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ΣΤΑΤΙΣΤΙΚΗ ΜΗΧΑΝΙΚΗ.— **QFT - Derivation of a conservative or dissipative measure - preserving flow operator in quantum statistical mechanics,** by *C. Syros**, διὰ τοῦ Ἀκαδημαϊκοῦ κ. Περ. Θεοχάρη.

The Lagrangian density of the Quantum Field Theory is considered as a generalized random, infinitely divisible field. This allows to derive from the evolution operator in QFT, U(t, t′) a statistical evolution operator, **U**(t, t′), which after a quantization exhibits conservative or dissipative properties. From this an averaging statistical evolution operator, **T**, has been derived. It is shown that **T** describes a measure-preserving flow with ergodic behavior. It allows to give a quantum definition of the temperature in the equilibrium or non-equilibrium state of the system.

1. INTRODUCTION

A derivation of Statistical Mechanics from Quantum Field Theory in Minkowski space proceeds, as a matter of fact, via changing the space metric. The usual methods to do that is by going over to the Eucledian geometry by means of a Wick rotation $t \rightarrow -it : e^{-iHt} \rightarrow e^{-Ht}$ [1], [2], [3], [4], [5].

Although this is very useful in practice, it has the unusual consequence — to name only one — of putting Statistical Field Theory to a fundamentally different world from that in which Quantum Field Theory is operable.

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* ΚΩΝΣΤΑΝΤΙΝΟΣ ΣΥΡΟΣ, **Καθορισμὸς συντηρητικοῦ ἢ ἀποσβεστικοῦ τελεστοῦ ροῆς διατηροῦντες τὸ μέτρον εἰς τὴν Κβαντικὴν Στατιστικὴν Μηχανικήν.**

The purpose of this note is to give a derivation of a statistical evolution operator which represents a flow, is measure-preserving and exhibits the property of asymptotic ergodicity.

Our goal will be obtained by considering the Lagrangian density, **L**, of the field as a generalized, infinitely divisible random field [6], [7]. The Lagrangian density structured in this way will be studied in the Minkowski space-time. This, in turn, is thought of as a lattice space with variable space- and time-spacings.

Our next fundamental assumption is that the system generally evolves via transitions (if it is microscopic, with microscopic boundary conditions, if it is macroscopic, with macroscopic boundary conditions obeyed by the wave functions).

The transition time fully determines the time- and space-spacing of the Minkowski lattice space-time.

The transitions of the system constituents induced by the interaction Hamiltonian, $H_I(t)$, are assumed to take place just at the lattice points, and the variation duration of the interaction Hamiltonian may be less or equal to the transition duration itself.

The set of the transition proper time intervals $\{\tau_\lambda^{tr}\}$ will be considered in evaluating the chronological products as a «point» — set of zero measure.

The time-variation (within τ^{tr}) of the interaction Hamiltonian, $H_I(t)$, will be assumed to be very steep.

These are mainly the basic assumptions of the present approach to the derivation of Statistical Mechanics from QFT.

In section 2 the derivation of the Random Field evolution operator $\mathbf{U}(t, t')$ is given. In section 3 the Random Quantum field evolution operator $\mathbf{U}^\sigma(t, t')$ is derived. $\mathbf{U}^\sigma(t, t')$ is dissipative for $\sigma = 1$, and conservative for $\sigma = 2$.

Section 4 presents the averaging statistical evolution operator, $\mathbf{T}_s$. It describes hypothetical particles behaving like the average of particles of the system partitioned in the sense of the state vector.

The averaging statistical evolution operator, $\mathbf{T}_s$, is shown in section 5 to have the properties of a flow operator wich is asymptotically measure-preserving. In section 6 the determination of the transition time is discussed. It turns out that the temperature depends for energy quantized systems on the Planck constant, while for non quantized systems it is $\frac{h}{2\pi}$-independent. Finally, in section 7 a general discussion of the obtained results and some conclusions are given.

2. THE RANDOM FIELD EVOLUTION OPERATOR

The Lagrangian density of the field $\mathbf{L}(\varphi, \partial_\mu \varphi)$ which is related to the Hamiltonian density $\mathbf{H}(x)$ by

$$\mathbf{L}(\varphi(x), \pi(x)) = \partial_0 \varphi(x)\, \pi(x) - \mathbf{H}(\varphi(x), \pi(x)). \tag{2.1}$$

$\varphi(x)$ is the field function, $\pi(x)$ is the conjugate momentum, $\pi = \partial \mathbf{L} / \partial_0 \varphi$, $x \in M^4$, the Minkowski space.

The evolution operator describing the change in time of the state vector in the Schroedinger picture is given by

$$U(t,t') = \mathbf{P} \exp[-i\left(\tfrac{h}{2\pi}\right)^{-1} \int_{t'}^{t} d^4x\, \mathbf{H}(x)], \tag{2.2}$$

where only the time integration is limited in [t, t'], while the other integrations are extended over $\mathbf{R}^3$. $\mathbf{P}$ is the chronological operator.

It will be a fundamental assumption of the present paper that the Langrangian of the system has the properties of a random [6] field. In the present paper $\mathbf{H}(x)$ is defined in the usual way on the space of the solutions of the Euler-Lagrange equation obtained by the variational principle applied to the action integral

$$I = \int d^4x\, \mathbf{L}(\varphi(x), \pi(x)). \tag{2.3}$$

The functions (φ, π) satisfy appropriate boundary conditions.

Therefore, the space to which $\varphi(x)$ and $\pi(x)$ belong will be assumed to be the L^2-space of the Euler-Lagrange solutions.

In addition, the validity of the Noether theorem implying a number of symmetries will be manifestly assumed. Also, the variation of the functions $\{\varphi\}$ will in the usual way include the variation of the parameters $\{h\}$ of the symmetry group, G(h), implying the conservation laws. Conceivably, if $\delta\varphi$ is implied by a symmetry parameter variation, then symmetry breaking occurs. The bigger the symmetry parameter variation (breaking), the lesser the probability for the corresponding path.

Definition I

The S_h is the parameter set of the general symmetry group following from the conservation laws according to the Noether theorem.

Definition II

The field function variation $\delta\varphi(x) = \varphi'(x') - \varphi(x)$, where

$$\varphi(x) = \varphi'(x') + \delta\varphi(x),$$

and

$$x^{\mu} \to x'^{\mu} = x^{\mu} + \delta x^{\mu} \tag{2.4}$$

is called a Noether variation and includes variation with respect both to the coordinates and to the symmetry group parameters, $\{h\}$.

