ΣΤΑΤΙΣΤΙΚΗ.— **Markovian models for the prediction of the service distribution in the grade in manpower systems,** *by P. C. G. Vassiliou**. Ἀνεκοινώθη ὑπὸ τοῦ Ἀκαδημαϊκοῦ κ. Ἰ. Ξανθάκη.

1. INTRODUCTION

The aim of this paper is to present models for the prediction and description of the service distribution-with respect to the grade- of staff in hierarchical graded manpower systems.

R. Drinkwater and O. Kane (1971) and Osmond (1972) following Young's suggestion of constant transition probabilities between different statuses tried to follow the service structure with respect to the grade, in the various grades of hierarchically graded manpower systems. One of the main reasons being the close relation between the service distribution with respect to the grade and the cost of the respective manpower systems. R. Drinkwater and O. Kane (1971) applied their studies to the civil service establishment while Osmont (1972) applied his studies to University establishments.

The organization in this paper is seen as a graded social system through which people move by recruitment, promotion and wastage. The laws governing movements are stochastic. Within each grade individuals could be classified according to the number of years that they served in their present grades. We will call these categories subgrades of the respective grades.

We have seen in Young and Vassiliou (1974) and Vassiliou (1974) that the number of people promoted from grade i to grade j depends on the expansion of the system, the number of vacancies to be filled and it is related to the relative differences on wastage in grade i. A problem which immediately arises is, from which subgrade and in what proportion, are those individuals to be promoted from grade i to grade j, are going to come from. In section 2 we model the phenomenon assuming that the expected number of people promoted from subgrade k of grade

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* Π. Χ. Γ. ΒΑΣΙΛΕΙΟΥ, **Μαρκοβιανὰ ὑποδείγματα διὰ τὴν πρόβλεψιν τῆς κατανομῆς τοῦ χρόνου παραμονῆς εἰς τὴν βαθμίδα ἑνὸς συστήματος ἀνθρωπίνου δυναμικοῦ.**

i at time t to grade j is proportional to the expected number of people promoted from grade i to grade j at the time t. This assumption is tested in section 3 with data from firm S in which there is no evidence to reject the hypothesis. With this analysis other interesting results emerge. It is possible to calculate the probability of an individual who belongs to subgrade k of grade i being promoted to grade j at time t and to compare it with the corresponding probabilities of individuals belonging to the rest of the subgrades of the same grade. For example in some cases it has been revealed that after a time t in the grade without getting a promotion the probability of getting one is decreasing. Thus the ecology of the grade is also somehow studied.

The second problem studied is that of wastage from the respective subgrades of each grade. In Vassiliou (1976) we have seen that the probability of an individual belonging to grade i leaving the system at time t is related with the cumulative acceleration of the system. In section 2 of this paper we model the phenomenon assuming that the probability, of an individual belonging to the subgrade k of grade i, resigning at time t is proportional to the probability of an individual belonging in grade i, resigning and to the proportion of the number of people in subgrade k to the total number in grade i. In section 3 we apply the model to data from firm S and we find the fit to be excellent.

Each grade thus has a number of subgrades. This number is not necessarily a finite number in theory, because there is nothing to prevent an individual staying indefinitely in the same grade. However, since the working life of an individual has a finite duration with an upper limit of about 40 years the number of subgrades is itself bounded. In practice because of small number statistics, and because usually we have a small period of time available as data, we will be bound to have a fairly small number of subgrades. In some cases certainly we will have to group together two or three years of service in the grade.

Internal movements in the grade between the subgrades naturally occur every year since individuals remaining in the grade are getting older in service. These are easy to compute, in the case where there is no grouping since those not promoted move up one subgrade at a time. In the case where we group two or more subgrades as one, we treat the individuals as having the same probability of leaving, or getting promoted from the subgrade.

In section 3 of this chapter we describe the application of the above models to the data from the firm S. The comparison of the expected number of people in different subgrades with the actual number of people in the subgrades shows a reasonable good fit.

2. THE EQUATIONS OF THE MODELS FOR THE FLOWS FROM THE SUBGRADES

We assume that the rate of expansion of the organization is known in advance so that the number of recruits $R(t)$ in the t^{th} interval of time is the sum of leavers plus the number of additional staff needed to reach the known, target number. It is further assumed that the population is grouped into n grades or statuses, grade 1 being the most senior and grade n the most junior. Each grade is assumed to be devided into m $(m = 1, 2, \ldots, m)$ subgrades, subgrade 1 being the subgrade containing the group of individuals with the shortest service in the grade.

