ΦΥΣΙΚΗ.— **Boltzmann Distributions and Focusing in the Galactic Plane,** *by C. Syros* *. Ἀνεκοινώθη ὑπὸ τοῦ Ἀκαδημαϊκοῦ κ. Καίσαρος Ἀλεξοπούλου.

Summary. From the Boltzmann equation an argument is obtained which gives a plausible explanation for the observed matter concentration in galactic plane. The same argument applies to the focusing of the gravitational waves in the same plane.

The question of the focusation of the gravitational waves reported by Weber [1] is now actively discussed [2, 3, 4]. Since focusing into the galactic plane has been in connexion with the distribution of matter in the galaxy, it is natural to inquire into the reasons for the observed matter distribution. In the present note a general argument is forwarded which might give the answer to both questions: The matter distribution and the anisotropy in the propagation of the gravitational waves. This argument can be derived from the Boltzmann equation. Here is considered the non-relativistic equation. It will be assumed for simplicity that the external force-field, $\underline{F}$, is constant.

First a stationary solution, f_R, of rotational character is given [5]. This distribution function satisfies also the Liouville equation.

$$f_R(\underline{x}, \underline{c}) = \varkappa \cdot \exp[-\lambda \underline{F} \cdot (\underline{x} - \underline{x}') \wedge c]. \qquad (1)$$

The quantities $\varkappa$, λ, $\underline{x}'$ are constants.

It is easy to see by using the relations [6]

$$\underline{c}_1' = \underline{c}_1 + 2 M_2 (\underline{g}_{21} \cdot \underline{k}) \underline{k}, \qquad \underline{c}' = \underline{c} - 2 M_2 (\underline{g}_{21} \cdot \underline{k}) \underline{k}$$

that the collision integral with f_R vanishes. On the other hand,

$$\underline{c} \cdot \nabla_x (\underline{F} \cdot (\underline{x} - \underline{x}') \wedge \underline{c}) = 0 \quad \text{and} \quad \underline{F} \cdot \nabla_c (\underline{F} \cdot (\underline{x} - \underline{x}') \wedge \underline{c}) = 0.$$

Hence $f_R(\underline{x}, \underline{c})$ satisfies rigorously the transport equation.

If the system particles possess a spin, nothing essential changes in the argument.

Incidentally, $\ln f_R$ is a summational invariant.

A rotational solution with $\underline{F} \equiv 0$ is known since long [6].

Next it is shown that there exist space directions in which the matter propagates with weaker attenuation with respect to all other

* Κ. ΣΥΡΟΥ, **Κατανομαὶ Boltzmann καὶ ἑστίασις εἰς τὸ γαλαξιακὸν ἐπίπεδον.**

directions. To see that, it is observed that nothing essential changes if it is assumed that $\underline{F} = 0$ in the central region of the galaxy. Taking the Laplace transformation with respect to the time, $\bar{f} = \mathcal{L}_t\{f\}$, we get the equation

$$\lambda \bar{f} + \underline{c} \cdot \nabla_x \bar{f} = \bar{\jmath} - f(\underline{x}, \underline{c}, 0). \qquad (2)$$

From Eq. (2) it follows that

$$\lambda = K/n(\underline{c}) - c \langle \cos(\underline{c}, \underline{n}) \rangle_S \, n_S(\underline{c}) / n(\underline{c}) - n_o(\underline{c}) / n(\underline{c}). \qquad (3)$$

Here, $n(\underline{c})$ is the velocity distribution, $n_S(\underline{c})$ is the integral of $\bar{f}(\underline{x}, \underline{c})$ taken over the system's surface, S, $K = \int \bar{f} \, d^3x$ and $\underline{n}$ is the outer normal to S at $\underline{x}$.

The average cosine is given by

$$\langle \cos(\underline{c}, \underline{n}) \rangle_S = [n_S(\underline{c}) S]^{-1} \int \cos(\underline{c}, \underline{n}) \bar{f} \, dS.$$

From Eq. (3) it is obvious now that, if C^3 is the velocity space, there exist a sub-space C_1^3 such that for all $\underline{c} \in C_1^3$ the right member of Eq. (3) becomes very small and for some $\underline{c} \in C_1^3$ there holds $\mathrm{Re}\lambda = 0$. It is evident that for all $\underline{c} \in C_1^3$, for which $f(\underline{x}, \underline{c}, t)$ does not vanish, the distributions of matter decays very slowly in time or it does not decay at all ($\mathrm{Re}\lambda = 0$). This sub-space, $C_\lambda^3 \subset C_1^3$, of velocities is such that the matter concentration is high relatively to that in the complement, $c_2^3 = c^3 - c_\lambda^3$. If λ takes purely imaginary values, then $f(\underline{x}, \underline{c}, t)$ behaves like a periodic function of time for certain velocities. The directions of these velocities appear as directions of «geodesic» expansion. Such expansion should appear not only in the space filled by gases but as well in solids, whenever the diffusing particles obey the Boltzmann equation. It follows from Eq. (3) that the form of the «geodesic» domains depends on the velocity spectrum and on the form of the system's boundary. The relativistic system can be treated in an analogous manner. It can be shown that there exist, for the gravitational waves too, domains of «geodesic» propagation. Consequently there are predicted at least two focusing actions for the graviational waves: The «geodesic» selectivity of propagation directions and the relativistic focusing effect. The second alone is not sufficient [7] to explain the expected intensity of the gravitational waves as gravitational synchrotron radiation.

