ΟΥΡΑΝΙΟΣ ΜΗΧΑΝΙΚΗ.— The Keplerian orbit of a projectile around the earth, after the thrust is suddenly removed**. By ***Dem. G. Magiros*** *. Ἀνεκοινώθη ὑπὸ τοῦ κ. Ἰωάνν. Ξανθάκη.

Introduction.

In the following we discuss the elements of the Keplerian orbit of a projectile around the earth, after the thrust is suddenly removed, in the cases of sudden or gradual application of the thrust, if the thrust acts continuously either for infinitesimal time t_0 or for non-infinitesimal time τ. Formulae are given for the elements of the Keplerian orbit in terms of the elements of the Keplerian orbit either the original or that which corresponds to time t_0. For the calculation of the elements of the Keplerian orbit when the thrust is removed, the position vector and the velocity vector at that time must be known. These vectors are given in a suitable form in a previous paper [1], «paper I», contained in the present volume. We treat first the case of infinitesimal time, then the case of non-infinitesimal time, if the thrust in both cases is suddenly or gradually applied. The numbers ε throughout the paper, if multiplied by 100, give the percentage of increment of the corresponding element.

I. *The case of infinitesimal time.*

If the thrust, acting for infinitesimal time t_0, ceases at the point M_0

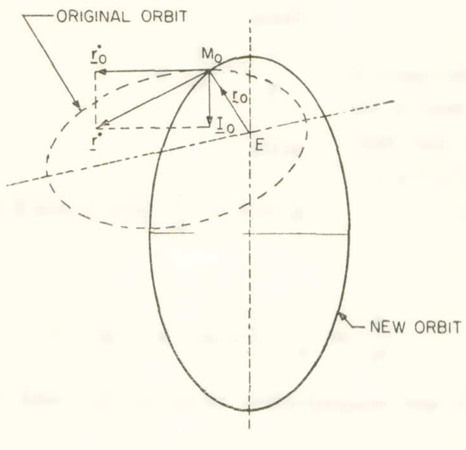

Fig. 1.— The velocity $\dot{\underline{r}}$, tangent to the new orbit at the point M_0, is the resultant of the initial velocity $\dot{\underline{r}}_0$ and the thrust $\underline{I}_0$. The original and the new orbits have the same focus E, the earth.

say, of the original orbit, Fig. 1, then the position vector and the velocity

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vector at M_0 are given, according to the formulae (10) and (11) or (10.1) and (11.1), of the «paper I», by:

$$\underline{r}(t_0) = \underline{r}_0, \quad \dot{\underline{r}}(t_0) = \dot{\underline{r}}_0 + \underline{I}_0. \tag{1}$$

After the remarks on the impulse $\underline{I}_0$ made in Chapter III of «Paper I», we proceed to calculate the elements of the orbit, after the thrust is suddenly removed at $t = t_0$, these elements being designated by the subscript I.

a. *The semi-major axis* a_I.

For the original and the new semi-major axis, a and a_I respectively, we have [2].

$$\frac{1}{a} = \frac{2}{r_0} - \frac{(\dot{\underline{r}}_0)^2}{\mu} \tag{2.1}$$

$$\frac{1}{a_I} = \frac{2}{r_0} - \frac{(\dot{\underline{r}}_0 + \underline{I}_0)^2}{\mu} \tag{2.2}$$

which give:

$$\frac{1}{a_I} = \frac{1}{a} - \frac{1}{\mu}\left(2\,\dot{r}_0 I_0 \cos\vartheta + I_0^2\right) \tag{2.3}$$

then:

$$\frac{1}{a_I} = \frac{1}{a}\,\psi \tag{2.4}$$

with

$$\psi = 1 - \frac{a}{\mu}\left(2\,\dot{r}_0 I_0 \cos\vartheta + I_0^2\right) \tag{2.5}$$