Definition III

The field functions $\{\varphi(x; h) \mid h \in S_h\}$ satisfying the Euler-Lagrange equation $\partial \mathbf{L}/\partial\varphi - \partial_{\mu}(\partial \mathbf{L}/\partial_{\mu}\varphi) = 0$ are subjected to such boundary conditions $\lim\limits_{x\to\infty}(\varphi(x; h) = 0$, that $(\varphi, \pi) \in L^2_{\varphi}$, and the integrals

$$\int\varphi^2 d^4x,\ \int\pi^2 d^4x \quad \text{are finite for} \quad h \in S_h.$$

Proposition I

Let

1o $\varphi(x)$, $\partial\varphi(x)$ $\in L^2_{\varphi}$,
2o $d\varphi = \partial_0\varphi(x).dt$,
3o $\lim_{h\to h'}$, $\delta\varphi(x) = d\varphi(x)$ (Noether variation).
Then, the series

$$\sum_{n=0}^{\infty} (n!)^{-1} [\prod_{j=0}^{n} \int_{\mathbb{R}^3} d^3x_j \int_{L^2_{\varphi}} d\varphi(x_j;h)\pi(x_j;h)] \text{ converges absolutely.}$$

Proof

From $\mu^{(n)}(t;h) = (n!)^{-1}\prod_i^n \int_{\mathbf{R}^3} d^3x_i \int_{\mathbf{L}^2_\varphi} d\varphi(x_i;h)\pi(x_i;h)$

it follows for the simplest case $n = 1$ and for the Noether [8] variation of $\varphi(x,h)$ that

$$\mu(t;h) = \int_{\mathbf{R}^3} d^3x \int_{L^2_\varphi} d\varphi(x;h)\partial_0\varphi(x;h) = \int_{\mathbf{M}^4_{(t,t')}} d^4x\partial_0\varphi(x;h)\pi(x;h)$$

where $\mathbf{M}^4(t,t') = \mathbf{R}^3 \times [t', t]$ is a sector of $\mathbf{M}^4$.
From definition I and from the fact that d^4x is Lorentz invariant it follows that the integral is positive and finite

$$0 < \int d^4[\partial_0\varphi(x;h)\pi(x)] < \infty.$$

Since $\mu^{(n)}(t;h) = [\mu(t;h)]^n$, $n = 1,2,..$, the proof is complete.

Proposition II

Let

1o $\mathbf{H}(x) = \partial_0\,\varphi(x)\pi(x) - \mathbf{L}(\varphi(x), \pi(x))$,

2o For $\mathbf{L} < \xi$, $\mathbf{L} \in \mathbf{R}$, a probability, $P(\xi)$, be given such that the conditions are satisfied:

a. $P(\xi_1) \leqq P(\xi_2)$, if $\xi_1 = \xi_2$,
b. $\lim_{\xi\to-\infty} P(\xi) = 0$, $\lim_{\xi\to\infty} P(\xi) = 1$,
c. $\lim_{\xi\to\alpha-0} P(\xi) = P(\alpha)$.

3o The Lagrangian density be an infinitely divisible, random field:

$$\mathbf{L} = \sum_{\nu=1}^{N} \mathbf{L}_\nu \quad \text{for every integer } N = 2, 3, \dots .$$

with all $\{\mathbf{L}_\nu\}$ varying mutually independent and having identical probability distributions, $P(\xi)$.

Then, the evolution of this system is described by $\left(\frac{h}{2\pi}=1\right)$

$$U(t, t') = \exp\{-i\int d^3x \int d\varphi(x)\pi(x)\exp[i\int d^4x \mathbf{L}(\varphi(x), \pi(x))]\}.$$

Proof

From 1o it follows that

$$U(t, t') = \mathbf{P}\exp\{[-i\int d^4x[\partial_0\varphi\pi - \mathbf{L}(\varphi, \partial_\mu \varphi)]\}.$$

$$\Big\{\sum_{\nu=0}^{\infty}(-i)^n/(n!)\mathbf{P}[\prod_{j=0}^{n}\int d^3x_j \int d\varphi(x_j;h)\pi(x_j;h)] \tag{2.5}$$

$$\cdot \exp[i\int d^4x \mathbf{L}(\varphi(x;h), \partial_\mu\varphi(x;h))]\Big\},$$

where the first bracket, $[\ldots]$ is equal to unity for $\nu = 0$.

In view of properties 2o and 3o, the last factor in (2.5) can be written as a sum of infinitely many terms with identical probabilities distributions [6]:

$$\mathbf{L}(\varphi(x;h), \partial_\mu\varphi(x)) = \sum_{j=1}^{n}\mathbf{L}(\varphi(x_j,h), \partial_\mu \varphi(x_j,h)) \tag{2.6}$$

for all positive integers, $n = 2, 3, \ldots$.

Putting (2.6) into (2.5) each time with n equal to the order of the corresponding term of the series and summing over all n using proposition I, we obtain the *random field evolution operator*

$$\mathbf{U}(t, t') = \exp\{-i\int d^3x \int d\varphi(x)\pi(x)\exp[i\int d^4x \mathbf{L}(\varphi(x),\pi(x))]\}, \tag{2.7}$$

and the assertion is proved.

Corollary I

The Feynman path integral.

$$F = (1/2\pi)\int \mathbf{D}p\mathbf{D}q \ \operatorname{expt}(i\int dt[pq - H(p, q)]), \tfrac{h}{2\pi}=1 \tag{2.8}$$

follows formally from (2.5) by using the substitution:

$$((n!)^{-1} \prod_{j=1}^{n} \int d^3x \int d\varphi(x_j)\pi(x_j) \rightarrow (1/2\pi) \prod_{j=1}^{n} \int\int d\varphi(x)d\pi(x_j)$$

$$(p \rightarrow \pi(x),\ q \rightarrow \varphi(x),\ n = \infty). \tag{2.9}$$

Remark I

The first part of the correspondence (2.9) would contradict the uncertainty principle on the quantum level.

Remark II

The appearance of infinity on the rhs due to the non-existence of the measure of the above relationship is prevented on the lhs by the factor $1/n!$ for $n = \infty$.

Remark III

The contribution to the evolution operator of the path integral in (2.9) corresponding to the Feynman integral vanishes.

Remark IV

While elimination of p is carried out in (2.8) under the assumption [8]

$$H(t) = p^2/2m + V(q), \tag{2.10}$$

no momentum integration is required in (2.7). This renders the method interesting for the quantization of more general gauge fields.

3. THE QUANTIZED RANDOM FIELD EVOLUTION OPERATOR

Energy renormalization was first introduced in thermodynamics by Gibbs in the form of the chemical potential. This kind of renormalization will follow spontaneously here from randomness and infinite divisibility of the Lagrangian density.

Definition IV

(First quantization condition [7])

$$I(t, t') = \int_{M^4_{(t,\, t')}} d^4x(\mathbf{L}'(\varphi'(x),\ \pi'(x)) = \frac{h}{2\pi}\begin{cases} n + 1/2, & \sigma = 1 \\ n, & \sigma = 2n, \ =1, 2, \ldots \end{cases}$$

$$= \frac{h}{2\pi}\,\Lambda(n, \sigma).$$

Proposition III

Let the field action satisfy $I(t, t') = \frac{h}{2\pi}\,\Lambda(n, \sigma)$. Then the evolution operator (2.7) becomes:

i) either time conservative [9] for $\sigma = 2$,

ii) or dissipative [9] for $\sigma = 1$.

iii) Without first quantization, the operator shows a complex behavior.

