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ΜΑΘΗΜΑΤΙΚΑ.— **Generalized structure sheaf envelopes of topological function algebra spaces,** *by Anastasios Mallios**. Ἀνεκοινώθη ὑπὸ τοῦ Ἀκαδημαϊκοῦ κ. Φίλωνος Βασιλείου.

1. Introduction. The purpose of the present note is to give detailed information concerning some earlier results of this author, as well as a complete proof of the main theorem included herewith, an early announcement of which has been given in Ref. [**10**]. At the same time, the said theorem constitutes the first systematic account towards an «abstract complex analytic space theory» within the context of topological algebra sheaves [**8**], the main motivation of the latter notion being so far the standard theory of analytic functions of several complex variables. An analogous study, although in another setting, has already begun by C. E. Rickart (cf., for instance, [**12**]), and besides another treatment, more contiguous to the present «sheaf-theoretic» context, however in a rather implicit way, has been recently given by R. M. Brooks [**2**]. In this respect, cf. also the recent work of D. S. Kim [**5**].

2. Preliminaries. We first recall from [**10**] (cf. also [**8**]) the notion of an (abstract) *topological algebra space.* By this we mean a pair (X, A) consisting of a Hausdorff topological space X and a *topological algebra sheaf* A over X [**8**]. In case A is a subsheaf of the sheaf C of germs of

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(complex-valued) continuous functions on X, we shall have, in particular, a *topological function algebra space*. Notice that we do not require in this latter case any special restriction, concerning the topology of the particular section (topological function) algebras under consideration.

Now, given a topological algebra space (X, A), its structure sheaf envelope $H(X)$ is defined by the relation:

$$H(X) :\underset{\text{Def.}}{=} \mathcal{M}(\Gamma(X, A)), \tag{2. 1}$$

i.e. as the spectrum of the topological algebra $\Gamma(X, A)$. of the global sections of A over X.

The preceding definition is in agreement with recent training, within the context of complex analytic space theory, since for the particular case of such a space (X, O), one defines the envelope of holomorphy of X, as the spectrum of the respective topological algebra $\Gamma(X, O)$, an idea which goes back explicitly for arbitrary Riemann domains at least to H. Rossi [**13**]. (For more recent developments in this concern, cf. for instance [**4**], as well as [**15**]).

Now, for purposes of applications in representation theory of topological algebras, and in particular of topological function algebras (cf., for instanse, [**11**]), it is of a special interest to consider that, which might be called the *generalized structure sheaf envelope* of a given topological algebra space (X, A), in analogy with the case of generalized spectra of topological algebras (cf. [**9**] and [**10**]), i.e. one has by definition:

$$H(X, F) := \mathcal{H}om_s(\Gamma(X, A), F), \tag{2. 2}$$

where by the second member of the preceding relation we mean the generalized spectrum of the topological algebra $\Gamma(X, A)$, with respect to a given topological algebra F [**9**].

On the other hand, in view of applications to vector-valued functions, one is also led to consider tensor products of topological algebra spaces. For simplicity, we shall consider in particular the case that the latter spaces are over the same topological space X.

Thus, given the topological algebra space (X, A) its «vectorization» with respect to a given topological algebra E, will be the space $(X, A\,\tau\, E)$, where the (topological algebra) sheaf $A\,\tau\,E$ is defined by the lemma which follows. Now, for technical reasons concerning topological tensor products, it will be more convenient to assume the topological algebras involved to be locally convex ones. But, we shall have first to fix our terminology concerning the formulation of Lemma 2. 1 below:

Given the locally convex (topological vector) spaces E and F, one has the following (canonical) exact sequence

$$0 \rightarrow E \otimes F \rightarrow \mathscr{L}(F'_s, E), \tag{2. 3}$$

where $\mathscr{L}(E'_s, F)$ denotes the space of continuous linear maps of F'_s (: the weak topological dual of F) into E. In case E and F are, in particular, locally convex algebras, by a *compatible topology* on $E \otimes F$ we mean a locally convex algebra topology τ on $E \otimes F$, which is induced on the latter algebra by the following topological version of (2. 3)

$$0 \rightarrow E \underset{\tau}{\otimes} F \rightarrow \mathscr{L}_{\mathsf{T}}(F'_{\mathsf{S}}, E), \tag{2. 4}$$

with $\mathsf{S} \geqslant s$. One may think, for instance, of T as the topology e of biequicontinuous convergence and S the Mackey topology on F', with E a nuchear locally convex algebra [**6**] (cf. also Corollary 2. 1 below).

Thus, given a topological algebra space (X, A), the contravariant functor

$$U \rightarrow \Gamma(U, A) \hat{\underset{\tau}{\otimes}} E, \tag{2. 5}$$

with $U \subseteq X$, open, defines a presheaf of locally convex (topological) algebras on X (cf. also [**8**]), where in the range of (2. 5) we mean the completion under τ (as in (2. 4)) of the respective locally convex algebra. (In this concern, we assume, unless otherwise stated, that the locally convex topological algebras involved have (jointly) continuous multiplication).

Now, we denote by $A \tau E$ the sheaf on X generated by the *topological algebra presheaf* defined by (2. 5).

Thus, we are now in the position to state the following lemma by which one guarantees (under certain conditions; cf. Corollary 2. 1) that *the presheaf defined by* (2. 5) *is actually a sheaf:* That is, we first have.

2. 1. Lemma. *Let (X, A) be a topological algebra space and* E *a locally convex algebra. Then, denoting by $\Gamma(U, A \tau E)$ the algebra of local sections over an open set $U \subseteq X$ of the sheaf $A \tau E$ generated by the presheaf*

$$(2.6) \qquad \{\Gamma(U, A) \underset{\tau}{\otimes} \mathrm{E} : U \subseteq X, \ open\},$$

one has the relation :

$$(2.7) \qquad \Gamma(U, A\tau \mathrm{E}) = \Gamma(U, A) \underset{\tau}{\otimes} \mathrm{E},$$

within a bijection, for every open set $U \subseteq X$.

