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ΠΡΟΕΔΡΙΑ ΚΩΝΣΤΑΝΤΙΝΟΥ ΤΡΥΠΑΝΗ

ΦΥΣΙΚΗ.— **Matter - antimatter asymmetry and Lamb shift,** *by C. Syros**, διὰ τοῦ 'Ακαδημαϊκοῦ κ. Καίσαρος 'Αλεξοπούλου.

ABSTRACT

The QED electron self energy equals to a high degree the internal (3→1, 2) transition energy of the H-atom. New quantum states of the hydrogen atom have been found. The Lamb shift (1077.63 MHz) is reproduced without parameters to an accuracy of .325% as transition between the Dirac states and the newly found states. Predicted are also the frequencies 10922 MHz (.236%), 4463 MHz (3.069%) and 187 MHz (.223%). No hyperfine interaction is required. Spectroscopic asymmetry between hydrogen and antihydrogen is also predicted. The method is applicable to any one-dimensional interaction Hamiltonian with the Dirac equation.

There are only few cases in theoretical particle physics in which a matter-antimatter asymmetry is implied. On the other hand, matter appears much more frequently in experiments than antimatter. The question is why antimatter appears separated from matter in the degree actually observed. This question will be examined in the case of the hydrogen and antihydrogen.

It is a well-known fact that the non-relativistic spectroscopic notation of the atomic states implies a certain degree of degeneracy. In this notation the first few states of the hydrogenic atoms are as follows: $S_{1/2}$, $2\,S_{1/2}$, $2\,P_{1/2}$, $2\,P_{3/2}$, $3\,S_{1/2}$, $3\,P_{1/2}$ corresponding to the total angular momentum

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number j=1/2, 1/2, 1/2, 3/2 and 1/2, 1/2. Thus the states 2 $S_{1/2}$, 2 $P_{1/2}$ and 3 $S_{1/2}$, 3 $P_{1/2}$, etc. appear as degenerate. In fact, however, the experimental states 2 $S_{1/2}$, 2 $P_{1/2}$ are found to a frequency distance of 1057.873 MHz from one another. This is the famous Lamb shift [**1**]. The interpretation given in QED mainly in terms of self energy of the electron and the vacuum polarization might be difficult to explain if the degeneracy in question were absent. It seems that in fact there are instances in which degeneracy is absent. Therefore, additional attempts to explain how the Lamb shift is to be understood in these cases are highly desirable. The lack of degeneracy in the H-atom follows from the energies of the atom with opposite k-values, where k is the Dirac angular momentum quantum number. These energies are by far not identical in the frame of the present theory, contrary to what is the case in the traditional spectral equation.

The purpose of the present note is threefold:

(A) To demonstrate that the relativistic hydrogen and antihydrogen states are not correspondingly identical to those in the spectroscopic notation and not of equal probabilities.

(B) To show that the set of the new hydrogen atom quantum states (and possibly of all atoms) is broader than that given by the traditional Dirac set of bispinors.

(C) To show on the basis of (B) that the Lamb shift is due 99,67% to transitions of the type $(k=3)\rightarrow(n'=1, k=2)$. Here (n', k) is a hydrogen state according to the Dirac solution theory and (n'+k) is the corresponding new hydrogen state found in the present work.

According to the bispinor solution of the Dirac equation found in the present work with the Coulomb interaction, $V(r)=e^2/4\pi\varepsilon_0 r$ alone the energy eigenvalues for positive k, ε_+, and for negative k, ε_-, satisfy the relation [**2**]

$$\varepsilon_- + \varepsilon_+ = 0. \tag{1}$$

They are obtained from the spectral equation

$$\varepsilon_\pm = \frac{-\sigma^2{}_\pm + \alpha^2 Z^2}{\sigma^2{}_\pm + \alpha^2 Z^2}, \tag{2}$$

where

$$\sigma_\pm = \sqrt{k^2 - \alpha^2 Z^2} \mp k. \tag{3}$$

In eq. (2) $\alpha = e^2/2\varepsilon_0 hc$ is the fine structure constant and Z is the atomic number (fig. 1).