Proof

From (2.1) and from (2.7) it follows that

$$\mathbf{U} = \mathbf{P}\exp[-\left(\tfrac{h}{2\pi}\right)^{-1}\int d^3x d(\varphi(x)\pi(x)\ \sin[\left(\tfrac{h}{2\pi}\right)^{-1}\int d^4x(\mathbf{H}(x) - \partial_0\varphi(x)\pi(x))]$$
$$-i\left(\tfrac{h}{2\pi}\right)^{-1}\int d^3x\int d\varphi(x)\pi(x)\cos[\left(\tfrac{h}{2\pi}\right)^{-1}\int d^4x(\mathbf{H}(x) - \partial_0\varphi(x)\pi(x))]]. \tag{3.1}$$

If we use Def. IV (3.1) becomes:

$$\mathbf{U}^{\sigma}(t, t') = \mathbf{P}\begin{cases} \exp[-\left(\tfrac{h}{2\pi}\right)^{-1}\int_{t'}^{t} dtH(t) \ -\Lambda(n, 1)], & \sigma = 1 \quad (3.2a) \\ \exp[-i\left(\tfrac{h}{2\pi}\right)^{-1}\int_{t'}^{t} dtH(t) - \Lambda(n, 2)], & \sigma = 2 \quad (3.2b) \end{cases}$$

where $H(t) = \int d^3x\mathbf{H}(x)$

Remark V

The Random Quantum Field evolution operator (3.2a) describes dissipative QFT-processes, while (3.2b) describes conservative processes. It is noticed that (3.2b) reduces the usual QFT evolution operator for $\Lambda(0, 2)$: $\mathbf{U}(t, t') = U(t, t')$

4. THE AVERAGING STATISTICAL EVOLUTION OPERATOR

The above result (3.2) is a consequence of randomness in QFT. Randomness alone does not suffice to deduce Statistical Mechanics. Averaging processes play an equally important role, and they have to be introduced explicitly. Before showing the properties of the averaging statistical evolution operator, the following definitions are required.

Definition V

$\mathbf{S}_\psi$ is the Hilbert space of the state vectors

$$|\Psi\{m\}\rangle = \sum_{\alpha=0} \frac{1}{\sqrt{\alpha!}} \prod_{v=1} \int dk_v^3 c_m^\alpha (\vec{k}_1, \ldots \vec{k}_\alpha\ ; t)\alpha^+(\vec{k}_v)|0\rangle, \qquad 4.1)$$

of the system whose evolution is described by $\mathbf{U}(t, t')$. $\mathbf{S}_\psi^c$, $\mathbf{S}_\psi^d$ are subspaces of $\mathbf{S}_\psi = \mathbf{S}_d \oplus \mathbf{S}_\psi^p$, where $c =$ conservative, and $d =$ dissipative.

Definition VI

The statistical evolution operator, $\mathbf{T}$, obtained from $\mathbf{U}$ is defined by the geometric mean of the product of the N steps

$$\mathbf{T}_s = [\mathbf{U}(t, \tau^{N-1} + t') \ \ldots \ \mathbf{U}(t' + \tau_1, \tau_2 + t')(t' + \tau_1, t')]^{1/N}. \qquad (4.2)$$

τ_n is the N—th transition time of the system, $t = t' + \sum_{n=1}^{N} \tau_n$, N is the total number of transitions accommodated in $[t, t']$ and

$$s = \left(\frac{h}{2\pi} N\right)^{-1} \sum_{\lambda=1}^{N} E_m(\lambda).\tau_{m,\,m-1}(\lambda).$$

Definition VII

A measure space $(\mathbf{S}_\psi, S, \mu)$ is constructed on :

i) $\mathbf{S}_\psi$, the Hilbert space of the state vectors describing the system.

ii) S, the σ-ring of any combination of basis elements of $\mathbf{S}_\psi$.

iii) μ, the measure on $\mathbf{S}_\psi$.

Definitition VIII

The Hamiltonian of the system, $\mathbf{H}, = \mathbf{H}_0, + \mathbf{H}_1(t)$ varies in a step-wise manner only during a very small fraction, I_t, of τ_λ. The time τ_λ is time-independent, $\mathbf{H}_1(t_\lambda) = \mathbf{H}_1^\lambda$, in the complement of each transition time-interval, I_0^λ . $\mathbf{H}_I^\lambda$ is a function of the spatial coordinates of the lattice space inside the volume, V = abc, of the system. The wave functions of the system. $\{c_m^\alpha | m = 1, 2, \ldots\}$ obey the equation

$$i \frac{h}{2\pi} \frac{\partial \varphi(q, t)}{\partial t} = -\left(\frac{h}{2\pi}\right)^2 \frac{1}{2m} \Delta_q \varphi(q, t) + H_I^\lambda(q, t)\varphi(q, t), \ t \epsilon I_t^\lambda \tag{4.3a}$$

$$-\left(\frac{h}{2\pi}\right)^2 \frac{1}{2m} \Delta_q \varphi_m^\lambda(q) + H_I^\lambda(q)(q)\varphi_m^\lambda(q) = E_m \varphi_m^\lambda(q), \quad t \in I_0^\lambda \tag{4.3b}$$

for $-a/2 \leqslant x \leqslant a/2$, $-b/2 \leqslant y \leqslant b/2$, $-c/2 \leqslant z \leqslant c/2$, and satisfy the boundary conditions

$$c_m^\alpha(\pm a/2, y, z) = c_m^\alpha(x, \pm b/2, z) = c_m^\alpha(x, y, \pm c/2) = 0 \tag{4.4}$$

$$\text{grad } c_m^\alpha(\pm a/2, y, z) = \text{grad } c_m^\alpha(x, \pm b/2) = \text{grad } c_m^\alpha(x, y, \pm c/2) = 0. \tag{4.5}$$

$\mathbf{H}_I$ (x) may contain W(q), an external field, and $W_I(q, t)$, a two-body interaction

$$W_I(q, t) = \int \Phi(q - q')\, \varphi^+(q', t)\, \varphi(q', t) dq',$$

which is treated as a perturbation.

5. THE MEASURE - PRESERVING PROPERTY.

Proposition IV

Let

1o The Hamiltonian $H(t) = H_0 + H_I(t)$ be structured as in Def. VIII.

2o $\lim_{t\to\infty} H_I(t) \to \widetilde{H}_\infty$ = constant.

Then,

i) $\mathbf{T}_s$ satisfies $\mathbf{T}_s . \mathbf{T}_{s'} = \mathbf{T}_{s+s'}$ (flow operator).

ii) $< \Psi \mid \mathbf{T}_s^{\text{asympt.}} \mid \Psi > \; = \; < \Psi \mid \mathbf{T}_{s+s'}^{\text{asympt.}} \mid \Psi >$ (measure-preserving).