Proof: (à la Bungart [**3**]). Without loss of generality we may take $U = X$, and let, for every open $U \subseteq X$,

$$(2.8) \qquad \theta_U : \Gamma(X, A) \to \Gamma(U, A)$$

be the (canonical) restriction map. Then, by definition of the sheaf $A\tau\mathrm{E}$ one gets a natural map

$$(2.9) \qquad \rho : \Gamma(X, A) \underset{\tau}{\otimes} \mathrm{E} \to \Gamma(X, A\tau\mathrm{E}),$$

which is an algebra homomorphism. We shall show that *the map ρ is a bijection:* Indeed, let $f \in \Gamma(X, A) \underset{\tau}{\otimes} \mathrm{E} \underset{\rightarrow}{\subset}$ (by (2.4)) $\mathcal{L}_{\mathsf{T}}(\mathrm{E}'_{\mathrm{S}}, \Gamma(X, A))$, with $\rho(f) = 0$. Hence, by taking an open basis U of X one obtains $\theta_U \circ f = 0$, for every open set $U \in \mathsf{U}$, i.e. for every $x' \in \mathrm{E}'$, one has $(\theta_U \circ f)(x') = \theta_U(f(x')) = 0$, for every $U \in \mathsf{U}$, and hence $f(x') = 0$, for every $x' \in \mathrm{E}'$, so that $f = 0$, i.e. *the map ρ is 1 — 1.* On the other hand, taking an element $f \in \Gamma(X, A\tau\mathrm{E})$, there exists an open basis U of X such that, for every $U \in \mathsf{U}$, one has $f|_U = f_U$, where

$$f_U \in \Gamma(U, A) \underset{\tau}{\otimes} \mathrm{E} \underset{\rightarrow}{\subset} \mathcal{L}_{\mathsf{T}}(\mathrm{E}'_{\mathrm{S}}, \Gamma(U, A)).$$

Moreover, for any $U, V \in \mathsf{U}$ with $V \subseteq U$, one obtains $\theta_{UV} \circ f_U = f_V = f|_V$, so that the family $(f_U)_{U \in \mathsf{U}}$ defines an element g in the domain of ρ, whose image under ρ is obviously f, that is ρ *is also an onto map,* and this finishes the proof of the lemma. ∎

The preceding provides also a stengthening, for the case considered herewith, of a similar result of L. Bungart (cf. [**3**; p. 328, Proposition 9.2, and its Corollary]), which also was the motivation to the present setting. We could continue the argument within the context of the preceding lemma, however for simplicity's sake we restrict ourselves to the particular case described by the following Corollary 2.1.

Thus, suppose that in the exact sequence (2.4), one has $\mathsf{T} = e$, the topology of biequicontinuous convergence, $\mathfrak{S} = \tau$, the Mackey topology on F′, so that if, moreover E is a complete nulear locally convex algebra and F complete, one has the following (topological) exact sequence:

$$(2.10) \qquad 0 \rightarrow E \hat{\otimes}_{\pi} F \rightarrow \mathscr{L}_e(F'_\tau, E) \rightarrow 0,$$

with $E \hat{\otimes}_{\varepsilon} F = E \hat{\otimes}_{n} F$ a complete locally convex algebra.

In particular, the following corollary is now an immediate consequence of the preceding Lemma 2. 1, so that we may omit the details of the proof. That is, we have.

2. 1. Corollary. *Let (X, A) be a topological algebra space such that the respective local sections define complete, nuclear locally convex algebras, and let* E *be a complete locally convex algebra. Then, the sheaf $A \varepsilon E$ generated by the presheaf*

$$(2.11) \qquad \{\Gamma(U, A) \hat{\otimes}_{\varepsilon} E : U \subseteq X, \text{ open}\}$$

is such that one has the relation

$$(2.12) \qquad \Gamma(U, A \varepsilon E) = \Gamma(U, A) \hat{\otimes}_{\varepsilon} E,$$

within an algebraic isomorphism, for every open set $U \subseteq X$. ∎

Now, by the relation (2. 12), we may consider the sheaf $A \varepsilon E$ of the preceding corollary as a sheaf, whose local sections define topological algebras, by considering (2. 12) as a topological isomorphism, so that *one actually obtains a topological algebra sheaf in the sense of Ref.* [**8**] (cf. also [**3**; p. 328, Corollary 9. 3]).

We finally note that a more general formulation of the above Corollary 2. 1 can also be given, within the context of Ref. [**8**; p. 218, Theorem 2. 1], but we shall leave it for another treatment within a different more natural setting.

3 The main theorem. We first comment on the terminology applied in the formulation of the next theorem.

Thus, given a topological algebra space (X, A), we shall say that a subset S of X is *A-convex* whenever one has the relation

$$(3.1) \qquad M(\Gamma(S, A)) = S,$$

within a homeomorphism, where S carries the relative topology of X and the first member of (3.1) denotes the spectrum of the topological algebra $\Gamma(S, \mathcal{A})$. In this concern, we recall that the topology of the latter algebra is defined by the relation

$$\Gamma(S, \mathcal{A}) = \varinjlim_{U \supseteq S} \Gamma(U, \mathcal{A}), \tag{3.2}$$

where, by the second member of (3.2), one means the inductive limit (locally convex) topological algebra (with (jointly) continuous multiplication) defined by a fundamental system of open neighborhoods U of S in X.

In particular, one has the relation (3.1) above in case of a Stein manifold $(X, \mathcal{O})$ and a compact holomorpically convex (i.e. $\mathcal{O}$-convex) subset S of X. Thus, motivated by this fundamental example, we shall also call the first member of (3.1) the *structure sheaf envelope* (or the *$\mathcal{A}$-convex hull*) of S in X, with respect to the topological algebra space $(X, \mathcal{A})$ under consideration.

Finally, by a *central morphism* between two unital algebras E and F we shall mean an algebra homomorphism $h: E \to F$ such that its image is a central subalgebra of F, i.e. $\mathrm{Im}(h) \subseteq F$ has a trivial center in F, which means that the latter is equal to $\mathbb{C} \cdot 1_F \cong \mathbb{C}$ (: the field of complex numbers), where 1_F denotes the unit element in F. In case the algebras involved are topological, by a morphism between two of them, we always mean a continuous one. For the rest of the terminology applied in the sequel, we refer to Ref. [**10**].

We are now in the position to state our main result, which also was the ultimate goal of the present note. For clarity's sake its formulation is given within the context of the preceding Corollary 2.1, however a more general statement in the framework of Lemma 2.1 could also be considered. Thus, we now have the following.