Eq. (2) is of different structure as compared to the Dirac formula [3]

$$\varepsilon\,(n',k) = \left[\,1 + \frac{\alpha^2 Z^2}{(n' + \sqrt{k^2 - \alpha^2 Z^2})^2}\,\right]^{-1/2} . \qquad (4)$$

Eq. (2) gives eigenvalues differring slightly (ε_+) or much (ε_-) from those given by eq. (4), with (n', k) as is shown in Table I. The largest difference between the Dirac values and the present work values calculated in Table I is $4.5.10^{-5}$ eV. These differences will be discussed below.

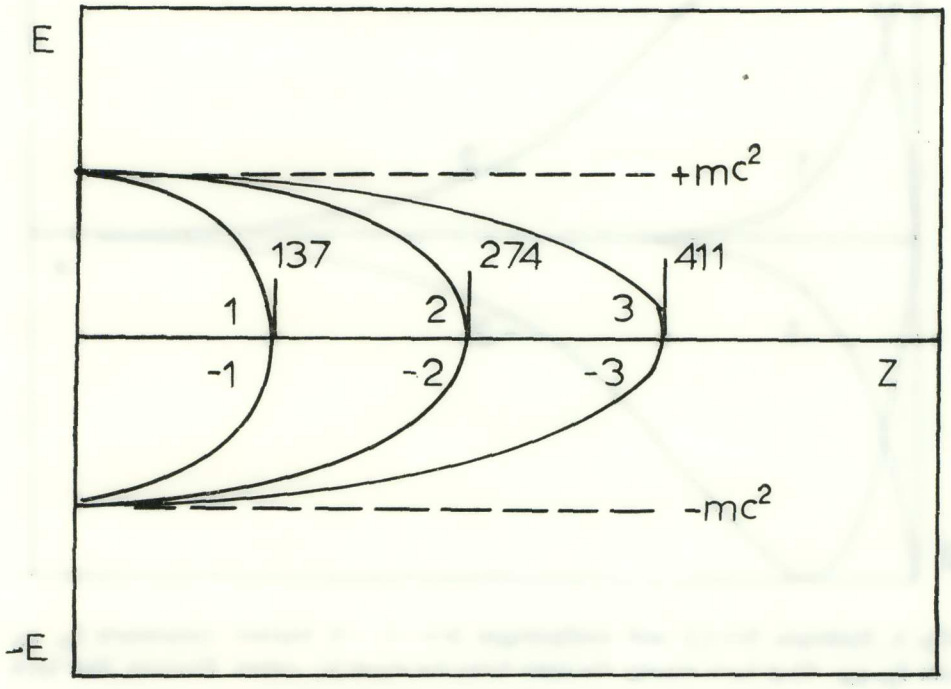

Fig. 1. The energies $\varepsilon_\pm$ (k, Z) as functions of the atomic number, Z, for hydrogen and antihydrogen atoms. For $|k|>1$ atoms or antiatoms with $Z>137$ may be found in states with finite probability.

Taking into account eq. (2) and using the spectroscopic notation [4] one finds the states for hydrogen and antihydrogen given in Table II. Comparing the columns 4 and 6 one observes that:

(i) In this notation the state $S_{1/2}$ for the antihydrogen is missing.

(ii) The states of both hydrogen and antihydrogen are of zero degeneracy.

(iii) The radial quantum number does not appear in the new states.

It follows, therefore, that in the spectroscopic notation the new hydrogen states are not symmetric to the antihydrogen states. The lowest antihydrogen state is in this notation the state $P_{1/2}$ with l=1, j=1/2 and k=—1. The energy of this state is $\varepsilon_{-}(-1)$. It is further shown that the relative proba-

