\frac{\sin\varphi}{2\cdot R + z} = \frac{\sin(\varphi + \varepsilon)}{2\cdot R} = \frac{\sin\varphi + \sin(\varphi + \varepsilon)}{4\cdot R + z} = \frac{\sin\varphi - \sin(\varphi + \varepsilon)}{z}$$

where z is defined by (11). From the above equation we have:

$$\frac{\sin\varphi + \sin(\varphi + \varepsilon)}{\sin\varphi - \sin(\varphi + \varepsilon)} = \frac{4\cdot R + z}{z}$$

or:

$$\frac{2\cdot\sin\left\{\varphi+\frac{\varepsilon}{2}\right\}\cdot\cos\frac{\varepsilon}{2}}{-2\cdot\cos\left\{\varphi+\frac{\varepsilon}{2}\right\}\cdot\sin\frac{\varepsilon}{2}}=\frac{4\cdot R+z}{z}$$

or:

$$\tan\left\{\varphi+\frac{\varepsilon}{2}\right\}\cdot\cot\frac{\varepsilon}{2}=-\frac{4\cdot R+z}{z}$$

And finally:

$$\varphi=\arctan\left\{-\frac{4\cdot R+z}{z}\cdot\tan\frac{\varepsilon}{2}\right\}-\frac{\varepsilon}{2}$$

which defines two values for the angle φ.

ii. Calculation of the distance (CV)

From the triangle HCV we have:

$$\frac{(CV)}{\sin\varepsilon}=\frac{(HC)}{\sin\left[\pi-(\varphi+\varepsilon)\right]}$$

or:

$$(CV)=\frac{2\cdot R\cdot\sin\varepsilon}{\sin(\varphi+\varepsilon)} \qquad (12)$$

which defines the distance (CV)

iii. Calculation of the limit angle a

From the triangle SCV wehave:

$$\frac{\sin\alpha}{(CV)}=\frac{\sin\left\{\pi-(\frac{\pi}{2}-\theta+\varphi+\alpha)\right\}}{(SC)}$$

or:

$$\frac{\sin\alpha}{(KV)} = \frac{\sin\left\{\frac{\pi}{2}+\theta-\varphi-\alpha\right\}}{(SC)} = \frac{\sin\alpha + \sin\left\{\frac{\pi}{2}+\theta-\varphi-\alpha\right\}}{(CV)+(SC)} = \frac{\sin\alpha - \sin\left[\frac{\pi}{2}+\theta-\varphi-\alpha\right]}{(CV)-(SC)}$$

and from (12):

$$\frac{2\cdot\sin\left[\frac{\pi}{4}+\frac{\theta-\varphi}{2}\right]\cdot\cos\left[\alpha-\frac{\pi}{4}-\frac{\theta-\varphi}{2}\right]}{2\cdot\cos\left[\frac{\pi}{4}+\frac{\theta-\varphi}{2}\right]\cdot\sin\left[\alpha-\frac{\pi}{4}-\frac{\theta-\varphi}{2}\right]} = \frac{\frac{2\cdot R\cdot\sin\varepsilon}{\sin(\varphi+\varepsilon)}+2\cdot R\cdot\sin\theta}{\frac{2\cdot R\cdot\sin\varepsilon}{\sin(\varphi+\varepsilon)}+2\cdot R\cdot\sin\theta}$$

or:

$$\tan\left\{\frac{\pi}{4}+\frac{\theta-\varphi}{2}\right\}\cdot\cot\left\{\alpha-\frac{\pi}{4}-\frac{\theta-\varphi}{2}\right\} = -\frac{\sin\varepsilon+\sin\theta\cdot\sin(\varphi+\varepsilon)}{\sin\varepsilon-\sin\theta\cdot\sin(\varphi+\varepsilon)}$$

And finally:

$$\alpha = \frac{\pi}{4}+\frac{\theta-\varphi}{2}+\arctan\left\{-\frac{\sin\varepsilon-\sin\theta\cdot\sin(\varphi+\varepsilon)}{\sin\varepsilon+\sin\theta\cdot\sin(\varphi+\varepsilon)}\cdot\tan\left[\frac{\pi}{4}+\frac{\theta-\varphi}{2}\right]\right\}$$

which defines two values for the angle α corresponding to the two values of the angle φ.