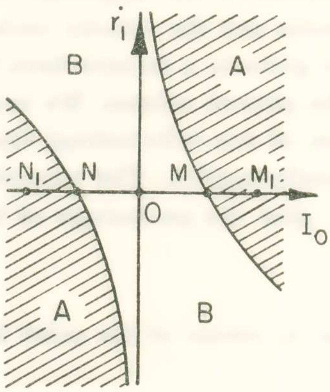

Fig. 2.— The points of the regions A give hyperbolic orbits, those of the region B elliptic orbits, and their boundary, the curve: $I_0^2 + 2 I_0 \dot{r}_1 - \frac{\mu}{a} = 0$, *parabolic orbits.*

We also can have:

$$\frac{1}{a_I} = \frac{1}{a}\,(1 + \varepsilon), \quad \varepsilon = \psi - 1. \tag{2.6}$$

In the above r_0, $\dot{r}_0$, I_0 are magnitudes of $\underline{r}_0$, $\dot{\underline{r}}_0$, $\underline{I}_0$; and ϑ the angle of $\dot{\underline{r}}_0$ and $\underline{I}_0$.

The conditions for the kind of the new orbit are:

$$I_0^2 + 2\,I_0\,\dot{r}_1 - \frac{\mu}{a} \lesseqgtr 0 \tag{3}$$

for elliptic, parabolic and hyperbolic ones, respectively, $\dot{r}_1$ is the projection of $\dot{\underline{r}}_0$ along $\underline{I}_0$, $\dot{r}_1 = \dot{r}_0 \cos\vartheta$. The graph of the conditions (3) is given in Fig. 2.

b. *The angular momentum vector* H_{I}, *and the angle between the original and the new orbits.*

The original and the new angular momentum vectors, $\underline{H}$ and $\underline{H}_{\mathrm{I}}$, are, by definition, given by the vector products:

$$\underline{H} = \underline{r}_o \times \underline{\dot{r}}_o, \quad \underline{H}_{\mathrm{I}} = \underline{r}_o \times (\underline{\dot{r}}_o + \underline{I}_o). \tag{4.1}$$

If $\Delta\underline{H}$ is the increment vector, we can write:

$$\underline{H}_{\mathrm{I}} = \underline{H} + \Delta\underline{H}, \quad \Delta\underline{H} = \underline{r}_o \times \underline{I}_o.$$

The length of $\underline{H}$ and $\Delta\underline{H}$ are:

$$H = r_o\, \dot{r}_o \sin \varphi_1, \quad \Delta H = r_o\, I_o \sin \varphi,$$

φ_1 being the angle of $\underline{r}_o$ and $\underline{\dot{r}}_o$, φ that of $\underline{r}_o$ and I_o, Fig. 3α. The vectors $\underline{H}$ and $\Delta\underline{H}$ are perpendicular to the $\underline{r}_o$, $\underline{\dot{r}}_o$ -plane and to the $\underline{r}_o$, $\underline{I}_o$ -plane, res-

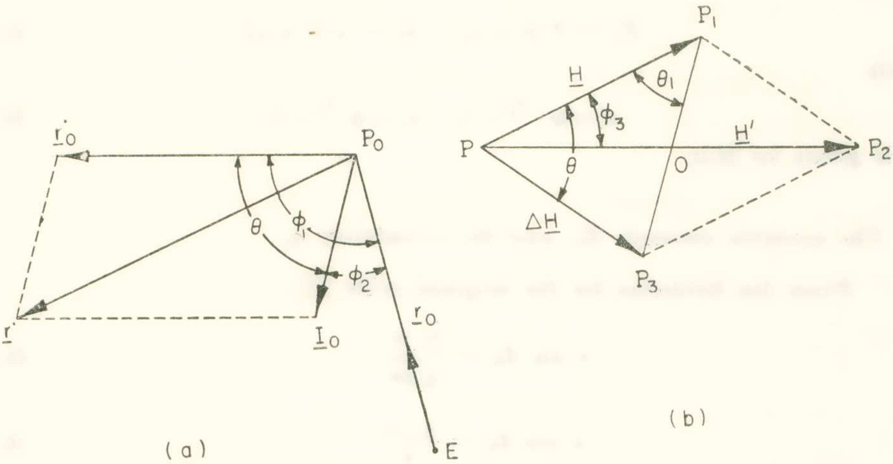

Fig. 3.—a) The vectors $\underline{r}_o$, $\underline{\dot{r}}_o$, $\underline{I}_o$ *define a trihedral angle with vertex* P_o. *b) The new angular momentum* $\underline{H}'$ *is the resultant of the original angular momentum* $\underline{H}$ *and its increment* $\Delta\underline{H}$.