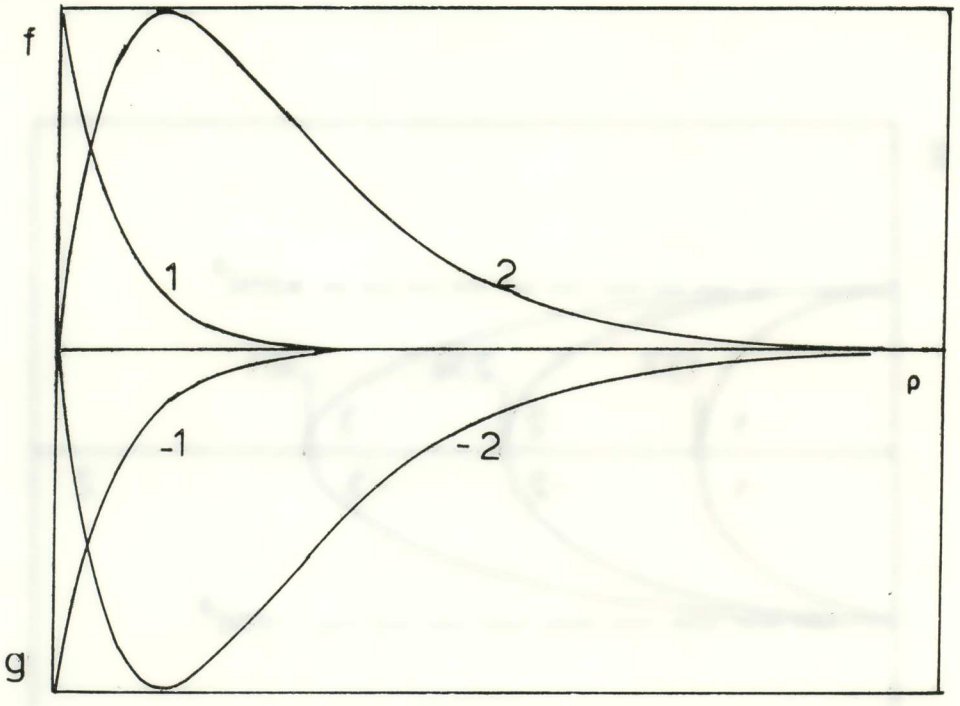

Fig. 2. Hydrogen (k=1,2) and antihydrogen (k=—1, —2) bispinor components f_H, g_H and $f_{\overline{H}}$, $g_{\overline{H}}$. They have exactly the same forms for equal |k| -values. However, they have very different normalization constants. They are proportional to: (k=1:.99998631, k=—1: $1.2755.10^{-5}$, k=2:9.803.10^{-5}, k=—2:3.189.10^{-10}). These tremendous differences give a measure for the relative abundances of the corresponding states of hydrogen and antihydrogen in the universe.

bilities for the states with $k=1$, -1 of the hydrogen and antihydrogen respectively are very unequal to the favour of the hydrogen.

This can be proved by using the components f, g of the bispinor. These have the form (Fig. 2):

$$f(\rho;k)=\left\{\cosh q+\left[k\ln\rho-\left(\rho\sqrt{1-\varepsilon^2}+\sqrt{\frac{1-\varepsilon}{1+\varepsilon}}\,\bar{v}(\rho)\right)\right]\frac{\sinh q}{q}\right\}\frac{N}{\rho}, \tag{5}$$

$$g(\rho;k)=\left\{-\sqrt{\frac{1-\varepsilon}{1+\varepsilon}}\cosh q+\left[\sqrt{\frac{1-\varepsilon}{1+\varepsilon}}k\ln\rho+\left((1-\varepsilon)\,\rho-\bar{v}(\rho)\right)\right]\frac{\sinh q}{q}\right\}\frac{N}{\rho}. \tag{6}$$

In eqs. (5-6) $q^2(\rho)$ is the «discriminator» (Fig. 3)

$$q^2(\rho)=k^2\ln^2\rho+\rho^2-(\varepsilon\rho-\bar{v}(\rho))^2, \tag{7}$$

$$\bar{v}(\rho)=\int H'(\rho)\,d\rho\quad,\quad \varepsilon=E/m_0c^2. \tag{8}$$

$\bar{v}(p)$ is the integrated Coulomb interaction and m_0 the rest mass.
It is seen from eqs. (5-6) that the bispinor components f and g have a common normalization factor. This factor, N, is the common factor of a constant bispinor $\binom{c_1}{c_2}$ which provides the solution of the Dirac equation with two arbitrary constants to be determined from the boundary conditions. The boundary conditions $f\to 0$ for $\rho\to 0$, $f\to 0$ for $\rho\to\infty$ have been applied and they provided eq. (2) for the energy eigenvalues. Therefore, the integrals give

$$P_+=\int(f^2(\rho;k)+g^2(\rho;k))d^3\rho \text{ and } P_-=\int(f^2(\rho;-k)+g^2(\rho;-k))\,d^3\rho \tag{9}$$

the measure of the probability for each of the matter state ($k\geqq 1$) and the antimatter state ($k\leqq -1$). In this definition of the relative probabilities there is no possibility for an ambiguity, because P_+ and P_- have no common factor and no free parameter. They result from each other only by the charge $e^-\to e^+$) and parity ($k\to -k$) reflexions. Therefore, the normalization constants, $N_\pm$, defined by

$$N_\pm\cdot\int(f^2(\rho;\pm k)+g^2(\rho;\pm k))\,d^3\rho=1 \tag{10}$$

provide the relative probabilities ($P_{\pm}=N_{\pm}^{-1}$) :

$$P_{\text{hydrogen}} = \frac{P_+}{P_+ + P_-} \quad \text{and} \quad P_{\text{antihydrogen}} = \frac{P_-}{P_+ + P_-} . \tag{11}$$

Calculations show that $P_{\pm}$ diminish very rapidly with increasing $|k|$.