But the angle α must fulfil also the relation:

$$2\cdot d\cdot\sin(\theta-\alpha+\varepsilon)=(\lambda+\Delta\lambda)$$

which with the Bragg equation gives:

$$\sin(\theta-\alpha+\varepsilon)=\left\{1+\frac{\Delta\lambda}{\lambda}\right\}$$

which defines another value for the angle α. For the calculation of the broadening we use the smaller value of the angle α.

iv. Calculation of the angle ρ

We have from the triangle OCV:

$$\frac{\sin\rho}{(CV)} = \frac{\sin\left\{\pi-(\varphi+\rho)\right\}}{(OC)}$$

And from this:

$$\frac{\sin\rho}{(CV)} = \frac{\sin(\varphi+\rho)}{R} = \frac{\sin\rho+\sin(\varphi+\rho)}{(CV)+R} = \frac{\sin\rho-\sin(\varphi+\rho)}{(CV)-R}$$

or:

$$\frac{2\cdot\sin\left\{\rho+\frac{\varphi}{2}\right\}\cdot\cos\frac{\varphi}{2}}{-2\cdot\cos\left\{\rho+\frac{\varphi}{2}\right\}\cdot\sin\frac{\varphi}{2}} = \frac{\frac{2\cdot R\cdot\sin\varepsilon}{\sin(\varphi+\varepsilon)}+R}{\frac{2\cdot R\cdot \text{sine}}{\sin(\varphi+\varepsilon)}-R}$$

and finally:

$$\rho=\arctan\left\{-\frac{2\cdot\sin\varepsilon+\sin(\varphi+\varepsilon)}{2\cdot\sin\varepsilon-\sin(\varphi+\varepsilon)}\cdot\tan\frac{\varphi}{2}\right\}-\frac{\varphi}{2}$$

which defines the angle ρ.

v. Calculation of the angle δ

From the same trianlge OCV we have:

$$\frac{(OV)}{\sin\varphi} = \frac{(CV)}{\sin\rho}$$

which gives:

$$(OV)=\frac{2\cdot R\cdot\sin\varepsilon}{\sin\varepsilon(\varphi+\varepsilon)}\cdot\frac{\sin\varphi}{\sin\rho}$$

From the triangle OVL we have:

$$(OL)=(OV)\cdot\cos(\theta-\alpha+2\cdot\varepsilon-\rho)$$

or:

$$(OL)=\frac{2\cdot R\cdot\sin\varepsilon}{\sin(\varphi+\varepsilon)}\cdot\frac{\sin\varphi}{\sin\rho}\cdot(\theta-\alpha+2\cdot\varepsilon-\rho) \qquad (13)$$

We have also from the triangle OLG:

$$\cos\delta = \frac{(OL)}{(OG)}$$

and from (13):

$$\cos\delta = \frac{2\cdot R\cdot\sin\varepsilon}{\sin(\varphi+\varepsilon)}\cdot\frac{\sin\varphi}{\sin\rho}\cdot\cos(\theta-\alpha+2\cdot\varepsilon-\rho)$$

which defines the angle δ. We see that:

$$\text{arc } \widehat{NG} = 2 \cdot \delta \cdot R$$

vii. Calculation of the broadening from the «right» ray SV

We have from the Fig. 3:

$$\text{arc}\widehat{FN} + \text{arc}\widehat{NG} + \text{arc}\widehat{GC} = 2 \cdot \theta \cdot R$$

or:

$$\text{arc}\widehat{FN} + \frac{\text{arc}\widehat{NG}}{2} + \left\{ \frac{\text{arc } \widehat{NG}}{2} + \text{arc } \widehat{GC} \right\} = 2 \cdot \theta \cdot R$$

or:

$$\text{arc}\widehat{FN} + \delta \cdot R + (\theta - \alpha + 2 \cdot \varepsilon) \cdot R = 2 \cdot \theta \cdot R$$

and finally:

$$\frac{\text{arc } \widehat{FN}}{R} = \theta + \alpha - 2 \cdot \varepsilon - \delta \qquad \text{Angular broadening from the «right» ray SV}$$

viii. Maximum broadening from the «right» ray

The broadening arc $\widehat{FN}$ becomes maximum when the ray VN is trangent to the focusing circle. Then $\delta = 0$, so that:

$$\left\{ \frac{\text{arc}\widehat{FN}}{R} \right\}_{max} = \theta + \alpha - 2 \cdot \varepsilon \qquad \text{Maximum angular broadening from a «right» ray}$$

ix. Perceptive broadening

From the triangle TCV (Fig. 2) we have for the perceptive angular broadening η:

$$n + \left\{ \varphi - \left(\frac{\pi}{2} - \theta\right) \right\} + \left\{ \pi - (\varphi + \varepsilon) + \frac{\pi}{2} - (\theta - \alpha + \varepsilon) \right\} = \pi$$

and finally:

$$\eta = 2 \cdot \varepsilon - \alpha \qquad \text{Perceptive broadening}$$

III. BROADENING FROM A "LEFT" RAY

We supose again that the front face of the bent crystal (Fif. 4) has a given length $2\cdot(CP)=2\cdot y$. The angle ε is then:

$$\varepsilon = \frac{y}{2 \cdot R}$$

We accept that the limit «left» ray penetrates approximatelly the crystal until the same crystal plane $(hkl)_n$ as the central ray SC. The limit «left» ray is then the SV. We have:

$$(CP)=(CM)=y, \quad (CE)=x, \quad (CK)=n.d=z$$

The distance z is defined by (11).

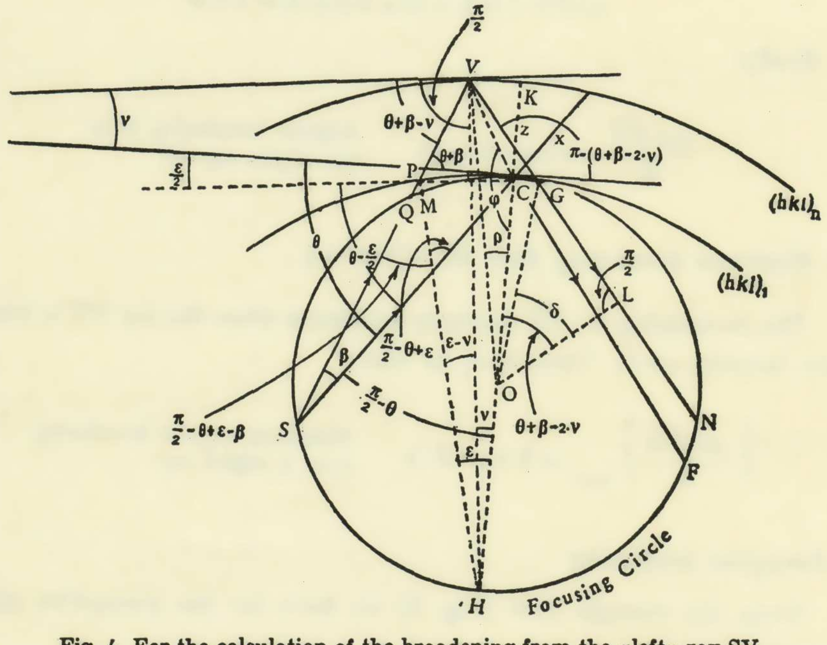

Fig. 4. For the calculation of the broadening from the «left» ray SV

i. Calculation of the limit angle β

From the triangle HPC we have:

$$\frac{(PC)}{2} = (HC) \cdot \sin\frac{\varepsilon}{2}$$

or:

$$(PC) = 4 \cdot R \cdot \sin\frac{\varepsilon}{2} \qquad (14)$$

From the triangle SPC we have:

$$\frac{\sin\beta}{(PC)} = \frac{\sin\left\{\pi - (\beta + \theta - \frac{\varepsilon}{2})\right\}}{(SC)}$$