pectively. If ϑ is the angle of these planes, ϑ must be the angle of $\underline{H}$ and $\Delta\underline{H}$. The new angular momentum vector $\underline{H}$ Is parallel to the $\underline{H}$, $\Delta\underline{H}$ -plane. Fig. 3b helps for calculation of the length H_{I} of $\underline{H}_{\mathrm{I}}$ and its angle φ_3 with $\underline{H}$. From the triangle $P_1\, P\, P_3$ we get:

$$(P_1\, P_3) = \varrho = r_o \{ (\dot{r}_o \sin \varphi_1)^2 + (I_o \sin \varphi_2)^2 - 2\, \dot{r}_o\, I_o \sin \varphi_1 \sin \varphi_2 \cos \vartheta \}^{1/2},$$

$$\sin \vartheta_1 = (\Delta H / P) \sin \vartheta.$$

From the triangle P_1PO we get:

$$(PO) = \left\{ H^2 + \left(\frac{\varrho}{2}\right)^2 + H\varrho \cos \vartheta_1 \right\}^{1/2}, \quad \sin \varphi_3 = ((P_1O) / (PO)) \sin \vartheta_1.$$

Then

$$H_I = 2\,(PO) = 2\left\{H^2 + \left(\frac{\varrho}{2}\right)^2 - H\varrho \cos\vartheta_1\right\}^{1/2} \quad (4.2)$$

$$\sin\varphi_3 = \frac{1}{2}\left\{\frac{1}{4} + \left(\frac{H}{\varrho}\right)^5 + \frac{H}{\varrho}\cos\vartheta\right\}^{-1/2} .\ \sin\vartheta_1 . \quad (4.3)$$

c. *The period P_I, and the mean angular motion n_I.*

We have:

$$P = \frac{2\pi}{K\sqrt{\mu}}\,\alpha^{3/2}, \quad n = \frac{2\pi}{P} = K\sqrt{\mu}\left(\frac{1}{\alpha}\right)^{3/2}$$

$$P_I = \frac{2\pi}{K\sqrt{\mu}}\,\alpha_I^{3/2}, \quad n_I = \frac{2\pi}{P_1} = K\sqrt{\mu}\left(\frac{1}{\alpha_I}\right)^{3/2},$$

then

$$P_I = P\,(1+\varepsilon_1), \quad n_I = n\,(1+\bar{\varepsilon}_1) \quad (5.1)$$

with

$$\varepsilon_1 = \psi^{-3/2} - 1, \quad \bar{\varepsilon}_1 = \psi^{3/2} - 1. \quad (5.2)$$

ψ is given by (2.5).

d. *The eccentric anomaly E_o, and the eccentricity e_I.*

From the formulae for the original orbit [3]:

$$e \sin E_o = \frac{\underline{r_o}\,\dot{\underline{r_o}}}{\sqrt{\mu\alpha}} \quad (6.1)$$

$$e \cos E_o = \frac{\alpha - r_o}{\alpha} \quad (6.2)$$

we get:

$$\tan E_o = \sqrt{\frac{\alpha}{\mu}} \cdot \frac{\underline{r_o}\,\dot{\underline{r_o}}}{\alpha - r_o}, \quad (6.3)$$

$$e^2 = \frac{(\underline{r_o}\,\dot{\underline{r_o}})^2}{\mu\alpha} + \frac{(\alpha - r_o)^2}{\alpha^2}. \quad (6.4)$$

Applying (6.3) we have for the eccentric anomaly E_{oI}:

$$\tan E_{oI} = \sqrt{\frac{\alpha}{\mu}} \cdot r_o \cdot \frac{\sqrt{\psi}}{\alpha - r_o\psi}\,(\dot{r}_o \cos\varphi_1 + I_o \cos\varphi_2), \quad (6.5)$$

φ_1 the angle of $\underline{r_o}$ and $\dot{\underline{r_o}}$, φ_2 that of $\underline{r_o}$ and $\underline{I_o}$.