ΠΕΡΙΛΗΨΙΣ

Ἐκ τῆς ἐξισώσεως τοῦ Boltzmann ἐπιτυγχάνεται ἐπιχείρημα ἐπιτρέπον ἑρμηνείαν διὰ τὴν παρατηρουμένην συγκέντρωσιν τῆς ὕλης περὶ τὸ γαλαξιακὸν ἐπίπεδον. Τὸ αὐτὸ ἐπιχείρημα ἐφαρμόζεται καὶ εἰς τὴν ἀπαιτουμένην ἐστίασιν τῶν κυμάτων βαρύτητος περὶ τὸ ἰσημερινὸν ἐπίπεδον τοῦ γαλαξίου.

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Ὁ Ἀκαδημαϊκὸς κ. **Κ. Ἀλεξόπουλος** παρουσιάζων τὴν ἀνωτέρω ἀνακοίνωσιν εἶπε τὰ ἑξῆς:

Εἰς τὴν γενικὴν θεωρίαν τῆς σχετικότητος, ὁ Einstein προεῖπε τὴν ὕπαρξιν κυμάτων βαρύτητος. Τὴν πειραματικὴν παρατήρησιν τῶν κυμάτων βαρύτητος ἀνέλαβε πρὸ ἐτῶν ὁ Weber εἰς τὸ Maryland. Μολονότι μέχρι σήμερον τὰ κύματα βαρύτητος δὲν παρετηρήθησαν κατὰ τρόπον ἐπιτρέποντα τελικὰ συμπεράσματα, ἡ ἀναμενομένη ἔντασις δὲν δύναται νὰ εἶναι ἰσότροπος εἰς ὅλον τὸν χῶρον. Ὁ λόγος πρὸς τοῦτο εἶναι ὅτι ἡ ἐνέργεια τῶν κυμάτων θὰ ἀπῄτει τὴν ἐξαΰλωσιν τεραστίων ποσοτήτων μάζης. Τοῦτο θὰ ἀντέφασκε πρὸς τὴν συνεχῆ ἐκπομπὴν κυμάτων βαρύτητος, δηλ. ἄνευ σημαντικῆς ἀποσβέσεως μετὰ τοῦ χρόνου. Ἡ ἐξήγησις τοῦ φαινομένου τῆς μὴ ἀποσβέσεως τῆς ἐντάσεως τῶν κυμάτων βαρύτητος ἀναζητεῖται εἰς κάποιον μηχανισμὸν ἑστιάσεως τούτων. Ἀπάντησιν εἰς τὸ ἐρώτημα τοῦτο δίδει ὁ κ. Σῦρος, ὅστις ἐργάζεται εἰς τὸ Ἐργαστήριον Ἀτομικῆς Ἐνεργείας τῆς Εὐρωπαϊκῆς Κοινότητος εἰς Βρυξέλλας. Οὗτος ἀπὸ ἐτῶν ἀσχολεῖται μὲ τὴν ἐξίσωσιν Boltzmann, ἔχει δὲ προβῆ σχετικῶς εἰς ἀνακοινώσεις εἰς τὴν Ἀκαδημίαν Ἀθηνῶν καὶ τὴν Βασιλικὴν Ἀκαδημίαν τοῦ Βελγίου. Εἰς τὴν παροῦσαν ἐργασίαν, τὴν ὁποίαν ἔχω τὴν τιμὴν νὰ παρουσιάσω εἰς τὴν Ἀκαδημίαν Ἀθηνῶν, ἀποδεικνύει ὅτι ἐκ τῆς ἐξισώσεως τοῦ Boltzmann προκύπτει ἡ δυνατότης νὰ ὑπάρξουν δυναμικοὶ λόγοι σχετιζόμενοι πρὸς τὴν δομὴν τῆς ἐξισώσεως, οἵτινες ἀναγκάζουν τόσον τὴν ὕλην, ὅσον καὶ τὰ κύματα βαρύτητος νὰ ἑστιάζωνται ἐντὸς καὶ περὶ τὸ ἰσημερινὸν ἐπίπεδον τοῦ γαλαξίου.