And from (14):

$$\frac{\sin\beta}{4 \cdot R \cdot \sin\frac{\varepsilon}{2}} = \frac{\sin\left\{\theta + \beta - \frac{\varepsilon}{2}\right\}}{2 \cdot R \cdot \sin\theta} = \frac{\sin\beta + \sin\left\{\theta + \beta - \frac{\varepsilon}{2}\right\}}{2 \cdot \sin\frac{\varepsilon}{2} + \sin\theta} =$$

$$= \frac{\sin\beta - \sin\left\{\theta + \beta - \frac{\varepsilon}{2}\right\}}{2 \cdot \sin\frac{\varepsilon}{2} - \sin\theta}$$

or:

$$\frac{2 \cdot \sin\left\{\beta + \frac{\theta}{2} - \frac{\varepsilon}{4}\right\} \cdot \cos\left\{\frac{\theta}{2} - \frac{\varepsilon}{4}\right\}}{-2 \cdot \cos\left\{\beta + \frac{\theta}{2} - \frac{\varepsilon}{4}\right\} \cdot \sin\left\{\frac{\theta}{2} - \frac{\varepsilon}{4}\right\}} = \frac{2 \cdot \sin\frac{\varepsilon}{2} + \sin\theta}{2 \cdot \sin\frac{\varepsilon}{2} - \sin\theta}$$

and:

$$\beta = \frac{\varepsilon}{4} - \frac{\theta}{2} + \arctan\left[-\frac{2 \cdot \sin\frac{\varepsilon}{2} + \sin\theta}{2 \cdot \sin\frac{\varepsilon}{2} - \sin\theta} \cdot \tan\left\{\frac{\theta}{2} - \frac{\varepsilon}{4}\right\}\right],$$

which defines the angle β.

ii. Calculation of the angle ν.

We have from the triangle HPV:

$$\frac{\sin\left[\pi-\left\{\frac{\pi}{2}-\theta+\varepsilon-\beta\right\}\right]}{(HV)}=\frac{\sin\left[\pi-\left\{\frac{\pi}{2}+\theta-\varepsilon+\beta+\varepsilon-\nu\right\}\right]}{(HP)}$$

or:

$$\frac{\cos\,(\theta+\beta-\varepsilon)}{2\cdot R+z}=\frac{\cos(\theta+\beta-\nu)}{2\cdot R}$$

and finally:

$$\nu=\theta+\beta-\arccos\left\{\frac{2\cdot R}{2\cdot R+z}\cdot\cos\,(\theta+\beta-\varepsilon)\right\}$$

which defines the angle ν.

But the angle ν must fulfil also the relation:

$$2\cdot d\cdot\sin\,(\theta+\beta-\nu)=\left\{1+\frac{\Delta\lambda}{\lambda}\right\}$$

which with the Bragg equation gives:

$$\sin(\theta+\beta-\nu)=\left\{1+\frac{\Delta\lambda}{\lambda}\right\}\cdot\sin\theta$$

It defines another value for the angle ν depending on the F.W.H.M. We omit the comment for all these values.

iii. Calculation of the angle φ

From the triangle HCV (Fig. 4) we have:

$$\frac{\sin\varphi}{(HV)}=\frac{\sin\left[\pi-(\varphi+\nu)\right]}{(HC)}$$

or:

$$\frac{\sin\varphi}{2\cdot R+z}=\frac{\sin\,(\varphi+\nu)}{2\cdot R}=\frac{\sin\varphi+\sin\,(\varphi+\nu)}{4\cdot R+z}=\frac{\sin\varphi-\sin\,(\varphi+\nu)}{z}$$

and:

$$\frac{2\cdot\sin\left\{\varphi+\frac{\nu}{2}\right\}\cdot\cos\frac{\nu}{2}}{-2\cdot\cos\left\{\varphi+\frac{\nu}{2}\right\}\cdot\sin\frac{\nu}{2}}=\frac{4\cdot R+z}{z}$$

or:

$$\varphi=\arctan\left\{-\frac{4\cdot R+z}{z}\cdot\tan\frac{\nu}{2}\right\}-\frac{\nu}{2}$$

which defines the angle φ.

iv. Calculation of the distance (CV)

We have from the triangle HCV:

$$\frac{(CV)}{\sin\nu}=\frac{(HC)}{\sin\left[\pi-(\varphi+\nu)\right]}$$

and:

$$(CV)=\frac{2\cdot R\cdot\sin\nu}{\sin(\varphi+\nu)} \qquad (15)$$

v. Calculation of the angle ρ

We have from the triangle OCV:

$$\frac{\sin\varphi}{(CV)}=\sin\left[\frac{\pi-(\rho+\varphi)}{(OC)}\right]$$

or:

$$\frac{\sin\rho}{(CV)}=\frac{\sin(\rho+\varphi)}{R}=\frac{\sin\rho+\sin(\rho+\varphi)}{(CV)+\rho}=\frac{\sin\rho-\sin(\rho+\varphi)}{(CV)-\rho}$$