Proof

First, (i) will be proved. In view of Def. VIII there holds

$$\Psi_s \mid \Psi\{m\} \rangle = \Psi\{n'\} \rangle$$

$$= \sum_{n=0} \frac{1}{\sqrt{n!}} \prod_{v=1}^{n} \int dk_v^3 \, e^{-s} \, c_m^n(k_1, \ldots \vec{k}_n ; t) \alpha^+(\vec{k_v}) \mid 0 \rangle.$$

By applying $\mathbf{T}_{s'}$ on $|\Psi\{m\}$ we see immediately that $\mathbf{T}_s \,.\, \mathbf{T}_{s'} = \mathbf{T}_{s+s'}$ and assertion (i) is true.

To show (ii) we recall that $H_I(t) = \widetilde{H}_\infty$ = constant fort $t \to \infty$, and the set of eigenvalues $\{E_m\}$ as well as the transition times $\{\tau_{m,\,m-1}\}$ vary asymptotically at most λ-independently. Hence, if we form the sum

$$s' = \left(\tfrac{h}{2\pi}N'\right)^{-1} \sum_{\lambda-1}^{N} E_m \,.\, \tau_{m,\,m-1}$$

with a different but large number of transitions, $\{N'\}$, the average value s′ will remain unchanged.

Consequently, if we define

$$\mathbf{T}_{\text{asympt.}} = \bar{\mathbf{U}} = [\mathbf{U}(t, \tau_{N-1} + t') \ldots \mathbf{U}(t' + \tau_2, \tau_1 + t')\ \mathbf{U}(t' + \tau_1, t')]^{1/M}.$$

then $\bar{\mathbf{U}}$ is N-independent for $M > N$, and

$$\mathbf{T}_s^{\text{asympt.}} = \bar{\mathbf{U}} = [\mathbf{U}(t' + \sum_{v}^{M}\tau_v, \sum_{v}^{M-1}\tau_v + t') \ldots$$
$$\mathbf{U}(t' + \sum_{v}^{N+3}\tau_v, \sum_{v}^{N+2}\tau_v + t')\mathbf{U}(t' + \sum_{v}^{N+2}\tau_v, \sum_{v}^{N+1}\tau_v + t')$$
$$\mathbf{U}(\sum_{v}^{N}\tau_v + t', \sum_{v}^{N-1}\tau_v + t')$$
$$\ldots \mathbf{U}(t' + \tau_2 + \tau_1, \tau_1 + t')\mathbf{U}(t' + \tau_1, t')]^{1/M}$$

where

$$M = N + N'.$$

Hence, the inner product

$$<\Psi\{m\} \mid \mathbf{T}_s^{\text{asympt.}} \mid \Psi\{m\} = <\ \Psi\{m'\} \mid \mathbf{T}_{s+s'}^{\text{asympt.}} \mid \Psi\{m'\}\rangle.$$

remains invariant, and this proves assertion (ii)

Proposition V

There are two types of dissipative evolution operators $\mathbf{U}$:

i) With $\lim_{t\to\infty} \mathbf{H}(t) = \widetilde{\mathbf{H}}_\infty$ = time-independent $\geqslant 0$.
ii) With $\lim_{t\to\infty} \mathbf{H}(t)$ = time dependent.

Proof

In case i) the proof follows directly from Proposition V. $\{E_m(\lambda)\}$ and $\{\tau_{m,\, m-1}(\lambda)\}$tend to limits, $s \to \mathbf{S}_\infty < \infty$, and the norm of $\mathbf{T}_s$ in $\mathbf{S}_\psi$ is conserved for $t \to \infty$.

In case ii), if $\lim_{t\to\infty} \mathbf{H}_I(t) =$ time dependent $\to 0$, then the expression

$$s = \lim_{N\to\infty} \left(\tfrac{h}{2\pi}N\right)^{-1} \sum_{\lambda=1}^{N} E_m(\lambda) \cdot \tau_{m,\, m-1}(\lambda)$$

has a finite limit, and $0 < \| \mathbf{T}_s \mid \Psi\{m\}\rangle \| < \infty$ for $t \to \infty$.
If $\lim_{t\to\infty} \mathbf{H}_I(t) =$ time dependent $\to\infty$, then three possibilities exist:

$$E_m(\lambda).\tau_{m.\, m-1}(\lambda) \to \begin{cases} \text{increases in time, then } \| \mathbf{T}_s \mid \Psi\{m\}\rangle\| \to 0 \text{ for } t \to \infty. \\ \text{constant » » » } 0 < \|\mathbf{T}_s \mid \Psi\{m\}\rangle\| < \infty \text{ for } t \to \infty. \\ \text{decreases» » » » } 0 < \|\mathbf{T}_s \mid \Psi\{m\}\rangle\| < \infty \text{ for } t \to \infty. \end{cases}$$

Remark VI

The above result allows a classification of the time-dependent interaction Hamiltonians with respect to their ergodic behavior. The proof of the following statement is trivial:

Proposition VI

Let the time-dependent interaction Hamiltonian be such that

$$s = \lim_{N\to\infty} \left(\tfrac{h}{2\pi}N\right)^{-1} \sum_{\lambda=1}^{N} E_m(\lambda) \cdot \tau_{m,\, m-1}(\lambda) = \text{constant for } t \to \infty.$$

Then the averaging statistical operator, $\mathbf{T}_s$, defined by

$$\mathbf{T}_s = [\mathbf{U}(t +, \tau_{N-1} + t') \ldots \mathbf{U}(t' + \tau_2, \tau_1 + t')\mathbf{U}(t' + \tau_1, t')]^{1/N},$$

$$\mathbf{U}\,(\sum_{\nu}^{N}\tau_\nu\, \mathbf{U} + t', \sum_{\nu}^{\nu-1}\tau_\nu + t') \ldots \mathbf{U}(t' + \tau_2 + \tau_1, \tau_1 + t')\, \mathbf{U}\,(t' + \tau_1, t')$$

where $\mathbf{U}\,(t, t')$

$$\mathbf{U}(t, t') = \mathbf{P}\exp\left\{-\left(\tfrac{h}{2\pi}\right)^{-1} \int_{t'}^{t} dt[H_0 + H\,(t)] - \Lambda(n, 2)\right\},$$

is ergodic.

Proof

According to a new formulation of the Ergodic Theorem [9], if there are no non-constant invariant functions, then there are no non-trivial invariant subsets of $\mathbf{S}_\psi$. Hence,it suffices to show that $\mathbf{T}_s \mid \Psi_{\{m\}} \rangle$ asymptotically

$$\lim_{t\to\infty} \mathbf{T}_s \mid \Psi_{\{m\}} \rangle = \lim_{t\to\infty} \sum_{n=0} \frac{1}{\sqrt{n!}} \prod_{\nu=1} \int dk^3_\nu \, e^{-s} \, c^n_m (\vec{k}_1, \ldots \vec{k}_n ;t)\, \alpha^+(\vec{k}_\nu) \mid 0\rangle$$

is constant in time.