Therefore:

$$\tan E_{oI} = (1+\varepsilon_2)\tan E_o \quad (6.6)$$

with

$$\varepsilon_2 = (\alpha - r_o)\frac{\sqrt{\psi}}{\alpha - r_o\psi}\left(1 + \frac{I_o}{\dot{r}_o}\frac{\cos\varphi_2}{\cos\varphi_1}\right) - 1. \tag{6.7}$$

For the new eccentricity e_I we get:

$$e_I = \frac{1}{\alpha} X^{1/2} \tag{7.1}$$

with:

$$X = \frac{\alpha}{\mu}\psi r_o^2 (\dot{r}_o \cos\varphi_1 + I_o \cos\varphi_2)^2 + (\alpha - r_o\psi)^2, \tag{7.2}$$

then:

$$e_I = e(1 + \varepsilon_3) \tag{7.3}$$

with:

$$\varepsilon_3 = \sqrt{\mu X}\left\{\alpha (r_o \dot{r}_o \cos\varphi_1)^2 + \mu(\alpha - r_o)^2\right\}^{-1/2} - 1. \tag{7,4}$$

e. *The «Perigee» q_I, «parameter» or «latus rectum» p_I and «true anomaly V_{oI}.*

For the perigee we have:

$$q = \alpha(1 - e), \quad q_I = \alpha_I(1 - e_I) = \frac{\alpha}{\psi}\left(1 - \frac{1}{\alpha}X^{1/2}\right),$$

then:

$$q_I = (1 + \varepsilon_4) q \tag{8.1}$$

$$\varepsilon_4 = \frac{\alpha - X^{1/2}}{\alpha(1-e)\psi} - 1. \tag{8.2}$$

For the parameter:

$$p = \alpha(1 - e^2), \quad p_I = \alpha_I(1 - e_I^2)$$

then:

$$p_I = (1 + \varepsilon_5) p, \tag{9.1}$$

$$\varepsilon_5 = \frac{\alpha^2 - X}{\alpha^2(1-e^2)\psi} - 1. \tag{9.2}$$

For the true anomaly V_{oI} we use the formula [4].

$$\cos V_o = \frac{\cos E_o - e}{1 - e\cos E_o}$$

which, with the help of (6.4), can be written as:

$$\cos V_o = \frac{\alpha - \alpha e - r_o}{r_o e}$$

when:

$$\cos V_{oI} = \frac{\alpha_I - \alpha_I e_I^2 - r_o}{r_o e_I} = \frac{\alpha(\alpha - r_o\psi) - X}{r_o \psi X^{1/5}},$$

then:

$$\cos V_{oI} = (1 + \varepsilon_6) \cos V_o \tag{10.1}$$

$$\varepsilon_6 = \frac{e\{\alpha(\alpha - r_o\psi) - X\}}{\psi X^{1/2}(\alpha - \alpha e^2 - r_o} - 1. \tag{10.2}$$

f. *Orientation cosines:* $\underline{P}_I$, $\underline{Q}_I$, $\underline{W}_I$.

The orientation cosines $\underline{P}$, $\underline{Q}$, W are unit vector; $\underline{P}$ the «perigee vector» from the earth towards the perigee; $\underline{Q}$ vector directed along the lactus rectum; $W = \underline{P} \times \underline{Q}$. The $\underline{P}$ and $\underline{Q}$ are given by: [5]

$$\underline{P} = \frac{\cos E_o}{r_o} \underline{r}_o - \sqrt{\frac{\alpha}{\mu}} \sin E_o . \dot{\underline{r}}_o \tag{11,1}$$

$$\underline{Q} = \frac{1}{\sqrt{1-e^2}} \left\{ \frac{\sin E_o}{r_o} \underline{r}_o + \sqrt{\frac{\alpha}{\mu}} (\cos E_o - e). \dot{\underline{r}}_o \right\}. \tag{11.2}$$