3.1. Theorem. *Let $(X, \mathcal{A})$ be a topological function algebra space whose local sections determine unital, complete, nuclear locally convex algebras, having locally equicontinuous generalized spectra, with respect to a unital complete topological algebra* F. *Moreover, let* E *be a unital, complete locally convex algebra with a locally equicontinuous generalized spectrum with respect to* F, *and finally let S be an $\mathcal{A}$-convex subset of X.*

Then, every central morphism $h \in Hom\,(\Gamma(S, \mathcal{A}\,\varepsilon\,\mathrm{E}), \mathrm{F})$ *can uniquely be expressed in the form*

$$h = \varrho \circ \delta_x , \tag{3.3}$$

where $\varrho \in Hom\,(\mathrm{E}, \mathrm{F})$ *and* δ_x *is the image of a point* $x \in S$ *under the generalized Dirac transform*

$$\delta : S \rightarrow Hom_s\,(\Gamma(S, \mathcal{A}\,\varepsilon\,\mathrm{E}), \mathrm{E}\,). \tag{3.4}$$

Moreover, ϱ *is also a central morphism. Finally, by applying the relation (3. 3) above, one obtains that the map*

$$\psi : Hom_s\,(\,\Gamma(S, \mathcal{A}\,\varepsilon\,\mathrm{E}), \mathrm{F}) \rightarrow Hom_s\,(\mathrm{E}, \mathrm{F}) \times S \tag{3.5}$$

defines a homeomorphism onto its range.

Proof: By (3. 2) one has the relation

$$\Gamma(\mathrm{S}, \mathcal{A}\,\varepsilon\,\mathrm{E}) = \varinjlim_{\mathrm{U} \supseteq \mathrm{S}} \Gamma(\mathrm{U}, \mathcal{A}\,\varepsilon\,\mathrm{E}), \tag{3.6}$$

where U ranges over a fundamental system of open neighborhoods of S in X. Hence, one obtains

$$\begin{aligned} \mathcal{H}om_s\,(\Gamma(\mathrm{S}, \mathcal{A}\,\varepsilon\,\mathrm{E}), \mathrm{F}) &= \mathcal{H}om_s\,(\varinjlim_{\mathrm{U} \supseteq \mathrm{S}} \Gamma(\mathrm{U}, \mathcal{A}\,\varepsilon\,\mathrm{E}), \mathrm{F}) \\ &= (\text{by Corollary 2.1})\ \mathcal{H}om_s\,(\varinjlim_{\mathrm{U} \supseteq \mathrm{S}} (\Gamma(\mathrm{U}, \mathcal{A}) \hat{\otimes}_{\varepsilon} \mathrm{E}), \mathrm{F}), \end{aligned} \tag{3.7}$$

so that, by Ref. [**10**; Part I, rel. (3. 13)], every element h in the first term of the preceding relations (3. 7) gives rise to a uniquely defined pair

$$(\rho, \phi) \in \mathcal{H}om_s\,(\mathrm{E}, \mathrm{F}) \times \mathcal{H}om_s\,(\Gamma(\mathrm{S}, \mathcal{A}), \mathrm{F}), \tag{3.8}$$

with $h = \rho \otimes \phi$. Now, since by hypothesis h is a central morphism, one concludes, by Ref. [**10**; Part I, Theorem 3. 2; cf., in particular, the rel. (3. 18)], that

$$\mathrm{Im}\,(\phi) \subseteq \mathrm{Im}\,(h) \cap (\,\mathrm{Im}\,(h)\,)' = \mathbb{C} \,.\, 1_{\mathrm{F}} \tag{3.9}$$

(where the «prime» in the last relation means the commutant subalgebra of Im (h) in F). Hence (ibidem; rel. (3. 16) and (3. 19)), the morphism $\phi \in \mathcal{H}om\,(\Gamma(\mathrm{S}, \mathcal{A}), \mathrm{F})$ is of the form

$$\phi = \chi \otimes 1_{\mathrm{F}}, \tag{3.10}$$

with $\chi \in \mathcal{M}\,(\Gamma(\mathrm{S}, \mathcal{A}))$, so that by the hypothesis for S, χ is uniquely determined by a point $x \in S$. Thus, by [**10**; Part I, rel. (3.13)] and the preceding, one gets a map $\psi : h \to (\rho, x)$, as in the rel. (3.5) in the statement of the theorem, which is a homeomorphism onto its range, defined by (3.5), as one concludes by the comments following the rel. (3.13) in Ref. [**10**]. Moreover, the map $\rho \in Hom(E, F)$ is also a central morphism by [**10**; Part I, Theorem 3.2]. Finally, concerning the relation (3.3), where δ_x is given by (3.4), in connection with the map (3.5), one obtains the desired result as a consequence of the following Lemma 3.1, and this will complete the proof of the theorem. ∎

That is, we also have.

3.1. Lemma. *With the hypothesis of the preceding Theorem 3.1, and the terminology applied therein, one concludes that: for every (section)*

$$\vec{f} \in \Gamma(S, A \varepsilon E), \tag{3.11}$$

the following relation is valid

$$h(\vec{f}) = \varrho(\vec{f}(x)), \tag{3.12}$$

where the pair (ϱ, x) is uniquely defined by (3.5) for any given central morphism $h \in Hom(\Gamma(S, A \varepsilon E), F)$, so that one actually obtains:

$$h = \varrho \circ \delta_x, \tag{3.13}$$

with the map δ_x given by (3.4).