It can easily be seen that the wave function $c^\alpha_m(\vec{k}, \ldots \vec{k}_\alpha\ ; t)$ becomes a constant because of Proposition IV, 2^0. For the same reason s takes a limiting value, because the terms added to s belong to the constant eigenvalue set of the limiting Hamiltonian. This limiting value of s is constant. Hence, the space $\mathbf{S}_\psi$ reduces under $\mathbf{T}_s$ for $t \to \infty$ to a constant, invariant sub-set.

Conversely, constant $c^\alpha_m (\vec{k}_1, \ldots \vec{k}_\alpha\ ; t)$ does not in general imply constant norm of $\mathbf{T}_s \mid \Psi_{\{m\}}\rangle$, if the premise is not fulfilled. According to the above formulation of ergodicity, $\mathbf{T}_s$ is ergodic if and only if every measurable invariant function is a constant. Since the expression

$$< \Psi_{\{m'\}} \mid \mathbf{T}^{\mathrm{asympt.}}_s \mid \Psi_{\{m\}}\rangle = \Psi_{\{m'\}} \mid \mathbf{T}^{\mathrm{asympt.}}_s \mid \Psi_{\{m\}},$$

is finite and constant in time, $\mathbf{T} \mid \Psi_{\{m\}}\rangle$ is measurable in the sense of Def. VII. This completes the proof.

6. ABOUT THE TRANSITION TIME AND THE TEMPERATURE

The determination of the temperature of an evolving system is a matter depending on the structure of the interaction Hamiltonian. This Hamiltonian determines the evolution in time of the transition time, and this in turn determines the evolution of the system. Hence, the transition time is itself function of the time, $\tau(t)$.

An exact determination of the transition times, $\tau(t)$, requires the exact solution of equation (4.3b). For interactions fulfilling certain conditions the solution can be obtained by perturbation theory. We shall content ourselves at this stage with perturbation theory.

Two distinct cases will be considered: a) Transitions to discrete states, and transition to the continuum. In addition, the interaction, H(t), is supposed to comply with Def. VII and with the fact that its variation time is very short. Also, in an evolving statistical system the interaction must act indefinitely (at each time-latice point) even if the system is in an equilibrium state. The difference with a system in a non-equilibrium state is that H(t) is time-dependent.

Since we are interested here in the transition time only, classical perturbation theory will be applied to find the transition probability per unit time, $w_{nm}(t)$. From this the transition time follows from [10]

$$\tau_{nm}(t) = [w_{nm}(t)]^{-1}$$
$$\frac{\partial}{\partial t}\,[\alpha^*_{nm}(t)\,.\,\alpha_{nm}(t)]. \tag{5.1}$$

If $H_I(t) \to \widetilde{H}_\infty$, then the transition amplitude is given by

$$\alpha_{nm}(t) = \left(\frac{h}{2\pi}\right)^{-1}\int \frac{\partial H_{I,\,nm}(t')}{\omega_{nm}\,.\,\partial t'}\,.\,e^{i\omega_{nm}t'}\,dt' - H_{I,\,nm}(t)\,\frac{e^{i\omega_{nm}t}}{\frac{h}{2\pi}\,.\,\omega_{nm}}, \tag{5.2}$$

where $\omega_{nm} = (E_n - E_m)\left(\frac{h}{2\pi}\right)^{-1}$.

For the interaction discussed above, $w_{nm}(t)$ is given by

$$w_{nm}(t) = \left(\frac{h}{2\pi}\omega_{nm}\right)^{-2}\frac{\partial\,[H_{I,nm}(t)]^2}{\partial t}$$

$$+\,2\,\mathrm{Im}\left\{\frac{H_{I,\,nm}(t)\,e^{i\omega_{nm}t}}{h^2\,\omega_{nm}\,\omega}\int^{t}\frac{\partial H_{I,nm}(t')}{\omega_{nm}\,\partial t'}\,.\,e^{-i\omega_{nm}t'}\,dt'.\right\}$$

$$+\,2\,\mathbf{Re}\left\{\frac{H_{I,\,nm}(t)\,e^{i\omega_{nm}t}}{h^2\,\omega_{nm}}\int_{\infty}^{t}\frac{\partial H_{I'\,nm}(t')}{\omega_{nm}\,\partial t'}\,.\,e^{-i\omega_{nm}t'}\,dt'\right). \tag{5.3}$$

If the evolution proceeds via states transitions continuous from a state characterized by the parameters (ξ, ζ, E) to the state with ($\xi' = \xi + d\xi$, $\zeta' = \zeta + d\zeta$ $E' = E + dE$), then the transition probability per unit time and per unit volume in the parameter space is given by

$$w(\xi, \zeta, E - \xi', \zeta', E') = \left(\frac{h}{2\pi}\right)^{-1} | H_{I;\ \xi,\ \eta,\ E} |^2 \rho(\xi, \eta, E), \tag{5.4}$$

where $\rho(\xi, \eta, E)$ is the density of the states in the interval between $[\xi, \zeta, E]$ and $[\xi' = \xi + d\xi,\ \zeta' = \zeta + d\zeta\ E' = E + dE]$.

Now, if the temperature during the transition $m \rightarrow n$ is defined by [7]

$$T_{nm} = \frac{\frac{h}{2\pi}}{k_B\, \tau_{nm}} \tag{5.5}$$

then it follows from (5.3) - (5.5) that T_{nm} depends in the case of the quantized states on the Planck constant, while in the case of transitions to the continuum, the Planck constant does not appear

$$T(\xi, \eta, E) = k_B^{-1}\ | H_{I;\ \xi,\ \eta,\ E} |^2 \rho(\xi, \eta, E). \tag{5.6}$$

If the interaction or the states density vanish, then the temperature vanishes too.

7. DISCUSSION AND CONCLUSIONS

Based on some simple principles, fundamental relations of Statistical Mechanics have been derived from Quantum Field Theory.

The stochastic behavior of a QFT system follows naturally from the assumption that the Lagrangian density of the field is a generalized, infinity divisible, random field. This made it possible to derive the statistical evolution operator, $\mathbf{U}$, from which by a quantization condition the dissipative evolution operator is obtained, or the conservative one in a more general energy renormalized form is obtained.

Using the statistical evolution operator, $\mathbf{U}$, we defined the averaging statistical evolution operator, $\mathbf{T}_s$, which reduces the state vector space, $\mathbf{S}_\psi$, to an asymptotically invariant sub-set of constant state vectors. It has been shown that $\mathbf{T}_s$ is a flow operator showing asymptotic ergodicity.

The temperature has been obtained as a functional of the interaction Hamiltonian. It is directly related with the frequency of transitions of the constituents of the system. This enables one to define the temperature in terms of microphysical observables in a way independent of equilibrium or non-equilibrium states of the system. So the temperature is definable for isolated systems without recourse to heat bath.