For the new $\underline{P}_I$ and $\underline{Q}_I$, by taking into account (1) (2.5), (6.1), (6.2), (7.1), (7.2), we get:

$$\underline{P}_I = \frac{\alpha - r_o\psi}{r_o X^{1/2}} \underline{r}_o - \frac{\alpha}{\mu X^{1/2}} r_o \dot{r}_o \cos\varphi_1 (\underline{r}_o + \underline{I}_o), \tag{12.1}$$

$$Q_I = \frac{\alpha}{\sqrt{\alpha^2 - X}} \left\{ \sqrt{\frac{\alpha\psi}{\mu X}} \cos\varphi_1 \, r_o . \underline{r}_o + \frac{1}{\sqrt{\alpha\mu\psi X}} \Big(\alpha(\alpha - r_o\psi) - X \Big) \Big(\dot{r}_o + I_o \Big) \right. \tag{12.2}$$

We can omit the subscript o from the formulae (12.1) and (12.2) since the orientation cosines do not vary with time, then they are independent of the position of the point M_o on the orbit.

g. *Orientation angles:* i_I, ω_I, Ω_I.

The «inclination» i is the angle between the plane of the orbit and that of the equator or of the ecliptic. The «argument of perigee» ω is the angle between the nodal line (to the direction of the ascending node) and the semi-major axis of the orbit (to the direction of the perigee). The «longitude of node» Ω is the angle (on the equator) of the nodal line to the ascending node and the intersection of equator - ecliptic.

For the new inclination i_I we notice that the inclinations are measured from the plane of equator and the angle φ_3 of the original and the new orbits from the plane of the original orbit. If i, i_I, φ_3 are positive in the same rotation as shown in Fig. 4, we can get the relationship:

$$i_I = i \pm \varphi_3 \tag{13}$$

φ_3 being given by the formula (4.3).

Now, for the new angles ω_I and Ω_I, take the orthogonal xyz-system as it is shown in Fig. 4. If P_{Ix}, P_{Iy}, P_{Iz} are the components of the new perigee vector $\underline{P}_I$ along the axes of this system, we can get the following formula: [3]

$$\begin{aligned} P_{Ix} &= \cos\omega_I . \cos\Omega_I - \sin\omega_I . \sin\Omega_I . \cos i_I, \\ P_{Iy} &= \cos\omega_I \sin\Omega_I + \sin\omega_I \cos\Omega_I \cos i_I, \\ P_{Iz} &= \sin\omega_I . \sin i_I . \end{aligned} \tag{14}$$

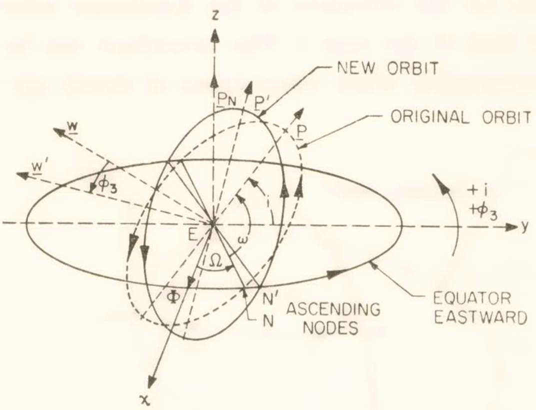

Fig. 4.—The location of the original and the new orbits with respect to the equator. The arrows in the orbits show the direction of the projectile.

We have the same formulae for the components P_x, P_y, P_z of the original perigee $\underline{P}$ by omitting the subscript I from (14).

By using the foamulae (12.1) and (13), the only unknowons in the system (14) are the angles ω_I and Ω_I. The last equation of (14) gives the new argument of perigee ω_I:

$$\sin\omega_I = P_{Iz}/\sin i_I . \tag{15}$$

Inserting the known value of ω_I into the first equation of (14), we can determine the new longitude of node Ω_I :

$$\cos\Omega_I = \frac{1}{\varrho^2+\cos^2\omega_I}\left\{P_{Ix}\cos\omega_I \pm \varrho\left(\varrho^2+\cos^2\omega_I - P^2_{Ix}\right)^{1/2}\right\}, \tag{16.1}$$

with :

$$\varrho = \sin \omega_I . \cos i_I . \qquad (16.2)$$

The second equation of (14) must be satisfied by the values found above, and this gives indication for the selection of plus or minus sign of the formula (16.1).