Proof: Suppose that we have an element $\vec{f} \in \Gamma(U, A \varepsilon E) = \Gamma(U, A) \hat{\otimes}_{\varepsilon} E$ (Corollary 2.1), with $U \supseteq S$, open which is a decomposable tensor, i.e. assume that $\vec{f} = g \otimes \vec{\alpha}$, with $g \in \Gamma(U, A)$ and $\vec{\alpha} \in E$. Now, for every central morphism h, as in Theorem 3.1, one has $h = \rho \otimes \phi$, where the pair (ϕ, ρ) is defined by (3.8) and the map ϕ is given by (3.10). Hence, concerning the particular $\vec{f}$ considered, one obtains:

$$\begin{aligned} h(\vec{f}) &= (\phi \otimes \rho)(g \otimes \vec{\alpha}) = \phi(g) \cdot \rho(\vec{\alpha}) = \chi(g) \cdot 1_F \cdot \rho(\vec{\alpha}) = \\ &= \chi(g)\, \rho(\vec{\alpha}) = g(x)\, \rho(\vec{\alpha}) = \rho(g(x)\vec{\alpha}) = \rho(\vec{f}(x)), \end{aligned}$$

i.e. the relation (3.12), so that one actually gets

$$h(\vec{f}) = \rho(\delta_x(\vec{f})) = (\rho \circ \delta_x)(\vec{f}),$$

where δ_x is defined by (3.4) for the given $x \in S$, namely one has the relation (3.13), for $\vec{f}$ as above. Now, the general case follows immediately by continuity and linearity, and this finishes the proof of the lemma. ∎

A form of the preceding results for Banach algebra-valued analytic functions defined on $\mathbb{C}^n$ has been given in Ref. [**11**], those considerations being also the initial motive to the present context.

4. Applications. It is an easy consequence of the preceding argumentation to realize that the whole of Section 3 in Ref. [**10**; Part II], concerning results of the Runge type, is valid within the context of topological (function) algebra spaces, so that we may omit the details.

On the other hand, we can indicate, for instance, the respective extension and improvement as well of the rel. (3.3) of that Ref. in the sense of the following application of Corollary 2.1 above. That is, one has:

4.1. Lemma. *Let (X, A) be a topological algebra space, with X a Hausdorff regular space whose local sections determine unital complete nuclear locally convex algebras having locally equicontinuous spectra, and* E *a unital complete locally convex algebra with a locally equicontinuous spectrum. Moreover, let K be a closed subset of X, which admits a denumerable decreasing fundamental system of open neighborhoods, consisting of A-convex subsets of X. Then, concerning the spectra of the particular topological algebras considered, one has the relation:*

$$M(\Gamma(K, A \varepsilon \mathrm{E})) = K \times M(\mathrm{E}), \tag{4.1}$$

within a homeomorphism of the topological spaces involved.

Proof: Let $(\mathrm{U}_n)_{n \in \mathrm{N}}$ be a fundamental system of neighborhoods of K as in the statement, so that one has the relations:

$$\mathrm{K} = \bigcap_n \mathrm{U}_n = \lim_{\substack{\longleftarrow \\ n}} \mathrm{U}_n, \quad \text{with} \quad M(\Gamma(\mathrm{U}_n, A)) = \mathrm{U}_n, \ n \in \mathrm{N}, \tag{4.2}$$

within a homeomorphism of the topological spaces indicated. Now, one has, by definition:

$$\Gamma(\mathrm{K}, \mathcal{A}\varepsilon\mathrm{E}) = \varinjlim_{\mathrm{U}\supseteq\mathrm{K}} \Gamma(\mathrm{U}, \mathcal{A}\varepsilon\mathrm{E}) = (\text{by } (4.2)) \varinjlim_{n} \Gamma(\mathrm{U}_n, \mathcal{A}\varepsilon\mathrm{E})$$

$$= (\text{Corollary } 2.1) \varinjlim_{n} (\Gamma(\mathrm{U}_n, \mathcal{A}) \hat{\otimes}_{\varepsilon} \mathrm{E}),$$

so that one obtains:

$$\mathcal{M}(\Gamma(\mathrm{K}, \mathcal{A}\varepsilon\mathrm{E})) = \mathcal{M}\big(\varinjlim_{n} (\Gamma(\mathrm{U}_n, \mathcal{A}) \hat{\otimes}_{\varepsilon} \mathrm{E})\big)$$

$$= \varprojlim_{n} \mathcal{M}(\Gamma(\mathrm{U}_n, \mathcal{A}) \hat{\otimes}_{\varepsilon} \mathrm{E}) = \varprojlim_{n} \big(\mathcal{M}(\Gamma(\mathrm{U}_n, \mathcal{A})) \times \mathcal{M}(\mathrm{E})\big)$$

$$= \big(\varprojlim_{n} \mathcal{M}(\Gamma(\mathrm{U}_n, \mathcal{A}))\big) \times \mathcal{M}(\mathrm{E})$$

$$= (\text{by } (4.2)) (\varprojlim \mathrm{U}_n) \times \mathcal{M}(\mathrm{E}) = \mathrm{K} \times \mathcal{M}(\mathrm{E}),$$

the equalities involved being valid within a homeomorphism of the respective topological spaces, and this finishes the proof of the lemma. ∎

In connection with the preceding lemma, we also notice that the requirements set forth therein are obviously satisfied by a closed subset K of a complex analytic space $(X, \mathcal{O})$, with X second countable, such that K admits a fundamental system of open neighborhoods, which are Stein subspaces of X. (In this concern, cf. also [**7**; p. 303, Theorem 2.1]). In particular, it follows by the preceding proof and the rel. (3.1), (3, 2) that K is $\mathcal{A}$-convex.

We finally remark that within the preceding framework it seems to fit more naturally results referring to «infinite-dimensional holomorphy», where the underlying space X is no more locally compact (cf. also the relevant comments in Ref. [**12**]), and besides the topological algebras involved are not Fréchet ones or not even barrelled (cf., for instance, Ref. [**1**; § 5, Theorem 1]).

On the other hand, the same considerations included herewith, emphasize also the role, which the *topological algebra spectrum functor* has in a similar context concerning complex analytic spaces in a finite or infinite number of dimensions. In this respect, cf. also the recent work of M. Schottenloher [**14**].

ΠΕΡΙΛΗΨΙΣ

Ἓν ζεῦγος (X, A) ἀποτελούμενον ἀπὸ ἕνα τοπολογικὸν χῶρον (Hausdorff) X καὶ μίαν δέσμην (sheaf) τοπολογικῶν ἀλγεβρῶν [**8**], τῶν ὁποίων τὰ στοιχεῖα εἶναι ἰδιαιτέρως συνεχεῖς μιγαδικαὶ συναρτήσεις ἐπὶ τοῦ X, καλεῖται *χῶρος τοπολογικῶν συναρτησιακῶν ἀλγεβρῶν* (topological function algebra space). Κατὰ ταῦτα, ἔχομεν ὑπὸ ἀφῃρημένην ἔποψιν τὸ πλαίσιον τῆς θεωρίας τῶν μιγαδικῶν ἀναλυτικῶν χώρων, ἡ ὁποία ἀποτελεῖ καὶ τὴν βασικὴν ἀφορμὴν διὰ τὰς ἐκτεθειμένας σκέψεις εἰς τὴν παροῦσαν ἐργασίαν.