Another point is that the path functions space variations must, for consistency, be in accordance with the Noether theorem.

ΠΕΡΙΛΗΨΙΣ

Καθορισμὸς συντηρητικοῦ ἢ ἀποσβεστικοῦ τελεστοῦ ροῆς διατηροῦντος τὸ μέτρον εἰς τὴν Κβαντικὴν Στατιστικὴν Μηχανικήν

Τὸ πρόβλημα τῆς ἑνοποιήσεως τῆς Στατιστικῆς Μηχανικῆς καὶ τῆς Θεωρίας τῶν Κβαντικῶν Πεδίων — τῆς Θεωρίας, ἡ ὁποία σήμερον πλέον ἐκτιμᾶται ὡς ἡ βασικωτέρα μέθοδος περιγραφῆς τῶν φυσικῶν φαινομένων εἰς τὸν Μικρόκοσμον — προσέκρουσεν πάντοτε εἰς ἀνυπέρβλητα ἐμπόδια.

Ἓν ἐκ τῶν σημαντικωτέρων ἐμποδίων συνίσταται εἰς τὸ γεγονός, ὅτι ἀπαιτεῖται μετασχηματισμὸς τῆς μεταβλητῆς τοῦ χρόνου t, μέσω τῆς στρέψεως Wick, ἡ δι' ἀναλυτικῆς συνεχίσεως τῆς μεταβλητῆς τοῦ χρόνου πρὸς τὸν φανταστικὸν ἄξονα τοῦ μιγαδικοῦ ἐπιπέδου,

$$t \to -it: \; e^{-iHt} \to e^{-Ht}$$

ἐκ τῶν φυσικῶν, πραγματικῶν τιμῶν, οἱ ὁποῖες εἶναι θεμελιώδους σημασίας εἰς ὅλην τὴν Φυσικήν, καὶ ὅλως εἰδικῶς εἰς τὴν Θεωρίαν τῆς Σχετικότητος, εἰς καθαρῶς φανταστικὲς τιμές.

Ἡ μέθοδος αὕτη συνεπάγεται ἀλλαγὴν τῆς μετρικῆς τοῦ χώρου τοῦ Minkowski εἰς μετρικὴν τοῦ Εὐκλειδείου χώρου, μέθοδος συνεπαγομένη τὴν ἀπώλειαν τοῦ ἀναλλοιώτου κατὰ Lorentz τῶν βασικῶν ἐξισώσεων τῶν κβαντικῶν πεδίων.

Κατ' αὐτὸν τὸν τρόπον, ἐμφανίζεται ἡ θερμοκρασία ὡς ἀντιστρόφως ἀνάλογος τοῦ φανταστικοῦ χρόνου. Ἡ ἐξάρτησις αὐτὴ δέον νὰ θεωρηθῇ ὡς μᾶλλον εἰδικὴ καὶ δὲν ἐπιτρέπει τὴν ἀναγωγὴν τῆς τιμῆς τῆς θερμοκρασίας εἰς οἱαδήποτε θεμελιώδη μεγέθη.

Ἄλλοι τρόποι εἰσαγωγῆς τῆς θερμοκρασίας συνίστανται εἰς τὴν κανονικοποίησιν τῆς σταθερᾶς ἀλληλεπιδράσεως, κυρίως εἰς τὸ πλαίσιον τῆς θεωρίας τῶν κρισίμων φαινομένων, ἢ μέσω τοῦ ὁλοκληρωτικοῦ παράγοντος τῆς ἐντροπίας. Οἱ ἄνω τρόποι

εἰσαγωγῆς τῆς ἐννοίας τῆς θερμοκρασίας, ἀπολύτως χρήσιμοι, δὲν ἐπιτρέπουν μίαν ἑνιαίαν καὶ θεμελιώδη φυσικὴν ἑρμηνείαν τοῦ μεγέθους αὐτοῦ ἀπορρέουσαν ἐκ βασικῶν φαινομένων.

Διὰ τῆς μεθόδου τῆς παρουσιαζομένης εἰς τὴν ἐργασίαν ταύτην οἱ ἄνω δυσκολίες δὲν ἐμφανίζονται, διότι ὁ ἐκθέτης, $-iH.t$, εἰς τὸν τελεστὴν ἐξελίξεως (evoluation operator, $U(t, t') = \mathbf{P}\exp[-iHt]$, γίνεται αὐτομάτως πραγματικὸς μέσω τῆς ἀπείρου διαιρετότητος (infinite divisibility)

$$L = \mathbf{L}_1 + L_2 \ldots + \mathbf{L}_\nu, \; \nu = 2, 3, \ldots$$

τοῦ στοχαστικοῦ πεδίου τῆς πυκνότητος Lagrange καὶ τῆς κβαντώσεως τοῦ ὁλοκληρώματος δράσεως αὐτοῦ

$$\left(\frac{h}{2\pi}\right)^{-1} \int L dt = -\left\{\begin{array}{ll} 2\pi n, & \text{συντηρητικὸν} \\ & \\ 2\pi\left(n + \dfrac{1}{2}\right) & \text{ἀποσβεστικόν.} \end{array}\right.$$

Ὁ στατιστικὸς τελεστὴς ἐξελίξεως, $U(t, t')$ διασπᾶται διὰ τῆς ἄνω κβαντώσεως εἰς δύο μέρη, ἑκάτερον τῶν ὁποίων εἶναι συντηρητικὸν (conservative) ἢ ἀποσβεστικὸν (dissipative).

Ἐκ τοῦ στατιστικοῦ τελεστοῦ ἐξελίξεως παράγεται ὁ μέσος στατιστικὸς τελεστὴς ἐξελίξεως μέσης τιμῆς, $\mathbf{T}_s$.

$$\mathbf{T} = [\mathbf{U}(t, t_N) \ldots \mathbf{U}(t_3, t_2)\, \mathbf{U}(t_2, t_1)]^{1/N}.$$

Ἡ μεγάλη σημασία τούτου ἔγκειται εἰς τὶς ἐξαιρετικὲς ἰδιότητές του νὰ περιγράφει φαινόμενα ροῆς (flow) καὶ νὰ διατηρεῖ τὸ μέτρον (measure-preserving). Οἱ ἰδιότητες αὐτὲς καθιστοῦν $\mathbf{T}$ ἐργοδικόν.

Βάσει τῶν ἀνωτέρω κατέστη δυνατὸς ὁ ὁρισμὸς τῆς θερμοκρασίας τοῦ συστήματος εἰς κατάστασιν θερμοδυναμικῆς ἰσορροπίας ἢ μὴ συναρτήσει τοῦ μέσου χρόνου μεταβάσεως, τ_{nm}, τῶν συστατικῶν τοῦ συστήματος ἐκ μίας εἰς ἄλλην μικροσκοπικὴν κατάστασιν

$$T_{nm} = \frac{\frac{h}{2\pi}}{k_B\, \tau_{nm}},$$

ὅπου $\frac{h}{2\pi}$, k_B οἱ σταθερὲς Planck καὶ Boltzmann ἀντιστοίχως.