II. *The case of non - infinitesimal time.*

We consider in this section the case of a thrust of special type suddenly or gradually applied to the projectile and suddenly removed either after infinitesimal time t_o, case I, or after time τ, case II.

The formulae for the elements of the Keplerian orbit in case II are given in terms of that of the case I. The procedure can be used as a model to treat the calculation when other types of thrust are given.

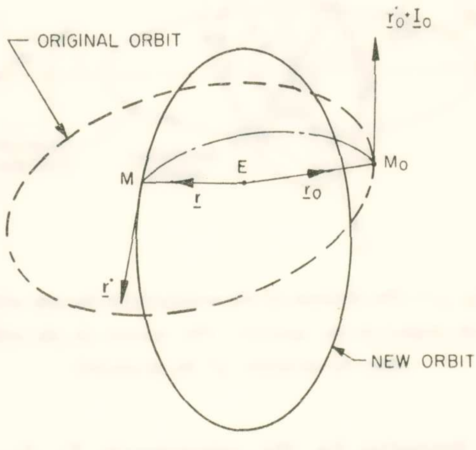

Fig. 5.— The dotted are M_o M shows the part of the orbit during the action of the thrust.

Take the direction of the thrust parallel to the initial velocity $\dot{\underline{r}}_o$, that is $\underline{T}(t) = \lambda(t)\,\dot{\underline{r}}_o$, λ being the factor of proportionality, and its magnitude constant for sudden application, or according to the law shown in Fig. 1 «Paper I», for gradual application.

The impulse $\underline{I}_o$ in the formulae (10) and (11) of «Paper I» is given by $\underline{I}_o = \frac{2}{3}\alpha t_o^{\,2/3}\,\dot{\underline{r}}_o$ in case of a gradual application with parabolic law in

$0 \leqq t \leqq t_0$; and by $\underline{I}_0 = \alpha t_0{}^{3/2}\,\underline{\dot{r}}_0$ in case of a sudden application of the constant thrust $\underline{T} = \alpha t_0{}^{1/2}\,\underline{\dot{r}}_0$. If we write: where $m=1$ for sudden application, and $m=2/3$ for gradual application of parabolic type, the formulae (10) and (11) of «paper I» can be written as:

$$\underline{r}(t) = \left\{ 1 - \frac{G}{2r_0{}^3}(t-t_0)^2 \right\} \underline{r}_0 + (t-t_0)\left\{1 + mt_0\lambda + \frac{1}{2}(t-t_0)\lambda\right\}\underline{\dot{r}}_0. \quad (17)$$

$$\underline{\dot{r}}(t) = -\frac{G}{r_0{}^3}(t-t_0)\,\underline{r}_0 + \left\{1 + mt_0\lambda + (t-t_0)\lambda\right\}\underline{\dot{r}}_0, \quad (18)$$

where $\lambda = \alpha t_0{}^{1/2}$. These formulae describe the motion of the projectile at time $t_0 \leqq t \leqq \tau$, when the arc of the orbit is represented by the broken arc of Fig. 5.

a. *The semi-major axes a_I, a_{II}.*

From the formulae (2) we have:

$$\frac{1}{a_I} = \frac{2\mu - (1+mt_0\lambda)^2 r_0 \dot{r}_0{}^2}{\mu r_0}, \quad \frac{1}{a_{II}} = \frac{2\mu - r_0 \dot{r}^2}{\mu r}$$

then we can write:

$$\frac{1}{a_{II}} = \frac{1}{a_I}(1 + \varepsilon_1) \quad (18.1)$$

with:

$$\varepsilon_1 = \frac{r_0}{r}\,\frac{2\mu - r\,\dot{r}_0}{2\mu - r_0\,\dot{r}_0{}^2(1+mt_0\lambda)^2} - 1 \quad (19.2)$$

r and $\dot{r}$ are the magnitudes of $\underline{r}$ and $\underline{\dot{r}}$, given by (17) and (18).