Define symbols as follows :

$N_i(t)$	the expected number of staff in grade i at the start of interval t ;
$N_i^k(t)$	the expected number of staff in subgrade k of grade i at the start of interval t ;
$p_{ij}(t)$	the probability that an individual moves from grade i to grade j during interval t ;
$N_{ij}(t)$	the expected number of staff to be promoted from grade i to grade j during interval t ;
$N_{ij}^k(t)$	the expected number of staff to be promoted from subgrade k of grade i to grade j during interval t ;
$c_i^k(t)$	the ecological coefficient of the promotion process for subgrade k of grade i (see equation (2.1) below) ;
$p_i^k(t)$	the ratio of the $N_i^k(t) / N_i(t)$;
$p_{iw}^R(t)$	the probability that an individual in grade i leaves the organization because of resignation or redundancy ;

$p^{R}_{iw}(k,t)$ the probability that an individual in subgrade k of grade i leaves the organization because of resignation or redundancy;

$p^{N}_{iw}(k,t)$ the probability that an individual in subgrade k of grade i leaves the organization because of retirement or death;

$\lambda_i(k,t)$ the ecological coefficient of the leaving process for subgrade k of grade i (see equation (2.3) below);

$p^{D}_{iw}(k,t)$ the probability that an individual in subgrade k of grade i leaves the organization because of discharge;

We will now describe the flows from the subgrades.

2.1. Promotions from the subgrades.

As we have discussed in section 1 we assume that the expected number of people promoted from subgrade k of grade i at time t to grade j is proportional to the expected number of people promoted from grade i to grade j at time t.

Thus

$$N^{k}_{ij}(t) = c^{k}_{i}(t)\, N_{ij}(t) \tag{2.1.1}$$

where $c^{k}_{i}(t)$ is the ecological coefficient of the promotion process for the subgrade k of grade i. The expected number of people promoted from subgrade k of grade i to grade j is also given by

$$N^{k}_{ij}(t) = p^{k}_{ij}(t)\, p^{k}_{i}(t)\, N_{i}(t) \tag{2.1.2}$$

The expected number of people $N_{ij}(t)$ is given by

$$p_{ij}(t)\, N_{i}(t) = N_{ij}(t) \tag{2.1.3}$$

So equation (2.1) could also be written in the from

$$p^{k}_{ij}(t) = c^{k}_{i}(t)\, p_{ij}(t)\left[p^{k}_{i}(t)\right]^{-1} \tag{2.1.4}$$

Although $c^{k}_{i}(t)$ does change with time, it will not vary too rapidly, hence it will be reasonable assuming that c^{k}_{i} is constant to estimate it from the available data. We tested this assumption with data from the records of firm S and there found no evidence in rejecting the hypothesis. This is described in section 3.

2.2. **Wastage from the subgrades.**

As in Vassiliou (1976) we will differentiate also the reasons of leaving. To do so we have to make some assumptions. Those are the following:

(1) The probability $p_{iw}^{N}(k, t)$ of an individual leaving from subgrade k of grade i because of retirement or medical retirement and death is constant. The null hypothesis thus is:

$$H_0 : p_{iw}^{N}(k, t) = p_{iw}^{N}(k) \quad \text{for given i, k and for every t.}$$

(2) The probability $p_{iw}^{D}(k, t)$ of an individual leaving from subgrade k of grade i because of discharge is constant. The null hypothesis thus is:

$$H_0 : p_{iw}^{D}(k, t) = p_{iw}^{D}(k) \quad \text{for given i, k and for every t.}$$

These hypotheses could be tested using the statistic introdused in Vassiliou (1976). We did so and it is described in the next section. We found no evidence in rejecting the hypothesis.

The probability $p_{iw}^{R}(k, t)$ of a person resigning from subgrade k of grade i at time t will be proportional to the ratio (proportion) of the number of people in subgrade k to the overall staff in the grade. Thus:

$$p_{iw}^{R}(k, t) = \lambda_i(k, t)\, p_{iw}^{R}(t)\, p_i^{k}(t) \tag{2.2.1}$$

where $\lambda_i(k, t)$ is the ecological coefficient for the leaving process for subgrade k of grade i.