Ὁ χρόνος μεταβάσεως ὑπολογίζεται εἰς τὴν Κβαντικὴν Μηχανικὴν ὡς Συναρτησιακὸν τοῦ τελεστοῦ Hamilton ἀλληλεπιδράσεως τῶν σωματίων τοῦ συστήματος.

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Ὁ Ἀκαδημαϊκὸς κ. **Περικλῆς Θεοχάρης** εἰς τὴν ἀνακοίνωσιν τοῦ κ. Κωνστ. Σύρου λέγει τὰ ἑξῆς:

Κύριε Πρόεδρε,

Ὅταν ἡ θαλπωρὴ τοῦ Ἡλίου μᾶς ζωογονῆ κατὰ τὸν χειμῶνα, ὅταν αἰσθανώμεθα τὴν εὐχάριστον ἐγγύτητα τῆς ἑστίας κατὰ τὶς παγερὲς ἡμέρες, ὅταν εἰς ὑψηλὰ ὄρη ὁ φρουρὸς τῆς πατρίδος ἢ ὁ χιονοδρόμος τυλίσσεται εἰς τὰ χονδρὰ ἐνδύματά του, ὅλοι ἐπιδιώκουν ἕνα σκοπόν: Τὴν διατήρησιν τῆς θερμοκρασίας των.

Τί εἶναι ὅμως αὐτό, τὸ ὁποῖον ὀνομάζομε θερμοκρασίαν; Πῶς ὁρίζεται ἐπιστημονικὰ καὶ ποία εἶναι ἡ φύσις του; Ὅλοι γνωρίζομεν, ὅτι διὰ νὰ θερμάνωμε τὸ ὕδωρ, φέρομε τὸ περιέχον τοῦτο δοχεῖον εἰς ἐπαφὴν μὲ θερμὸν σῶμα: φλόγα ἢ μικροκύματα κ.τ.τ. Γενικῶς: πρὸς μίαν πηγὴν θερμότητος ὡρισμένης θερμοκρασίας.

Κατὰ τὸν ἄνω τρόπον μετετοπίσθη τὸ ἐρώτημα περὶ τῆς θερμοκρασίας τυχόντος σώματος πρὸς ἐκεῖνο τῆς θερμοκρασίας τῆς πηγῆς θερμότητος. Ἀλλὰ τὸ ἐρώτημα παραμένει ἀκόμη ἀκέραιον: «Πῶς ὁρίζεται ἡ θερμοκρασία τῆς πηγῆς θερμότητος;»

Ἂν καὶ τὸ ἐρώτημα δὲν ἔχει μεγάλην πρακτικὴν σημασίαν, ἂν καὶ ἡ θερμοκρασία σχετίζεται πρὸς τὴν μέσην ἐνέργειαν τῶν μορίων, π.χ., ἑνὸς ἀερίου, ἐν τούτοις, τὸ πρόβλημα εἶναι θεμελιώδους θεωρητικῆς σημασίας καὶ δὲν εἶναι δυνατὸν νὰ παραγνωρισθῆ. Ὁ ὑπολογισμὸς τῆς μέσης ἐνεργείας τῶν μορίων τοῦ ἀερίου προϋποθέτει σχέσιν εἰς τὴν ὁποίαν προϋπάρχει ἤδη ἡ θερμοκρασία.

Τὸ πρόβλημα τοῦ ὁρισμοῦ τῆς θερμοκρασίας λαμβάνει ἔτι πολυπλοκωτέραν μορφήν, δοθέντος ὅτι εἰς τὸ πλαίσιον τῆς Θεωρίας τῶν Κβαντικῶν Πεδίων — μιᾶς θεωρίας, ἡ ὁποία σήμερον πλέον ἐκτιμᾶται ὡς ἡ βασικωτέρα μέθοδος περιγραφῆς τῶν φυσικῶν φαινομένων εἰς τὸν Μικρόκοσμον — αὐτὴ ἐξισοῦται πρὸς τὸν ἀντίστροφον φανταστικὸν χρόνον. Τοιουτοτρόπως συνάγονται σχέσεις τῆς Στατιστικῆς Θερμοδυναμικῆς.

Ὁ συσχετισμὸς τῆς θερμοκρασίας πρὸς τὸν χρόνον πραγματοποιεῖται μαθηματικῶς διὰ τοῦ μετασχηματισμοῦ τῆς μεταβλητῆς τοῦ χρόνου μέσω τῆς στρέψεως Wick, ἢ δι' ἀναλυτικῆς συνεχίσεως τῆς μεταβλητῆς τοῦ χρόνου εἰς τὸ σύνολον τῶν μιγαδικῶν ἀριθμῶν.

Ἡ μέθοδος αὐτή, ὅμως, συνεπάγεται τὴν δημιουργίαν ἀδιαπεράτου τοίχους μεταξὺ τῆς θεωρίας τῶν Κβαντικῶν Πεδίων καὶ τῆς Στατιστικῆς Μηχανικῆς, διότι προκαλεῖ τὴν ἀλλαγὴν τῆς μετρικῆς τοῦ χώρου Minkowski, εἰς τὸν ὁποῖον μελετῶνται τὰ Κβαντικὰ Πεδία, εἰς μετρικὴν Εὐκλειδείου χώρου, εἰς τὸν ὁποῖον διατυποῦται ἡ Στατιστικὴ Μηχανική. Ἀλλὰ ἡ ἀλλαγὴ αὐτὴ μετρικῆς ἔχει περαιτέρω ὡς συνέπειαν τὴν ἀπώλειαν τῆς ἰδιότητος τοῦ ἀναλλοιώτου τῶν βασικῶν ἐξισώσεων τῶν κβαντικῶν πεδίων κατὰ Lorentz.

Κατ' αὐτὸν τὸν τρόπον, ἐμφανίζεται ἡ θερμοκρασία ὡς ἀντιστρόφως ἀνάλογος τοῦ φανταστικοῦ χρόνου ἑπομένως ἡ ἐξάρτησις αὐτὴ δέον νὰ θεωρηθῆ ὡς τελείως εἰδικὴ μὴ ἐπιτρέπουσα τὸν ὑπολογισμὸν τῆς τιμῆς τῆς θερμοκρασίας βάσει οἱωνδήποτε θεμελιωδῶν μεγεθῶν.

Τοιουτοτρόπως, ὁ ὁρισμὸς τῆς θερμοκρασίας μεταπίπτει εἰς πρόβλημα ἑνοποιήσεως τῆς Στατιστικῆς Μηχανικῆς καὶ τῆς θεωρίας τῶν Κβαντικῶν Πεδίων, καθ' ὅσον ὁ τελεστὴς ἐξελίξεως ἑνὸς καὶ τοῦ αὐτοῦ φυσικοῦ συστήματος ἀπαιτεῖ ἀφ' ἑνὸς μὲν μετρικὴν Εὐκλειδείου χώρου διὰ τὴν Στατιστικὴν Μηχανικὴν καὶ ἀφ' ἑτέρου μετρικὴν κατὰ Minkowski διὰ τὰ Κβαντικὰ Πεδία.