b. *The angular momentum vectors $\underline{H}_I$, $\underline{H}_{II}$.*

For the angular momentum vectors $\underline{H}$, $\underline{H}_I$, $\underline{H}_{II}$ we have:

$$\underline{H} = \underline{r}_0 \times \underline{\dot{r}}_0, \quad \underline{H}_I = (1+mt_0\lambda)^2\,\underline{r}_0 \times \underline{\dot{r}}_0, \quad \underline{H}_{II} = \underline{r} \times \underline{\dot{r}},$$

then for their magnitudes:

$$H = r_0 \dot{r}_0 \sin\varphi_1, \quad H_I = (1+mt_0\lambda)^2\, r_0 \dot{r}_0 \sin\varphi_1, \quad H_{II} = r\dot{r}\sin\varphi,$$

φ_1 being the angle of r_0, $\underline{\dot{r}}_0$ and φ that of $\underline{r}$, $\underline{\dot{r}}$.

We can write:

$$H_2 = H_1 (1+\varepsilon_8) \tag{20.1}$$

with:

$$\varepsilon_8 = \frac{r \dot{r} \sin \varphi}{H (1+mt_o\lambda) \sin \varphi_1} - 1 \tag{20.2}$$

The plane of the original and the new orbits are the same, then we have the same direction for the original and the new angular momentum vectors, and the inclination remains constant.

c. *The periode P_I, P_{II} and the mean angular motions n_I, n_{II}.*

For the new periods we have:

$$P_{II} = \frac{2\pi}{K\sqrt{\mu}} \alpha_{II}^{3/2}, \quad P_I = \frac{2\pi}{K\sqrt{\mu}} \alpha_I^{3/2}$$

then:

$$P_{II} = P_I \left(\frac{\alpha_{II}}{\alpha_I}\right)^{3/2} = P_I (1+\varepsilon_7)^{3/2} = P_I \left(1 - \frac{3}{2}\varepsilon_7\right) \tag{21}$$

for small ε_7, which is given by: (19.2).

In the same way for the new mean angular motion we can get:

$$n_{II} = n_I \left(1 + \frac{3}{2}\varepsilon_7\right). \tag{22}$$

d. *The eccentric anomalies E_{oI}, E_{oII}, and the eccentricities e_I, e_{II}.*

By using (6.3) we get:

$$\tan E_{oI} = \sqrt{\frac{\alpha_I}{\mu}} \; \frac{r_o \dot{r}_o (1+mt_o \lambda) \cos \varphi_1}{\alpha_I - r_o}$$

$$\tan E_{oII} = \sqrt{\frac{\alpha_{II}}{\mu}} \; \frac{r \dot{r} \cos \varphi}{\alpha_{II} - r} = \sqrt{\frac{\alpha_I (1+\varepsilon_7)^{-1}}{\mu}} \cdot \frac{r \dot{r} \cos \varphi}{\alpha_I (1+\varepsilon_7)^{-1} - r},$$

then:

$$\tan E_{oII} = (1+\varepsilon_9) \tan E_{oI} \tag{23.1}$$

with:

$$\varepsilon_9 = \left(1 - \frac{1}{2}\varepsilon_7\right) \frac{r \dot{r} \cos \varphi}{\dot{r}_o r_o (1+mt_o \lambda) \cos \varphi_1} \cdot \frac{\alpha_I - r_o}{\alpha_I (1-\varepsilon_7) - r} - 1. \tag{23.2}$$