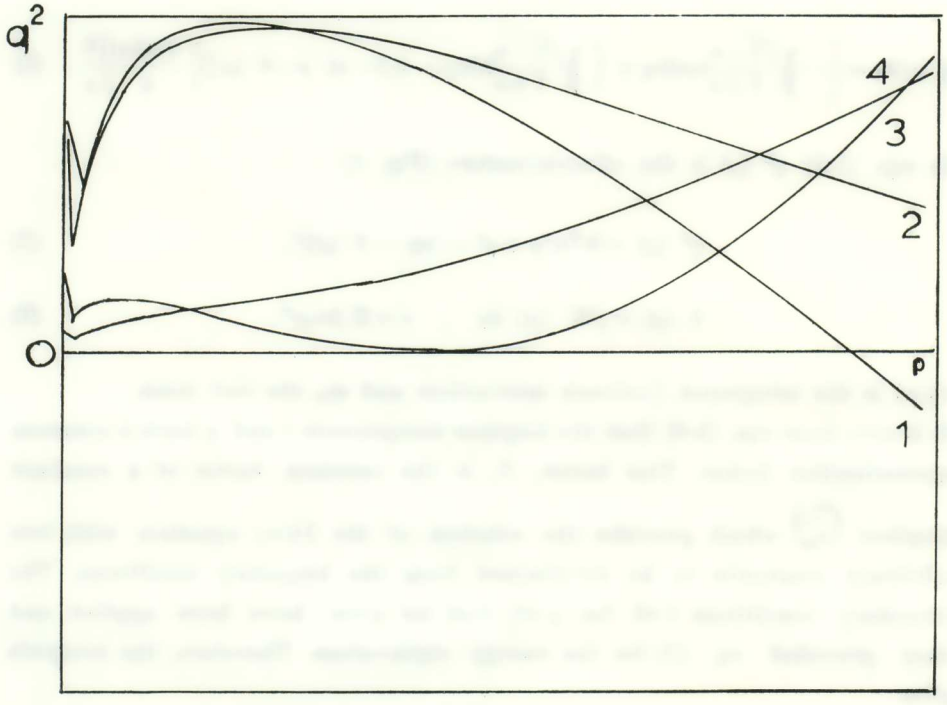

Fig. 3. The discriminator $q^2(\rho)$ determines the character of the spinor as bound ($q^2>0$) or scattering ($q^2<0$) states. The curve 1 ($\varepsilon=1$) gives a scattering state. The curves 2 ($\varepsilon=.999933$), 3 ($\varepsilon=.999973$), 4 ($\varepsilon=.99992$) give bound states of the hydrogen atom.

It suffices, therefore, for the evaluation of the relative matter-antimatter probabilities to find the $P_{\pm}(|k|=1)$ only. This is the ground state probability. The results are

$$\begin{aligned} P_{\pm}\ (k=1) &= .99998668699 \\ P_{-}\ (k=-1) &= .00001331300 \end{aligned} \tag{12}$$

This proves the matter-antimatter asymmetry in the universe, where the Dirac equation is valid.

Next, the proof of the second assertion (B) will be given. In view of the mathematical exactness of both the Dirac solution and of eqs. (5-6) it follows that these represent additional states of the hydrogenic atoms which were unknown previously.

The reasons why they have not all been discovered by the optical spectroscopy is twofold:

(i) In transitions of the type $(k+1)\rightarrow(k)$ the wave lengths are identical to the wave lengths of the transitions between the Dirac states of the type $(0, k+1)\rightarrow(0, k)$.