Although $\lambda_i(k, t)$ does change with time, it will not vary too rapidly, hence it will be reasonable assuming that $\lambda_i(k)$ is constant to estimate it from the available data. We tested the fit of this model with data from the records of firm S and we found the fit to be excellent. This is described in section 3.

2.3. **Recruits.**

The recruits in the grade will belong to the subgrade which contains the individuals with the shortest service in the grade. Obviously there will not be any recruits to the other subgrades of the same grade.

3. APPLICATION

We have tested these models in the case of firm S. The number of grades that we distinguished is described in Young and Vassiliou (1974). Grade 5 was the lowest of the levels of management. We distinguished 3 subgrades. The first contained the individuals with 3 years maximum service in the grade the second those between 3 and 6 and finally the third those with 6^+ years of service. In grade 4 the next in the hierarchy we distinguished two subgrades. The first one contained these with less than 6 years service in the grade, while the second contained those with 6^+ years of service in the grade. In the higher 1, 2 and 3 we did not distinguished any subgrades because of the small numbers involved.

First in tables I and II we present the flows of promotion from the corresponding subgrades of grades 4 and 5. The maximum likelihood estimate of $\hat{c}_i(k)$ is given by:

$$\hat{c}_i(k) = \frac{\sum_t N_{ij}^k(t)}{\sum_t N_{ij}(t)} \tag{3.1}$$

The tables contain the flows for each year of the years 1959-66 and they are presented in the form of contingency tables. The correction for continuity (Yates correction) was applied in the cases where the expected values were rather small (< 5). The statistics $\chi^2(i, j)$ which are presented in the last column of the tables have $8-1=7$ degrees of freedom and are used to test if $c_i(k, t)$ is independent of time. The statistics $\chi^2(k, t)$ is used to test the homogeneity among the columns of the tables and are presented in the last rows. On the null hypothesis the values of $\chi^2(k, t)$ come from a chi-square distribution with $(k-1)$ degrees of freedom where k is the number of subgrades in the grade. In table I which contains the flows from grade 4 the values of the

χ^2-statistic shows no evidence against the hypothesis. In table II which contains the flows from grade 5 the values of the statistics $\chi^2(i, j)$ are not significant. From the values of $\chi^2(k, t)$ only 63 value is significant at the 2.5% level, something to be expected due to the reorganizations which the firm went through during that period. Summarising the results there appears to be no strong evidence against the hypothesis.

In table III we present the flows from the subgrades which correspond to (a) retirement or medical retirement and death, and (b) to, discharge for each of the years 1959 - 1966. We also test the two hypothesis which were made in section 2. 2 of this paper, that the probabilities $p_{iw}^{N}(k, t)$ and $p_{iw}^{D}(k, t)$ are independent of time. The maximum likelihood estimate of $p_{iw}^{N}(k, t)$ is given by

$$\hat{p}_{iw}^{N}(k, t) = \frac{\sum_t N_{iw}^{N}(k, t)}{\sum_t N_{iw}^{N}(t)} \tag{3.2}$$

where $N_{iw}^{N}(t)$, $N_{iw}^{N}(k, t)$ are the number of people who left the grade i, respectively the subgrade k of i, at time interval t because of reason (a). Anequivalent formula expresses the maximum likelihood estimate of $p_{iw}^{D}(k, t)$. On the null hypothesis the $\chi^2(i, j)$ see Vassiliou (1976) have a chi-square distribution with $T-1$ degrees of freedom, where T is the number of years available. In table III in the last column are given the values of $\chi^2(1, j)$ where in the cases where the expected numbers were really small ($\ll 5$) the χ^2- statistic was used only as a measure of agreement. Only the value $\chi^2(2, T)$ for grade 4 was significant at 0.5% level due almost entirely on the 1966 flow when the major reorganization took place. Summarising the results, there appears to be no strong evidence against the two hypotheses.

In table IV we present the flows of the leavers from the subgrades of grade 4 and 5 because of resignation. We also give in brackets the expected number of leavers estimated from equation (2. 2. 1) on the assumption that $\lambda_i(k, t)$ is constant. In the last column we give the usuall χ^2-statistic for those predictions. The χ^2 values on this table shows that the fit is very good.

For the purpose of comparing the predictions of the expected number of people in the various subgrades we have carried out the following

Using as data the period 1959 to 1963 we have been able to predict up to 3 years ahead. Similarly using the 1960 to 1964 experience we have been able to predict 2 years ahead from 1964; using the 1961 to 1965 experience we have been able to predict 1 year ahead from 1965, and thus compare several predictions with actual outcomes. In table V we present those results and also give values of the χ^2-statistic- used in this context as a measure of agreement. The fit is reasonably good.