Οὕτω παρέχομεν μίαν συστηματικὴν ἐφαρμογὴν τῆς ἐννοίας τῶν δεσμῶν τοπολογικῶν ἀλγεβρῶν (αὐτόθι), προκειμένου νὰ λάβωμεν ἕνα γενικευμένον τύπον, δίδοντα τὴν «θήκην ὁλομορφίας» (: θήκην δομικῆς δέσμης) ἑνὸς καταλλήλου ὑποσυνόλου εἰς ἕνα χῶρον, ὡς ἀνωτέρω (πρβλ. τὰς σχέσεις (3. 3) καὶ (3. 12) εἰς τὰ προηγούμενα).

Τὰ ἀνωτέρω ἐπεκτείνουν, διὰ τὴν θεωρουμένην περίπτωσιν, τὴν κλασσικὴν ἔννοιαν τῆς θήκης ὁλομορφίας (envelope of holomorphy) ἑνὸς ὑποσυνόλου μιᾶς μιγαδικῆς ἀναλυτικῆς πολλαπλότητος (X, O), ἡ ὁποία δύναται νὰ ὁρισθῇ μέσῳ τοῦ φάσματος τῆς ἀντιστοίχου τοπολογικῆς ἀλγέβρας Γ(X, O), τὸ ὁποῖον καὶ ὑποδεικνύει ἐπίσης μίαν βασικὴν ἐφαρμογὴν τῆς τελευταίας ἐννοίας (σχετικῶς πρβλ. ἐπίσης [**4**], [**2**], καθὼς καὶ [**12**], [**14**]).

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Λαβὼν τὸν λόγον ὁ Ἀκαδημαϊκὸς κ. **Φίλων Βασιλείου** εἶπε τὰ ἑξῆς:

Κύριε Πρόεδρε,

Ἐργασίαν, συντεταγμένην ἀγγλιστί, τοῦ τακτικοῦ καθηγητοῦ τοῦ Πανεπιστημίου Ἀθηνῶν κ. Ἀναστασίου Μάλλιου, ἔχω τὴν τιμὴν νὰ ἀνακοινώσω εἰς τὴν Ἀκαδημίαν Ἀθηνῶν, ἐργασίαν ἔχουσαν τὸν συμπληρωμένον τίτλον «Γενικευμέναι θῆκαι (ἢ καλύμματα) δομικῶν δεσμῶν, χώρων τοπολογικῶν συναρτησιακῶν Ἀλγεβρῶν».

Καλοῦμεν χῶρον τοπολογικῶν συναρτησιακῶν Ἀλγεβρῶν κάθε ζεῦγος ἀποτελούμενον ἀπὸ ἕνα τοπολογικὸν χῶρον Hausdorff καὶ ἀπὸ μίαν δέσμην τοπολογικῶν Ἀλγεβρῶν, τῶν ὁποίων τὰ στοιχεῖα εἶναι, ἰδιαιτέρως, συνεχεῖς μιγαδικαὶ συναρτήσεις ἐπὶ τοῦ ὡς ἄνω τοπολογικοῦ χώρου. Οὕτω πως ἔχομεν, ὑπὸ ἀφῃρημένην μορφήν, τὸ πλαίσιον τῆς θεωρίας τῶν μιγαδικῶν ἀναλυτικῶν χώρων, θεωρίας ἀποτελούσης, κατὰ τὸν συγγραφέα, τὸν βασικὸν λόγον ἐξ οὗ ἀφωρμήθη οὗτος διὰ τὴν συγγραφὴν τῆς παρούσης ἀνακοινώσεως.

Ὁ κ. Μάλλιος παρέχει εἰς τὴν ἀνακοίνωσιν αὐτὴν συστηματικὴν ἐφαρμογὴν τῆς ἐννοίας τῶν δεσμῶν τοπολογικῶν Ἀλγεβρῶν, μὲ σκοπὸν ὅπως λάβῃ γενικευμένον τύπον, δίδοντα τὴν λεγομένην «θήκην ἢ κάλυμμα ὁλομορφίας (envelope of holomorphy)» ἢ «θήκην δομικῆς δέσμης», καταλλήλου ὑποσυνόλου τοῦ ὡς ἄνω θεωρουμένου τοπολογικοῦ χώρου.

Κατὰ τὴν παρατήρησιν τοῦ συγγραφέως, τὰ ἀνωτέρω ἐπεκτείνουν, διὰ τὴν θεωρουμένην περίπτωσιν, τὴν κλασσικὴν ἔννοιαν τῆς θήκης ὁλομορφίας ἑνὸς ὑποσυνόλου μιᾶς μιγαδικῆς ἀναλυτικῆς πολλαπλότητος. Ἡ πολλαπλότης αὕτη ἠμπορεῖ νὰ ὁρισθῇ μέσῳ τοῦ φάσματος ἀντιστοίχου τοπολογικῆς Ἀλγέβρας, φάσματος τὸ ὁποῖον καὶ ὑποδηλώνει, κατὰ τὸν συγγραφέα, βασικὴν ἐφαρμογὴν τῆς ἐννοίας τῆς θήκης ὁλομορφίας.

Διὰ περισσοτέρας λεπτομερείας, διὰ τὸ περιεχόμενον καὶ τὴν χρησιμοποιουμένην ἐδῶ ὁρολογίαν, παραπέμπομεν τὸν ἐνδιαφερόμενον εἰς τὰ Πρακτικὰ τῆς Ἀκαδημίας.

ΜΑΘΗΜΑΤΙΚΑ.— **On a problem of spectral synthesis in regular Banach Algebras,** *by E. Galanis**. Ἀνεκοινώθη ὑπὸ τοῦ Ἀκαδημαϊκοῦ κ. Φίλωνος Βασιλείου.

Let R be a semi-simple commutative, regular, self-adjoint Banach algebra with unit, with M_R as carrier space. Suppose G is a compact group, which operates continuously on M_R that is to say that for all $g \in G$ there exists a homeomorphism

$T_g : M_R \rightarrow M_R$ such that

(i) $T_g \circ T_h = T_{gh}$

(ii) T is continuous for the product topology $G \times M_R$.