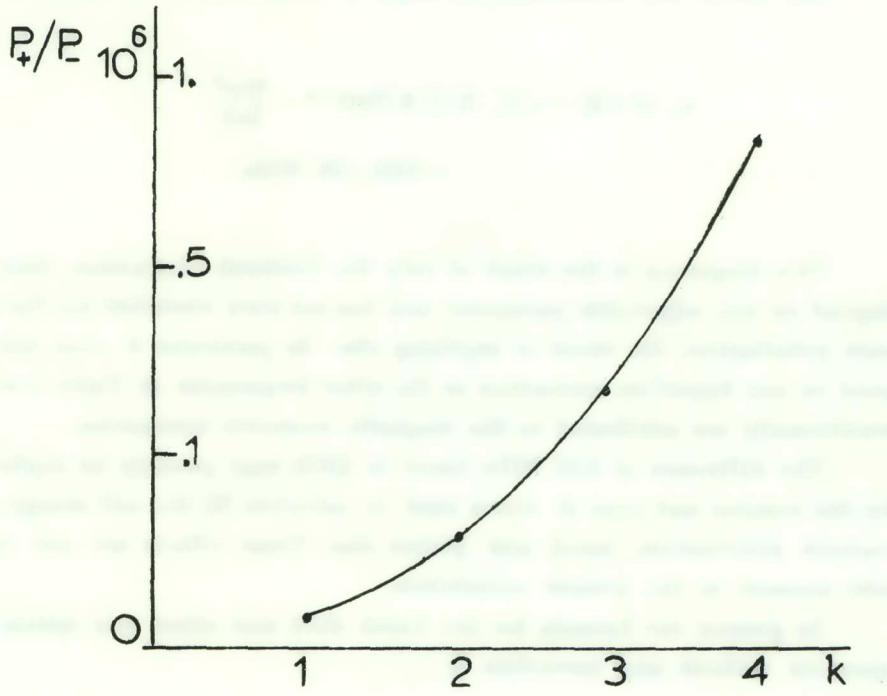

Fig. 4. The points give the ratio $P_+/P-$ of the relative probabilities for the H- and the anti-H-atoms. The axis of the abscissae gives the relativistic quantum numbers. A tremendous decrease is seen of the probability for anti-matter in the universe where the Dirac equation is valid. Even in the ground state of the H- and anti-H-atoms the probability ratio is more than 75100.

(ii) The wave lengths of transitions between present work states and Dirac states of the type $(n'+k)\rightarrow(n', k)$ are of the order of 1 cm. Transitions of this second type take in fact place and this will be shown in the assertion below.

The discussion of the properties of these new states is, however, mathematical in nature and, although it has important physical repercussions it will not be pursuited here anymore.

Finally, assertion (C) explains the main part of the Lamb shift assigned in QED to the self-energy of the electron.

From Table I it is seen that for $n'>0$ there are small differences between the energy levels $\varepsilon(n', k)$ and $\varepsilon(n'+k)$. These differences are all of the order of magnitude $h\nu_{Lamb}$, where h is the Planck constant and ν_{Lamb} the Lamb frequency.

The traditional Lamb shift corresponds (Table I) to the difference

$$\varepsilon_+ (1+2) - \varepsilon (1, 2) \widehat{=} 8.7510^{-12} \cdot \frac{m_0 c^2}{2\pi h}$$

$$= 1081.195 \text{ MHz.} \qquad (13)$$

This frequency is the result of only the Coulomb interaction, does not depend on any adjustable parameter and has not been corrected for the vacuum polarization, the recoil or anything else. In particular it does not depend on any hyperfine interaction as the other frequencies in Table I which traditionally are attributed to the magnetic moments interaction.

The difference of 3.55 MHz found in QED may possibly be explicable by the number and type of states used to calculate [5] the self energy, the vacuum polarization, recoil and proton size. These effects are not taken into account in the present calculation.

In general the formula for the Lamb shift and other non optical frequencies without any correction is

$$\nu = \frac{m_0 c^2}{h}\left\{\frac{\alpha^2-(k-\sqrt{k^2-\alpha^2})^2}{\alpha^2+(k-\sqrt{k^2-\alpha^2})^2} - \frac{1}{\sqrt{1+\dfrac{\alpha^2}{(n'+\sqrt{k^2-\alpha^2})^2}}}\right\}. \qquad (14)$$

Eq. (14) predicts the frequencies given in Table I. The experimental data of the last column have been taken from the references 8, 9 and 10. It should be pointed out that the results of the present work in Table I do not contain any correction. The Lamb shift «due» to self-energy is 1077.64 MHz. The comparison with the predicted frequency of 1081.195 MHz gives an agreement to 99.675%. It is understood, of course, that no parameter at all is contained in the present theory.