and substituding the (CV) from (15):

$$\rho=\arctan\left\{-\frac{2\cdot\sin\nu+\sin(\varphi+\nu)}{2\cdot\sin\nu-\sin(\varphi+\nu)}\cdot\tan\frac{\varphi}{2}\right\}-\frac{\varphi}{2}$$

vi. Calculation of the angle δ

From the same triangle OCV (Fig. 4) we have:

$$\frac{(OV)}{\sin\varphi}=\frac{(CV)}{\sin\rho}$$

and according to (15):

$$(OV)=\frac{2\cdot R\cdot\sin\nu}{\sin(\varphi+\nu)}\cdot\frac{\sin\varphi}{\sin\rho} \qquad (16)$$

From the triangle OVL we have:

$$(OL)=(OV)\cdot\cos(\theta+\beta-2\cdot\nu+\rho)$$

and according to (16):

$$(OL)=\frac{2\cdot R\cdot\sin\nu}{\sin(\varphi+\nu)}\cdot\frac{\sin\varphi}{\sin\rho}\cdot\cos(\theta+\beta-2\cdot\omega+\rho) \qquad (17)$$

From the trangle OLV we have:

$$\cos\delta=\frac{(OL)}{(OG)}$$

And according to (17):

$$\cos\delta=\frac{2\cdot R\cdot\sin\nu}{\sin(\varphi+\nu)}\cdot\frac{\sin\varphi}{\sin\rho}\cdot\cos(\theta+\beta-2\cdot\omega+\rho)$$

We have also: $\text{arc}\widehat{NG}=2\cdot\delta\cdot R$

vii. Calculation of the broadening $\frac{arc\,\widehat{FN}}{R}$

From the Fig. 4 we have:

$$\text{arc}\,\widehat{FN}+\frac{\text{arc}\widehat{NG}}{2}+\left\{\frac{\text{arc}\,\widehat{NG}}{2}+\text{arc}\,\widehat{GC}\right\}=2\cdot\theta\cdot R$$

or:

$$\text{arc}\widehat{FN}+\delta\cdot R+(\theta+\beta-2\cdot\nu)\cdot R=2\cdot\theta\cdot R$$

and finally:

$$\frac{\text{arc}\,\widehat{FN}}{R}=\theta-\beta+2\cdot\nu$$

Angular broadening from the «left» ray SV

ΠΕΡΙΛΗΨΙΣ

Ἡ ἐπίδρασις τοῦ βάθους διεισδύσεως τῆς ἀκτινοβολίας Roentgen εἰς τὸ σφάλμα ἑστιάσεως τοῦ φασματογράφου Johann.

Ὑπολογίζεται ἐν προκειμένῳ τὸ σφάλμα ἑστιάσεως εἰς τὸν φασματογράφον Johann ἀκτίνων Roentgen, τὸ ὀφειλόμενον εἰς τὸ βάθος διεισδύσεως τῆς ἀκτινοβολίας εἰς τὸν κρύσταλλον φασματοσκοπικῆς ἀναλύσεως. Τὸ βάθος διεισδύσεως τῆς ἀκτινοβολίας, τὸ προκαλοῦν τὴν διαπλάτυνσιν τῶν γραμμῶν, ἐξαρτᾶται ἀπὸ τὸ μέσον πλάτος τῆς χρησιμοποιουμένης γραμμῆς Roentgen. Ἐξετάζονται αἱ διαπλατύνσεις αἱ προκαλούμεναι ὑπὸ τῆς κεντρικῆς ἀκτῖνος καὶ ὑπὸ τῶν ἑκατέρωθεν αὐτῆς ἀκτίνων.

REFERENCES

1. C. N. Koumelis, C. A. Londos, Z. I. Kavogli, D. K. Leventouri A. B. Vasilikou and G. E. Zardas, Canadian J. Phys. **60**, 1241, (1982)
2. H. H. Johann, Zeitschr. f. Fysik **69**, 185, (1931)
3. Y. Cauchois, J. de Physique **3**, 320, (1932)
4. C. N. Koumelis and C. J. Sparks Jr, Trans. Acad. of Athens **61**, 315, (1986).