Looking at (i) we see that no confusion will arise if we write $T_g x = gx$ $[g \in G,\ x \in M_R]$ and we shall do this. Now suppose $r \in R$. We shall define a function

$$r_g : M_R \rightarrow \mathbb{C} \quad \text{by} \quad r_g(x) = r(gx)$$

clearly $r_g \in C(M_R)$ but equally clearly it is not automatic that $r_g \in R$. Let us therefore impose the restriction $r_g \in R$ for all $r \in R$, $g \in G$. Clearly it is of interest to consider the fixed points of R under G that is to say to consider $R_1 = \{r \in R : r_g = r \text{ for all } g \in G\}$. R_1 is a Banach algebra under the $\|\cdot\|_R$ norm. The maximal ideal space (carrier space) is easily identified. Recall that the *orbit* of an element $x \in M_R$ is given by $O_x = \{gx : g \in G\}$.

By definition $R_1 = \{f \in R \text{ s.t } f|O_x \text{ is a constant for each } x\}$.

We can thus guess and quickly confirm (note e.g. that if $f \in R_1$ and $f|O_x \neq 0$ for all x then $f^{-1} \in R$ and $f^{-1}|_{O_x}$ is constant for each $x \in M_R$) that the maximal ideal space M_{R_1} of R_1 is $M_R | \sim = \{O_x : x \in M_{R_1}\}$ where $\sim$ is the equivalence relation *«belong to the same orbit»*. We note that the quotient and Gelfand topologies on M_{R_1} coincide so that no ambiguity arises in talking about the natural topology on M_{R_1}. This topology is the only one we shall consider. In particular the map $\tilde{\pi} : M_R \rightarrow M_{R_1}$, $\tilde{\pi} x = O_x$ is continuous.

* Ε. ΓΑΛΑΝΗ, **Περὶ ἑνὸς προβλήματος φασματικῆς συνθέσεως εἰς κανονικὰς Ἀλγέβρας Banach.**

Let R represented as an algebra of functions on its maximal ideal space M_R. For a closed set $E \subset M_R$ we denote by

$$I(E) = \{ f \in R;\ f^{-1}(0) \supset E \}$$

$$I_0(E) = \{ f \in R;\ f^{-1}(0) \text{ is a neighborhood of } E \}.$$

Definitions.

A closed set $E \subset M_R$ is called a *set of spectral synthesis* (an S-set) for the algebra R if $\overline{I_0(E)} = I(E)$.

A closed set $E \subset M_R$ is called a *Ditkin set* (a D-set), if for every $f \in I(E)$ there exists a sequence $g_n \in I_0(E)$ $n = 1, 2, \ldots$ such that $\lim_{n \to \infty} \| g_n f - f \|_R = 0$ clearly, every D-set is an S-set.

In this paper we shall prove the following theorems.

Theorem 1. Suppose E is a closed subset of M_{R_1}. Then E is of synthesis whenever $\tilde{\pi}^{-1}(E)$ is.

Remark. J. E. Björk gives in [**1**] a counter example which shows that the converse is false.

Theorem 2. Suppose E is a closed subset of M_{R_1}. Then E is Ditkin set whenever $\tilde{\pi}^{-1}(E)$ is.

Remark. We do not know if the converse of the *theorem-2* is true or false.

We define 2 mappings which by analogy with those of Herz we call the P and M mappings.

Lemma 1. (i) If $f \in R_1$ then writing $Mf(x) = f(\tilde{\pi} x)$ for all $x \in M_R$ we have $Mf \in R$.

(ii) If $x \sim y$, $r \in R$ then $\int r(gx)\, d\mu(g) = \int r(gy)\, d\mu(g)$ (where μ is a Haar measure on G) so that $P_r(\tilde{\pi} x) = \int r(gx)\, d\mu(g)$ is well defined.

(iii) Taking P_r as in (ii) we have $P_r \in R_1$.

(iv) If $f_1 \in R_1$, $f \in R$ we have $P(f_1 f) = f_1 P f$.

Proof: (i) Follows from the definition of R_1.

(ii) We have $x = hy$ for some $h \in G$ and so
$\int r(gx)\, d\mu(g) = \int r(ghy)\, d\mu(g) = \int r(gy)\, d\mu(g)$ (since is right invariant).

(iii) Observe that by elementary results on integration in Banach algebras $f = \int r_g\, d\mu(g)$ and $r_{g_a} \to r_g$ whenever $g_a \to g$ and by (ii) f is constant in orbits. (This is where we need the conditions $r_g \in R$ for all $r \in R$, $g \in G$).

(iv) Obvious from the definition.

Taking P and M as above we thus have mappings $M : R_1 \to R$, $P : R \to R_1$.

Lemma 2. (v) M is an isometry
(vi) $\| P \| \leqslant 1$
(vii) $P \circ M = I\partial_{R_1}$

Proof: (v) Is obvious.

(vi) $\| \int r_g\, d\mu(g) \| \leqslant \int \| r_g \|_R\, d\mu(g) = \int \| r \|_R\, d\mu(g) = \| r \|_R$. So that $\| P_r \|_{R_1} \leqslant \| r \|_R$.

(vii) $(P \circ M(r))\,(\widetilde{\pi} x) = \int r\,(\widetilde{\pi}(gx))\, d\mu(g) = \int r\,(\widetilde{\pi} x)\, d\mu(g) = r\,(\widetilde{\pi} x)$, for all $r \in R_1$, $\widetilde{\pi} x \in M_{R_1}$.

The key property of P is that it is local in the following sence.

Lemma 3. If $r \in R$ then $\mathrm{supp}\,(P_r) \subset \widetilde{\pi}\,(\mathrm{supp}\ r)$.

Proof: Suppose Q is an open set in R then gQ is open (since T_g is an homeomorphism) and so $\{x : x \sim y,\ y \in Q\} = \bigcup_{g \in G} gQ$ is open. Thus $\widetilde{\pi}$ is an open mapping.