4. CONCLUSIONS

We believe that the present models provide a good description of the flows of staff from the various grades and subgrades for various reasons and they gain considerable insight into the phenomenon in question. Moreover, we believe that these models succeed in predicting the service distribution in the grade which is very important for the costing of the manpower system.

TABLE I

Promotion flows from the subgrades of grade 4 for the period 1959 - 1966

	59	60	61	62	63	64	65	66	Total	$\chi^2(i, j)$
$N_{ij}(t)$	10	4	10	12	22	14	12	39	123	
$N^1_{ij}(t)$	5 (7.15)	4 (2.86)	5 (7.15)	9 (8.58)	14 (15.74)	12 (10.01)	8 (8.58)	31 (27.9)	38	1.767
$N^2_{ij}(t)$	5 (2.68)	0 (1.07)	5 (2.68)	3 (3.22)	8 (5.90)	2 (3.75)	4 (3.21)	8 (10.46)	33	4.359
$\chi^2(k, T)$	1.62	0.78	1.62	0.02	0.75	0.42	0.03	0.61		

1. Brackets contain expected numbers.

TABLE II

Promotion flows from the subgrades of grade 5 for the period 1959 - 1966

	59	60	61	62	63	64	65	66	Total	χ^2 (i, j)
$N_{ij}(t)$	13	35	39	35	76	52	56	113	419	
$N^1_{ij}(t)$	6 (3.63)	11 (9.77)	9 (10.89)	10 (9.77)	14 (21.22)	9 (14.52)	20 (15.64)	38 (31.55)	117	7.17
$N^2_{ij}(t)$	5 (3.69)	15 (9.94)	18 (11.08)	9 (0.94)	16 (21.58)	18 (14.77)	18 (15.91)	20 (32.09)	119	12.25
$N^3_{ij}(t)$	2 (5.68)	9 (15.28)	12 (17.03)	16 (15.28)	46 (33.19)	25 (22.71)	18 (24.45)	55 (49.36)	183	11.83
χ^2 (k, T)	2.92	4.38	5.08	0.09	8.12 *	2.37	2.55	5.83		

1. Brackets contain expected numbers.
2. * Denotes significance at 2.5 % level.

TABLE III

Retirement and discharges from the subgrades of grades 4 and 5 for the period 1959 - 1966

Grade 5	59	60	61	62	63	64	65	66	Total	$\chi^2(i, j)$
$N_5^3(t)$	244	265	249	260	259	240	228	225	1970	
$N_5^N(3, t)$	16 (19.32)	21 (20.98)	24 (19.71)	32 (20.6)	15 (20.5)	16 (19)	20 (18)	12 (17 82)	156	11.88
$N_5^1(t)$	269	273	226	207	196	261	282	284	1998	
$N_5^N(1, t)$	0 (1.21)	1 (1.23)	1 (1.02)	0 (0 93)	3 (0.88)	2 (1.17)	1 (1.23)	1 (1.27)	9	3.60
$N_5^D(1, t)$	1 (1.62)	1 (1.64)	1 (1.36)	1 (1.244)	2 (1.18)	2 (1.57)	4 (1.69)	0 (1.70)	12	3.41
$N_5^2(t)$	145	160	210	192	191	144	128	113	1283	
$N_5^N(2, t)$	1 (1.47)	0 (1.62)	3 (2.12)	0 (1.94)	2 (1.93)	1 (1.46)	4 (1.30)	2 (1.144)	13	5.60

1. Brackets contain expected numbers.

Table III (cont'd.)

Grade 4	59	60	61	62	63	64	65	66	Total	$\chi^2(i, j)$
$N_4^2(t)$	76	90	88	84	84	73	66	74	635	
$N_4^N(2, t)$	2	8	11	10	8	7	4	6	56	6.24
	(6.7)	(7.93)	(7.76)	(7.41)	(7.41)	(6.44)	(5.82)	(6.52)		
$N_4^1(t)$	126	111	127	143	148	198	223	268	1344	
$N_4^N(1, t)$	4	1	0	2	0	1	1	13	22	20.87
	(2.06)	(1.82)	(2.06)	(2.34)	(2.422)	(3.24)	(3.65)	(4.39)		