Summarizing this note the following three general remarks may be made:

(i) The hydrogen-antihydrogen asymmetry in the spectroscopic notation is not described by the Dirac bispinor solution, because it depends on the absolute value |k| of the relativistic angular momentum quantum number. This means essentially that the Dirac equation has been solved [**6**] in 1928 only for positive k-values. There exist, however, further solutions with physical meaning [**2**], and these are given by eqs. (5-6). This bispinor complements Dirac's bispinor and no one separately describes fully the behaviour of the hydrogenic atoms.

(ii) Since the Lamb shift is not due to the self energy of the electron, one will have to think about the way to understand the self energy in QED.

(iii) Since no adjustable parameter at all is contained in the present Lamb shift theory, and since in our calculated frequencies no correction whatsoever is included one will have to understand how to calculate better the neces-

TABLE I

This table gives the energy eigenvalues of the H-atom in column 3 according to the usual Dirac formula, E (n′, k). Column 4 gives E (n′+k) according to the present work. For n′=0 the differences of the two series of eigenvalues are equal to zero. For higher values of n' the differences are very small. They give the fine structure frequencies of the H-atom. These results have been calculated in double precision with the UNIVAC 1100/60 computer. The maximum difference amounts to the energy. 000045 eV. It corresponds to the known hyperfine frequency 10947.836 MHz.

The second difference gives the Lamb shift 1081.195 MHz. This number contains only the Coulomb part; It is emphasized that the frequencies in column 6 are the direct result of the new quantum states of the H-atom. They have not been corrected for the vacuum polarization or for the recoil effects. In addition they do not contain any adjustable

parameters and they depend only on universal consta nts. The symbols have the following meanings:

k=relativistic angular momentum quantum number.

n′=radial quantum number of the Dirac solution theory. It is absent in the new spinor solutions. The relation of the two sets of states is k→n′+k.

m=rest mass of the electron.

c=velocity of light.

n′	k	E(n′, k) m. c^2	E(n′+k) m. c^2	Difference m. c^2	Predicted MHz	Observed MHz
0	1	.99997337406068	.99997337406068	0	0	-
	2	.99999334358163	.99999334358163	0	0	-
	3	.99999704159731	.99999704159731	0	0	-
	4	.99999833589956	.99999833589956	0	0	-
	5	.99999893497604	.99999893497604	0	0	-
1	1	.99999334349301	.99999334358163	$8.86.10^{-11}$	10947.836	10922
	2	.99999704158855	.99999704159730	$8.75.10^{-12}$	1081.195	1057.873
	3	.99999833589771	.99999833589956	$1.85.10^{-12}$	227.359	-
	4	.99999893497547	.99999893497604	$5.67.10^{-13}$	70.061	74
	5	.99999926039993	.99999926040015	$5.47.10^{-13}$	27.061	-
2	1	.99999704156230	.99999704159730	$3.50.10^{-11}$	4324.766	4463
	2	.99999833589402	.99999833589956	$5.54.10^{-12}$	678.370	558
	3	.99999893497452	.99999893497603	$1.51.10^{-12}$	186.583	187
	4	.99999926039959	.99999926040015	$5.47.10^{-13}$	69.589	-
	5	.99999945662033	.99999945662057	$2.36.10^{-13}$	29.161	-
3	1	.99999833588294	.99999833589956	$1.66.10^{-11}$	1051.175	-
	2	.99999893497263	.99999893496039	$1.22.10^{-11}$	1512.431	-
	3	.99999926039906	.99999926040015	$1.09.10^{-12}$	135.179	-
	4	.99999945662013	.99999945662057	$4.33.10^{-13}$	53.503	-
	5	.99999983974942	.99999958397515	$2.10.10^{-13}$	25.948	-

sary corrections in order to achieve agreement with the observed values. Also, the frequencies 74.187, 558 and 4463 MHz are attributed to the hyperfine structure interaction. This assignment will have to be reexamined because the present results which agree very well with the above frequencies do not take account of any other interaction besides the Coulomb one.

The corresponding remark holds true also for the calculations of the muonium [**7,8**] Lamb shift. Finally, the fact that all frequencies of the presen work deviate from all but one, 4463 MHz, corresponding experimental values less than .33% might indicate that an error may be contained in the assignment of this value.