The set $\widetilde{\pi}\,(\mathrm{supp}\ r)$ is closed in $M_R \sim$ and given any $x \notin \widetilde{\pi}\,(\mathrm{supp}\ r)$ we can find an open neighborhood V of x with $V \cap \widetilde{\pi}\,(\mathrm{supp}\ r) = \varnothing$. Suppose $\widetilde{\pi} y \in .V$ Then $\widetilde{\pi} y \notin \widetilde{\pi}\,(\mathrm{supp}\, r)$ i.e. $y \not\sim x$ for all $x \in \mathrm{supp}\, r$ in other words $r_g(y) = r(gy) = 0$ for all $g \in G$. In particular $P_r(\widetilde{\pi} y) = \int r_g(y)\, d\mu(g) = 0$. Then lemma follows.

Proof of Theorem 1. Suppose now $f \in I(E)$. Then $Mf \in I(\widetilde{\pi}^{-1}(E))$. Since $\widetilde{\pi}^{-1}(E)$ is of synthesis we can find $g_1, g_2, \ldots \in R$ such that

$$\text{(i)} \quad g_n \in I_o(\widetilde{\pi}^{-1}(E))$$

$$\text{(ii)} \quad \| g_n - Mf \|_R \xrightarrow[n \to \infty]{} 0$$

Let us write $f_n = Pg_n$ so that $f_n \in R_1$. By *lemma 2* we know (i)′ f_n is zero in a neighborhood of $E = \tilde{\pi}\tilde{\pi}^{-1}(E)$ and by *lemma 2* (ii)′ $\| f_n - f \|_{R_1} = \| Pg_n - PMf \|_{R_1} \leqslant \| g_n - Mf \|_R \xrightarrow[n \to \infty]{} 0$. This completes the proof.

Proof of Theorem 2. Let $f \in I(E)$. Then $Mf \in I(\tilde{\pi}^{-1}(E))$ and therefore since $\tilde{\pi}^{-1}(E)$ is Ditkin, there exists a sequence $(g_n) \in I_0(\tilde{\pi}^{-1}(E))$, $n = 1, 2, \ldots$ Such that

$$\| g_n Mf - Mf \|_R \xrightarrow[n \to \infty]{} 0 .$$

Therefore

$$\| P(g_n Mf) - P(Mf) \|_{R_1} \xrightarrow[n \to \infty]{} 0$$

which by (iv) of *lemma 1* and (vii) of *lemma 2* is equivalent to

$$\| (Pg_n) f - f \|_R \xrightarrow[n \to \infty]{} 0 .$$

But *lemma 3* implies $Pg_n \in I_0(E)$ and this completes the proof.

ΠΕΡΙΛΗΨΙΣ

Θεωροῦμεν μίαν ἡμιαπλῆν, ἀντιμεταθετικήν, ὁμαλὴν καὶ αὐτοσυζυγῆ ἄλγεβραν τοῦ Banach R, τῆς ὁποίας ὁ χῶρος τῶν μεγίστων ἰδεωδῶν ἔστω M_R. Ὑποθέτομεν ὅτι ἡ συμπαγὴς ὁμὰς G δρᾷ συνεχῶς ἐπὶ τῆς R καὶ ὁρίζομεν τὴν συνάρτησιν $r_g : M_R \to \mathbb{C}$ διὰ τῆς σχέσεως $r_g(x) = r(gx)$ $[g \in G,\ x \in M_R]$.

Θεωροῦμεν τὴν ἄλγεβραν $R_1 = \{ r \in R : r_g = r,\ \forall g \in G \}$, ἡ ὁποία μὲ τὴν νὸρμ τῆς R εἶναι ἐπίσης μία ἄλγεβρα τοῦ Banach, μὲ χῶρον μεγίστων ἰδεωδῶν, ἔστω M_{R_1}. Θεωροῦμεν τὴν συνεχῆ ἀπεικόνισιν $\tilde{\pi} : M_R \to M_{R_1}$, ὁριζομένην διὰ τῆς σχέσεως $\tilde{\pi} x = O_x$, ὅπου $O_x = \{ gx : g \in G \}$ ἡ τροχιὰ τοῦ στοιχείου $x \in M_R$.

Εἰς τὴν παροῦσαν ἐργασίαν ἀποδεικνύομεν ὅτι, ἂν $E \subset M_{R_1}$ εἶναι κλειστὸν ὑποσύνολον τοῦ M_{R_1} καὶ $\tilde{\pi}^{-1}(E)$ εἶναι ἁρμονικῆς συνθέσεως (ἀντιστ. Ditkin), τότε καὶ τὸ E εἶναι ἁρμονικῆς συνθέσεως (ἀντιστ. Ditkin).

REFERENCES

1. J. E. Björk, Compact Groups Operating on Banach Algebras, Math. Ann. 205 (1973), 281 - 297.
2. N. Th. Varopoulos, Tensor Algebras and Harmonic Analysis, Acta Math. 119 (1967), 51 - 112.

★

Ὁ Ἀκαδημαϊκὸς κ. **Φίλων Βασιλείου**, παρουσιάζων τὴν ἀνωτέρω ἀνακοίνωσιν, εἶπε τὰ ἑξῆς:

Ἔχω τὴν τιμὴν νὰ ἀνακοινώσω εἰς τὴν Ἀκαδημίαν Ἀθηνῶν ἐργασίαν, συντεταγμένην ἀγγλιστί, τοῦ κ. Εὐστρατίου Γαλανῆ, διδάκτορος τοῦ Πανεπιστημίου Cambridge (Ἀγγλίας) καὶ προσφάτως ἐκλεγέντος ἐπικουρικοῦ καθηγητοῦ τοῦ Ε. Μ. Πολυτεχνείου, ἐργασίαν ἔχουσαν τὸν ἀνεπτυγμένον τίτλον «Περὶ ἑνὸς προβλήματος ἁρμονικῆς ἢ φασματικῆς συνθέσεως εἰς κανονικὰς Ἀλγέβρας Banach».