TABLE IV

Resignations from the subgrades of grades 4 and 5 during the period 1959 - 1966

Grade 5	59	60	61	62	63	64	65	66	χ^2
Subg. 1	20	14	4	11	9	14	24	31	1.83
	(17.21)	(11.69)	(5.78)	(10.50)	(10.40)	(13.60)	(25.96)	(31.17)	
Subg. 2	10	9	9	17	17	7	10	5	1.82
	(8.60)	(6.91)	(8.60)	(15.53)	(16.99)	(7.12)	(9.21)	(8.49)	
Subg. 3	1	0	3	4	8	4	5	9	4.81
	(4.48)	(3.48)	(2.11)	(5.23)	(5.63)	(3.92)	(5.37)	(6.18)	
Grade 4 Subg. 1	1	5	3	5	5	3	6	8	2.27
	(0.737)	(2.89)	(2.64)	(3.76)	(6.16)	(4.04)	(7.89)	(11.63)	

1. Brackets contain expected number.

TABLE V

Comparisons of the predictions with the actual numbers

1. Forecasts from 1959 - 63 experience.

	1964		1965		1966	
	A	Pr	A	Pr	A	Pr
Grade 1	33	33.69	33	35.38	37	33.10
Grade 2	47	49.10	44	49.50	58	51.92
Grade 3	78	87.40	81	83.46	83	83.35
Grade 4 Subg. 1	223	237.36	268	259.53	331	310.53
Grade 4 Subg. 2	66	65.49	74	56.30	62	57.06
Grade 5 Subg. 1	282	279.26	284	280.31	350	336.22
Grade 5 Subg. 2	128	124.87	113	119.116	141	171.29
Grade 5 Subg. 3	228	218.10	225	230.88	163	183.53
χ^2		254		7.18		11.16

Table V (cont'd.)

2. Forecasts from 1960 - 1964 experience.

	1965		1966	
	A	Pr	A	Pr
Grade 1	33	34.41	37	31.91
Grade 2	44	48.24	58	51.54
Grade 3	81	76.73	83	79.45
Grade 4 Subg. 1	268	254.1	331	306.7
Grade 4 Subg. 2	74	59	62	59.79
Grade 5 Subg. 1	284	290.45	350	350.28
Grade 5 Subg. 2	113	122.98	141	165.49
Grade 5 Subg. 3	225	229.09	163	179.65
χ^2		6.26		8.95

3. Forecasts from 1961 - 1965 experience.

	1966	
	A	Pr
Grade 1	37	30.42
Grade 2	58	48.97
Grade 3	83	82.6
Grade 4 Subg. 1	331	313.10
Grade 4 Subg. 2	62	73.56
Grade 5 Subg. 1	350	342.45
Grade 5 Subg. 2	141	152.16
Grade 5 Subg. 3	163	183.55
χ^2		9.17

REFERENCES

1. A. Young and G. Almond, Predicting Distributions of Staff Computer. Journal 3, 246, 1961.
2. A. Young and P. - C. G. Vassiliou, A non-linear model on the promotion of staff. Jl. R. Statist. Soc., Series A., 137, 584, 1974.
3. P. - C. G. Vassiliou, Mathematical models in manpower systems. Ph. D. Thesis the new university of Ulster. Coleraine. G. Britain, 1974.
4. ——, A. Markov Chain Model for Wastage in Manpower Systems. Opl. Res. Q. Vol. 27, 1, i, pp. 57 to 70, 1976.
5. G. B. H. Osmond, The effect of Incremental Scales and Varying Growth rates on University Salary Bills. Report. University of London, 1972.
6. R. W. Drinkwater and O. P. Kane, «Rolling up» a number of Civil Service classes. In Manpower and Management Science. D. J. Bartholomew and A. R. Smith (Editors). E; U; P; London, 1971.

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Ὁ Ἀκαδημαϊκὸς κ. **Ἰ. Ξανθάκης** κατὰ τὴν ἀνακοίνωσιν τῆς ἀνωτέρω ἐργασίας εἶπε τὰ κάτωθι:

Διὰ τὴν λύσιν πολλῶν προβλημάτων εἰς τὴν ἐφηρμοσμένην Στατιστικὴν κατασκευάζομεν στοχαστικὰ ὑποδείγματα. Ἓν μαθηματικὸν ὑπόδειγμα εἶναι ἓν σύνολον ἐξισώσεων, διαφορικῶν ἢ ὁλοκληρωτικῶν ἢ διαφορῶν κλπ. Βασικὴ προϋπόθεσις δι' ἓν μαθηματικὸν ὑπόδειγμα εἶναι νὰ ἐπιτυγχάνεται ἰσομορφισμὸς μεταξὺ τῶν λύσεων τῶν ἐξισώσεων αὐτοῦ, ὑπὸ διαφορετικὰς ἀρχικὰς συνθήκας, καὶ τῶν κυρίων χαρακτηριστικῶν τοῦ πραγματικοῦ συστήματος. Ὅταν εἰς τὰς ἐξισώσεις τοῦ μαθηματικοῦ ὑποδείγματος περιέχωνται τυχαῖαι μεταβληταί, τότε τὸ ὑπόδειγμα καλεῖται στοχαστικόν. Ἐπίσης, ὅταν τὸ στοχαστικὸν ὑπόδειγμα εἶναι μία Μαρκοβιανὴ ἀνέλιξις, τότε καλεῖται Μαρκοβιανὸν στοχαστικὸν ὑπόδειγμα.

Θεωρουμένου ἑνὸς ἱεραρχικοῦ συστήματος ἀνθρωπίνου δυναμικοῦ διακρίνομεν βαθμίδας εἰς τὰς ὁποίας ἀνήκουν τὰ μέλη τοῦ συστήματος. Διὰ τῆς κατασκευῆς ἑνὸς μὴ γραμμικοῦ στοχαστικοῦ ὑποδείγματος ἔχει λυθῆ τὸ πρόβλημα τῆς ἀναλύσεως καὶ προβλέψεως τῶν μετακινήσεων τῶν μελῶν τοῦ συστήματος μεταξὺ τῶν διαφόρων βαθμίδων. Ἐπίσης διὰ τῆς κατασκευῆς διαφόρων Μαρκοβιανῶν στοχαστικῶν ὑποδειγμάτων ἔχει λυθῆ τὸ πρόβλημα τῆς ἀπωλείας μελῶν τοῦ συστήματος ἀπὸ τὰς διαφόρους βαθμίδας. Τὸ πρόβλημα τὸ ὁποῖον τίθεται εἶναι ἡ πρόβλεψις τῆς κατανομῆς τοῦ χρόνου παραμονῆς τῶν μελῶν τοῦ συστήματος εἰς ἑκάστην βαθμίδα. Διὰ τὴν λύσιν τοῦ προβλήματος τούτου ἀπαιτεῖται νὰ εὑρεθῇ ἡ λύσις τοῦ προβλήματος τῶν μετακινήσεων τῶν μελῶν τοῦ συστήματος μεταξὺ τῶν ὑποβαθμίδων ἑκάστης βαθμίδος, καθὼς καὶ τὸ πρόβλημα τῆς προβλέψεως τῆς ἀπωλείας τῶν μελῶν τοῦ συστήματος ἀπὸ τὰς διαφόρους ὑποβαθμίδας ἑκάστης βαθμίδος.

Ὁ κ. Βασιλείου εἰς τὴν ἐργασίαν ταύτην δίδει κατὰ πρῶτον τὴν λύσιν τοῦ προβλήματος τῆς μετακινήσεως τῶν μελῶν τοῦ συστήματος μεταξὺ τῶν διαφόρων ὑποβαθμίδων ἑκάστης βαθμίδος καὶ τῶν ἄλλων βαθμίδων. Ἡ λύσις ἐπιτυγχάνεται διὰ τῆς κατασκευῆς ἑνὸς Μαρκοβιανοῦ στοχαστικοῦ ὑποδείγματος καταλλήλου διὰ τὸ σύστημα. Ἀκολούθως δίδει τὴν λύσιν τοῦ προβλήματος τῆς προβλέψεως τῆς ἀπωλείας τῶν μελῶν τοῦ συστήματος ἀπὸ τὰς διαφόρους ὑποβαθμίδας. Ἡ λύσις ἐπιτυγχάνεται διὰ τῆς στατιστικῆς θεμελιώσεως διαφόρων Μαρκοβιανῶν στοχαστικῶν ὑποδειγμάτων διὰ κάθε λόγον ἀπωλείας χωριστά. Διὰ αὐτοῦ τοῦ τρόπου κατόπιν μεταβαίνει εὐθέως εἰς τὴν λύσιν τοῦ προβλήματος τῆς κατανομῆς τοῦ χρόνου παραμονῆς εἰς ἑκάστην βαθμίδα.