For the new eccentricities, by applying (6.4) we have:

$$e_I^2 = \frac{\{r_o \dot{r}_o (1+mt_o \lambda) \cos \varphi_1\}^2}{\mu \alpha_I} + \frac{(\alpha_I - r_o)}{\alpha_I^2}$$

$$e_{II}^2 = \frac{(r \dot{r} \cos \varphi)^2}{\mu (1-\varepsilon_7) \alpha_I} + \frac{\{\alpha_I (1-\varepsilon_7) - r\}}{(1-2\varepsilon_7) \alpha_I^2},$$

then:

$$e_{II}^2 = e_I^2 (1 + \varepsilon_{10}) \tag{24.1}$$

with:

$$\varepsilon_{10} = \frac{(1-\varepsilon_7)\,\alpha_I (r\dot{r}\cos\varphi)^2 + \mu\,[(1-\varepsilon_7)\,\alpha_I - r]^2}{(1-2\,\varepsilon_7)\,[\{r_o\,\dot{r}_o\,(1+mt_o\,\lambda)\cos\varphi_1\}^2\,\alpha_I + \mu(\alpha_I - r_o)^2} - 1 \tag{24.2}$$

e. *The perigees q_I, q_{II}, and the parameter p_I, p_{II}.*

For the perigees we have:

$$q_I = \alpha_I (1 - e_I),\ q_{II} = \alpha_{II} (1 - e_{II}) = \alpha_I (1 - \varepsilon_7) \left\{ 1 - e_I (1 + \frac{1}{2}\varepsilon_{10}) \right\},$$

then:

$$q_{II} = (1 - \varepsilon_7)\, q_I - \frac{1}{2}\varepsilon_{10}\,\alpha_I\, e_I; \tag{25}$$

and for the parameters:

$$p_I = \alpha_I (1 - e_I^2),\ p_{II} = \alpha_{II} (1 - e_{II}^2) = \alpha_I (1 - \varepsilon_7) \left\{ 1 - e_I^2 (1 + e_{10}) \right\},$$

when, for small ε_7 and ε_{10}, we can write:

$$p_{II} = (1 - \varepsilon_7)\, p_I - \varepsilon_{10}\,\alpha_I\, e_I^2. \tag{26}$$

ΠΕΡΙΛΗΨΙΣ

Ἡ παροῦσα ἐργασία ἀποτελεῖ συμπλήρωμα καὶ συνέχειαν προηγουμένης ἐργασίας περιεχομένης εἰς τὸν παρόντα τόμον τῶν Πρακτικῶν σελ. 96 – 103. Εἰς τὴν προηγουμένην μελέτην μελετᾶται ἡ κίνησις ὀχήματος πέριξ τῆς γῆς ὑπὸ τὴν ἐπίδρασιν τῆς ἑλκτικῆς δυνάμεως τῆς γῆς καὶ μιᾶς ὠστικῆς δυνάμεως. Εἰς τὴν παροῦσαν ἐξετάζονται τὰ στοιχεῖα τῆς Κεπλερείου τροχιᾶς τοῦ ὀχήματος, ὅταν ἡ ὠστικὴ δύναμις παύσῃ ἀποτόμως νὰ ἐνεργῇ ἐπὶ τοῦ ὀχήματος. Δίδονται τύποι συνδέοντες τὰ στοιχεῖα τῆς Κεπλερείου τροχιᾶς πρὸς τὰ στοιχεῖα τῆς Κεπλερείου τροχιᾶς εἴτε τῆς ἀρχικῆς (ὅταν ἤρχισε ἐνεργοῦσα ἡ ὠστικὴ δύναμις), εἴτε τῆς ἀντιστοιχούσης εἰς τὸν χρόνον t_o. Οἱ ἀριθμοὶ ε εἰς τοὺς διδομένους τύπους, ἑκατονταπλασιαζόμενοι, δίδουν τὸ ποσοστὸν ἐπὶ τοῖς ἑκατὸν τῆς αὐξήσεως τῶν ἀντιστοιχούντων στοιχείων.

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ΓΛΩΣΣΟΛΟΓΙΑ.— Γραπτὴ ἀπόδοσις τριῶν φθόγγων τῆς συγχρόνου νεοελληνικῆς γλώσσης, *ὑπὸ* ***Στεφ. Μακρυμίχαλου****. Ἀνεκοινώθη ὑπὸ τοῦ κ. Σωκρ. Κουγέα.

* Θὰ δημοσιευθῇ κατωτέρω.