Concluding, it is found that QED is much more preciser than it was believed hitherto, but one has to interpret it still better. Also work to understand the hyperfine structure of the atoms appears as necessary in view of the fact that many spectral lines attributed to the hyperfine interaction are simply due to transitions of the hydrogen atom between levels in the Coulomb field. This conclusion is imposed by the reproduction of the frequencies 10947, 70, 187 etc. besides the Lamb shift [**9,10**] without any interaction of the magnetic moments.

TABLE II

A classification of the hydrogen and the antihydrogen states without the radial quantum number, n'. The hydrogen and antihydrogen states are non-symmetrically distributed if the classical spectroscopic notation is used. The $S_{1/2}$ state of the antihydrogen does not appear.

Quantum number k	Total spin j	Hydrogen l	(l)j	Antihydrogen l	(l)j
± 1	1/2	0	$S_{1/2}$	1	$P_{1/2}$
± 2	3/2	1	$P_{3/2}$	2	$D_{3/2}$
± 3	5/2	2	$D_{3/2}$	3	$F_{5/2}$
± 4	7/2	3	$F_{5/2}$	4	$G_{5/2}$ etc.

ΠΕΡΙΛΗΨΙΣ

'Ασυμμετρία Ύλης - 'Αντιύλης καὶ μετατόπισις Lamb

'Η εἰς τὴν Κβαντικὴν 'Ηλεκτροδυναμικὴν ὑπολογιζομένη αὐτο-ἐνέργεια (self-energy) τοῦ ἠλεκτρονίου συμπίπτει κατὰ 99.5% περίπου μὲ τὴν ἐνέργειαν μεταβάσεως τοῦ ἀτόμου τοῦ 'Υδρογόνου ἐκ μίας καταστάσεως k_α εὑρεθείσης εἰς τὴν παροῦσαν ἐργασίαν καὶ τῆς καταστάσεως (n'. k_τ) τῆς προβλεπομένης ὑπὸ τῆς γνωστῆς σχετικιστικῆς θεωρίας τοῦ ἀτόμου τοῦ 'Υδρογόνου, k_α καὶ k_τ ἡ ἀρχικὴ καὶ τελικὴ τιμὴ τοῦ σχετικιστικοῦ κβαντικοῦ ἀριθμοῦ τῆς στροφορμῆς καὶ n' ὁ ἀκτινικὸς κβαντικὸς ἀριθμός. 'Η μετατόπισις Lamb καὶ οἱ ἄλλες μὴ ὀπτικὲς συχνότητες τοῦ ἀτόμου τοῦ 'Υδρογόνου ἀποδίδονται λίαν ἱκανοποιητικῶς ὑπὸ τῶν εἰς τὴν παροῦσαν ἐργασίαν εὑρεθεισῶν κβαντικῶν καταστάσεων τοῦ 'Υδρογόνου. Οἱ εὑρεθεῖσες συχνότητες μὴ λαμνανομένων ὑπ' ὄψιν τῶν ἄλλων συνεισφορῶν, οἱ ὁποῖες δὲν ὑπολογίζονται εἰς τὴν παροῦσαν ἐργασίαν, εἶναι: 10947.8 MHz, 1081. 19 MHz, 70.06 MHz, 4324.76 MHz, 186.58 MHz κ.ἄ. 'Η ἀκρίβεια ὑπολογισμοῦ τῶν ἄνω συχνοτήτων εἶναι ἄνω τοῦ 99.65%. 'Επὶ πλέον διεπιστώθη ἡ ἀσυμμετρία ὕλης-ἀντιὕλης εἰς τὴν φασματοσκοπικὴν παράστασιν τοῦ 'Υδρογόνου καὶ τοῦ ἀντί-'Υδρογόνου. 'Η ἀναπτυχθεῖσα μέθοδος πρὸς λύσιν τῆς ἐξισώσεως τοῦ Dirac ἐφαρμόζεται εἰς ὅλες τὶς σφαιρικὰ συμμετρικὲς ἢ μονοδιάστατες Χαμιλτονιανὲς ἀλληλεπιδράσεως. Οἱ συχνότητες 10947.83 MHz καὶ 1081.19 MHz ἀντιστοιχοῦν εἰς τὰ μήκη κύματος $\lambda_1 = 0.02738$m καὶ $\lambda_2 = 0.2773$m.

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