Ἡ ἔννοια Ἀλγέβρας Banach προέκυψεν, ὡς γνωστόν, ἀπὸ τὴν ἐφαρμογὴν τῶν θεμελιωδῶν ἐργασιῶν τοῦ μαθηματικοῦ S. Banach εἰς τὴν Συναρτησιακὴν Ἀνάλυσιν. Ὡς βάσις τῆς ἐν λόγῳ ἐννοίας ἐχρησίμευσεν ἡ ἔννοια τοῦ μετρικοῦ χώρου, εἰσαχθεῖσα τὸ 1906 ἀπὸ τὸν Γάλλον μαθηματικὸν M. Frechet. Ἐξ ἄλλου οἱ μετρικοὶ χῶροι θεμελιοῦνται ἐπὶ τῆς ἐννοίας τῶν λεγομένων διανυσματικῶν χώρων. Ἰδιαιτέρως, καλοῦμεν εἰς τὰ Μαθηματικὰ χῶρον Banach, διανυσματικὸν χῶρον, πεπερασμένης ἢ μὴ διατάξεως, ὡς πρὸς τὸ σῶμα τῶν πραγματικῶν ἢ τῶν μιγαδικῶν ἀριθμῶν, χῶρον διὰ τὸν ὁποῖον ἰσχύουν κατ' ἀρχὰς τὰ ἑξῆς: Ὑπάρχει μονοσήμαντος ἀπεικόνισις, συντόμως γενικευμένον μέτρον ἢ norm, τοῦ ὑπ' ὄψιν διανυσματικοῦ χώρου, εἰς τὸ σύνολον τῶν πραγματικῶν ἀριθμῶν, ἀπεικόνισις τοιαύτη, ὥστε: 1) ἡ εἰκὼν κάθε στοιχείου τοῦ διανυσματικοῦ χώρου νὰ μὴ εἶναι ἀρνητική, 2) ἡ εἰκὼν στοιχείου τοῦ ἐν λόγῳ χώρου τότε καὶ μόνον νὰ εἶναι μηδέν, ὅταν τὸ ἀρχέτυπόν του εἶναι τὸ μηδενικὸν στοιχεῖον τοῦ διανυσματικοῦ χώρου, 3) ἡ εἰκὼν τοῦ γινομένου τυχόντος βαθμωτοῦ ἐπὶ τυχὸν στοιχεῖον τοῦ χώρου, νὰ εἶναι τὸ γινόμενον τῆς ἀπολύτου τιμῆς τοῦ βαθμωτοῦ ἐπὶ τὴν εἰκόνα τοῦ θεωρουμένου στοιχείου τοῦ διανυσματικοῦ χώρου, 4) νὰ ἰσχύῃ ἡ λεγομένη «τριγωνικὴ ἀνισότης». Τὸ ἐν λόγῳ γενικευμένον μέτρον (norm) παρέχει «ἀπόστασιν» διὰ κάθε ζεῦγος στοιχείων (χ, ψ) τοῦ διανυσματικοῦ χώρου, ἔχουσαν τὰς ἰδιότητας: Ἡ ἀπόστασις τοῦ ζεύγους (χ + ζ, ψ + ζ), διὰ τυχὸν ζ τοῦ διανυσματικοῦ χώρου, νὰ εἶναι ἡ ἀπόστασις τοῦ ζεύγους (χ, ψ) καὶ ἡ ἀπόστασις τοῦ ζεύγους (λχ, λψ), διὰ τυχὸν βαθμωτὸν λ, νὰ εἶναι ἡ ἀπόλυτος τιμὴ τοῦ λ ἐπὶ τὴν ἀπόστασιν τοῦ ζεύγους (χ, ψ). Πρόδηλον εἶναι ὅτι ὁ ἔχων γενικευμένον μέτρον χῶρος, ἠμπορεῖ νὰ θεωρηθῇ πάντοτε ὑπὸ τὴν ἀναφερθεῖσαν ἔννοιαν ὡς «μετρικὸς χῶρος» διὰ τὴν θεωρηθεῖσαν ἀπόστασιν. Ἐκ τῶν χώρων τούτων «πλήρεις» εἶναι ἐκεῖνοι, διὰ τοὺς ὁποίους κάθε ἀκολουθία Cauchy στοιχείων τοῦ χώρου εἶναι συγκλίνουσα εἰς στοιχεῖον τοῦ ἰδίου χώρου. Μὲ τὴν ἰδιότητα τοῦ πλήρους, ὡς 5ην συνθήκην, προστιθεμένην εἰς τὰς ἀναφερθείσας τέσσαρας διὰ τὸ γενικευμένον μέτρον συμπληρώνομεν τὸν ὁρισμὸν τοῦ χώρου Banach.

Ὁ συγγραφεὺς τῆς παρούσης ἀνακοινώσεως ἀναχωρεῖ ἀπὸ μίαν, οὕτω λεγομένην, ἡμιαπλῆν ἀντιμεταθετικήν, ὁμαλὴν καὶ αὐτοσυζυγῆ Ἄλγεβραν Banach, καθὼς ἐπίσης καὶ ἀπὸ τὸν χῶρον τῶν μεγίστων αὐτῆς ἰδεωδῶν. Ἀπὸ τὴν Ἄλγεβραν αὐτὴν μορφώνει μὲ τὸ αὐτὸ γενικευμένον μέτρον ἑτέραν Ἄλγεβραν, ἀποδεικνυομένην ἐπίσης ὡς Ἄλγεβραν Banach. Ὁ συγγραφεὺς θεωρεῖ ἔπειτα κατάλληλον συνεχῆ ἀπεικόνισιν τοῦ χώρου τῶν μεγίστων ἰδεωδῶν τῆς πρώτης Ἀλγέβρας εἰς τὸν χῶρον τῶν μεγίστων ἰδεωδῶν τῆς δευτέρας, βάσει δὲ τῶν δεδομένων αὐτῶν ἀποδεικνύει ὅτι κλειστὸν ὑποσύνολον τοῦ χώρου τῶν μεγίστων ἰδεωδῶν τῆς ὡς ἄνω δευτέρας Ἀλγέβρας, ἀντιστοιχοῦν εἰς ὑποσύνολον ἁρμονικῆς συνθέσεως τοῦ ἀπεικονιζομένου χώρου, εἶναι ἐπίσης ἁρμονικῆς συνθέσεως.

Λεπτομερείας ἐπὶ τῆς ἀποδείξεως, ὡς ἐπίσης καὶ τῆς εἰς αὐτὴν χρησιμοποιουμένης ὁρολογίας, θέλει εὕρει ὁ ἐνδιαφερόμενος εἰς τὰ Πρακτικὰ τῆς Ἀκαδημίας.