Ὑπάρχουν, βεβαίως, καὶ ἄλλοι τρόποι εἰσαγωγῆς τῆς ἔννοιας τῆς θερμοκρασίας ὡς π.χ., ἡ μέθοδος κατὰ τὴν ὁποίαν ἡ θερμοκρασία θεωρεῖται ὡς ὁ ὁλοκληρωτικὸς παράγων τῆς ἐντροπίας εἰς θερμοδυναμικὰ συστήματα ἢ καὶ ἡ μέθοδος διὰ τῆς κανονικοποιήσεως τῆς σταθερᾶς ἀλληλεπιδράσεως, κυρίως εἰς τὸ πλαίσιον τῆς θεωρίας τῶν κρισίμων φαινομένων εἰς τὴν συμπυκνωμένην ὕλην.

Σημαντικὴ πρόοδος ἐσημειώθη εἰς τὸ πρόβλημα τοῦ ὁρισμοῦ τῆς θερμοκρασίας, διὰ τῆς χρήσεως τῆς ὁριακῆς συνθήκης Kubo-Martin-Schwinger, ἀλλὰ καὶ κατ' αὐτὴν τὴν μέθοδον, εἴτε προϋποτίθεται τὸ στατιστικὸν σύνολον τοῦ Gibbs, ἢ τοῦτο προκύπτει δι' ἑνὸς μετασχηματισμοῦ Fourier. Καὶ ἐνταῦθα ἡ θερμοκρασία παραμένει ὡς μακροσκοπικὴ παράμετρος, ἄσχετος πρὸς βασικὰ καὶ θεμελιώδη φυσικὰ μεγέθη τοῦ μικροσυστήματος καθὼς ἐπίσης καὶ ὡς πρὸς τὸν μηχανισμὸν παραγωγῆς της.

Αἱ ἀνωτέρω μέθοδοι εἰσαγωγῆς τῆς ἐννοίας τῆς θερμοκρασίας, καίτοι ἀπολύτως χρήσιμοι, δὲν ἐπιτρέπουν ἑνιαίαν καὶ θεμελιώδη φυσικὴν ἑρμηνείαν τοῦ μεγέθους αὐτοῦ, θεμελιουμένην ἐπὶ βασικῶν φαινομένων.

Διὰ τῆς μεθόδου τῆς ἀναπτυσσομένης εἰς τὴν παροῦσαν ἐργασίαν οἱ ὡς ἄνω δυσκολίαι ἀποφεύγονται, διότι εἰς αὐτὴν ὁ χρόνος λαμβάνει αὐτομάτως τὴν ἐπιθυμητὴν μορφὴν διὰ τῆς κβαντώσεως τοῦ ὁλοκληρώματος δράσεως. Ὁ στατιστικὸς τελεστὴς ἐξελίξεως διασπᾶται διὰ τῆς ἄνω κβαντώσεως εἰς δύο μέρη, ἑκάτερον τῶν ὁποίων εἶναι εἴτε συντηρητικὸν (conservative), εἴτε ἐξασθενητικὸν (dissipative). Τῇ βοηθείᾳ τοῦ στατιστικοῦ τελεστοῦ ἐξελίξεως ἐπιτυγχάνεται ὁ στατιστικὸς τελεστὴς ἐξελίξεως τῆς μέσης τιμῆς. Ἡ μεγάλη σημασία τοῦ τελεστοῦ τούτου ἔγκειται εἰς τὶς ἐξαιρετικὲς ἰδιότητές του νὰ περιγράφει φαινόμενα ροῆς (flow) καὶ διατηρήσεως τοῦ μέτρου (measure-preserving). Οἱ ἰδιότητες αὐτὲς καθιστοῦν τὸν στατιστικὸν τελεστὴν μέσης τιμῆς ἐργοδικόν.

Βάσει τῶν ἀνωτέρω κατέστη δυνατὸς ὁ ὁρισμὸς τῆς θερμοκρασίας τοῦ συστήματος, τόσον εἰς κατάστασιν θερμοδυναμικῆς ἰσορροπίας ὅσον καὶ εἰς τοιαύτην μὴ θερμοδυναμικῆς ἰσορροπίας, συναρτήσει τοῦ μέσου χρόνου μεταβάσεως τῶν συστατικῶν τοῦ συστήματος ἐκ μιᾶς εἰς ἄλλην μικροσκοπικὴν κατάστασιν.

Ἐπειδὴ ὁ χρόνος μεταβάσεως ὑπολογίζεται εἰς τὴν Κβαντικὴν Μηχανικὴν ὡς Συναρτησιακὸν τοῦ τελεστοῦ ἀλληλεπιδράσεως κατὰ Hamilton τῶν σωματίων τοῦ συστήματος, ἕπεται τὸ λίαν σημαντικὸν ἀποτέλεσμα κατὰ τὸ ὁποῖον ἡ θερμοκρασία ἐκφράζεται ὡς συναρτησιακὸν τῆς ἀλληλεπιδράσεως ταύτης.

Ἐν κατακλείδι τῆς ἀναλύσεως ταύτης δύναται νὰ λεχθῇ, ὅτι τὰ σημαντικώτερα ἀποτελέσματα τῆς παρούσης ἐργασίας συνοψίζονται εἰς:

i) τὴν ἀπαλοιφὴν τῆς ἀνάγκης χρήσεως φανταστικοῦ χρόνου,

ii) τὴν ἀναγωγὴν τῆς θερμοκρασίας εἰς θεμελιώδη φυσικὰ μεγέθη, καὶ

iii) τὴν ὑπαγωγὴν τῆς Στατιστικῆς Μηχανικῆς εἰς τὴν Θεωρίαν τῶν Κβαντικῶν Πεδίων τῆς μετρικῆς κατὰ Minkowski.

Ὅλα αὐτὰ ὑπὸ τὴν ἁπλῆν προϋπόθεσιν, ὅτι τὸ πεδίον τῆς πυκνότητος κατὰ Lagrange ἀποτελεῖ στοχαστικόν, ἀπείρως διαιρετὸν Κβαντικὸν Πεδίον. Τέλος ἀναφέρεται, ὅτι τὴν ἄπειρον διαιρετότητα τῆς συναρτήσεως Lagrange εἶχε ἤδη χρησιμοποιήσει σιωπηρῶς ὁ μέγας φυσικὸς Feynman εἰς τὴν κατασκευὴν τοῦ περιφήμου ὁλοκληρώματος ἀτραποῦ, ἐκ τοῦ ὁποίου συνάγεται ἡ Κβαντικὴ Θεωρία.