Example: As an example we take the time-intensity rainfall curve in Athens area, which is found to be:

$$i = \frac{40}{20+t} \text{ in mm/minute.}$$

Assuming that $h_a = 6$ m/m and that usually the expected unfavourable duration time of rainfall, for the design of storm drains, is about 25′ we get:

$$t_a = 6\,\frac{40}{40} = 6'.$$

Adding to this value the above accepted minimum inlet time $t_i + t_a = - 12'$, which should correspond to the values usually accepted for the inlet time (to in textbooks), and given in the table I. It follows therefrom that in the majority of cases of table I the inlet time has been selected rather small, while the values suggested in textbooks should be limited to the lower value ($t = 15'$); consequently the values between 15′-20′ should be considered rather high for greater cities. (Thickly built up).

Γ. ΑΡΒΑΝΙΤΑΚΗ.— *Τὸ σύγχρονον ρωσικὸν ἡμερολόγιον.*

Μ. ΚΑΛΟΜΟΙΡΗ. — *Παρουσίασις μουσικοῦ δράματός του «Ἀνατολή» καὶ ἀνάλυσις μερικῶν τεχνικῶν καινοτομιῶν εἰς τὸ μουσικὸν δρᾶμα.*

ΜΑΘΗΜΑΤΙΚΗ ΑΝΑΛΥΣΙΣ.— **Generalization of some linear differential equations of Mathematical Physics**, *by Spyridon B. Sarantopoulos**. Ἀνεκοινώθη ὑπὸ κ. Παν. Ζερβοῦ.

1.— It is known[1] that all the linear differential equations which occur in certain branches of Mathematical Physics are confluent forms of a differential equation of the second order which has every point except a_1, a_2, a_3, a_4 and ∞ as an ordinary point. These five points are regular points and the difference of the two exponents α_σ, β_σ at a_σ ($\sigma = 1, 2, 3, 4$) and of the two exponents at ∞ is $^1/_2$. Such confluent forms are the linear differential equations of Legendre, Bessel, Stokes, Weber, Hermite, Mathiew and Lamé.

In a work which I hope to be published in a short time, I make some generalization of certain of these equations. I present here only some rusalts of my research.

* ΣΠΥΡΙΔΩΝΟΣ Β. ΣΑΡΑΝΤΟΠΟΥΛΟΥ, Γενίκευσις γραμμικῶν τινων διαφορικῶν ἐξισώσεων τῆς Μαθηματικῆς Φυσικῆς.

[1] See E. T. WITTAKER, Modern Analysis, 1920, p. 203.

The Legendre's equation

$$(1) \qquad (1-z^2)\frac{d^2u}{dz^2}-2z\frac{du}{dz}+\eta(\eta+1)u=0,$$

when η is a positive integer, is satisfied by the Legendre polynomial which is given by the Rodrigues' formula

$$(2) \qquad P_\eta(z)=\frac{1}{2^\eta.\eta!}\cdot\frac{d^\eta(z^2-1)^\eta}{dz^\eta},$$

or also by the Schläfli's integral formula

$$(3) \qquad P_\eta(z)=\frac{1}{2\pi i}\int_c \frac{(t^2-1)^\eta}{2^\eta(t-z)^{\eta+1}}dt,$$

where c is a contour which encircles the point z once counter-clockwise. The expression (3) is a solution of the differential equation (1) even when η is not a positive integer provided that c is a contour such that $(t^2-1)^{\eta+1}(t-z)^{-\eta-2}$ resumes its original value after describing c.

2.— I have proved the following more general proposition.

The linear differential equation of the μ order

$$(4) \qquad \sum_{\lambda=0}^{\mu}\Bigg[-f(z)\,\sigma_1^{(\lambda)}(z)\mathcal{E}_\mu^\lambda+\sum_{\nu=1}^{\lambda}\Delta_{\eta+\mu-\nu+1}^{\nu-1}\mathcal{E}_{\mu-\nu}^{\lambda-\nu}\Bigg[\frac{(\eta+1)\nu-(\eta+\mu)}{\nu!}\overset{(\nu)}{f}(z)+$$

$$+\frac{[f(z)\varphi'(z)]^{(\nu-1)}}{(\nu-1)!}\Bigg]\overset{(\lambda-\nu)}{\sigma_1}(z)\Bigg]\frac{d^{\mu-\lambda}u}{dz^{\mu-\lambda}}=0,$$

where it is

$$(5) \qquad \mathcal{E}_\mu^\lambda=\frac{\mu(\mu-1)\ldots(\mu-\lambda+1)}{1.2.3\ldots\lambda},\ \Delta_{\eta+1}^{\varrho}=(\eta+1)(\eta+2)\ldots(\eta+\varrho),\ \Delta_{\overline{\eta+1}}^{0}=1,$$

$$\eta+1\neq 0,\ (\mu+\eta\neq 0),$$

$$(6) \qquad f(z)=\alpha_0 z^{\mu+1}+\alpha_1 z^\mu+\ldots+\alpha_\mu z+\alpha_{\mu+1},$$

$$(7) \qquad \varphi(z)=\int_{z_0}^{z}\frac{a_0z^\mu+a_1z^{\mu-1}+\ldots+a_{\mu-1}z+a_\mu}{\alpha_0z^{\mu+1}+\alpha_1z^\mu+\ldots+\alpha_\mu z+\alpha_{\mu+1}}dt,$$

$$(8) \qquad (\mu\eta+1)\alpha_0+a_0=0,$$

$\alpha_0, \alpha_1, \ldots, \alpha_{\mu+1}, a_1, a_2, \ldots, a_\mu$, η *are arbitrary numbers, and* $\sigma_1(z)=\frac{1}{\sigma(z)}$ *an arbitrary function, it satisfied by the expression*

$$(9) \qquad u=\frac{\sigma(z)}{2\pi i\alpha}\int_c\frac{e^{\varphi(t)}[f(t)]^\eta}{(t-z)^{\eta+1}}dt,\quad (\alpha=\text{arbitrary constant}),$$

where c, *the contour of integration, is either a closed contour in the* t-*plane such that the function* $\Phi(t)$:

$$\Phi(t) = \frac{e^{\varphi(t)}\,[f(t)]^{\eta+1}}{(t-z^{\eta+\mu}} \tag{10}$$

resumes its initial value after t *has described it, or else is a simple curve such that* $\Phi(t)$ *has the same value at its termini.*

3.—We can distinguish different particular cases of the differential equation (4). We mention immediately below some of them.

1° Suppose that $\sigma(z)=1$. Then the differential equation (4) reduces to the forme

$$-f(z)\frac{d^{\mu}u}{dz^{\mu}} + \sum_{\nu=1}^{\mu} \Delta^{\nu-1}_{\eta+\mu-\nu+1}\left[\frac{(\eta+1)\nu-(\eta+\mu)}{\nu!}f^{(\nu)}(z) + \frac{[f(z)\,\varphi'(z)]^{(\nu-1)}}{(\nu-1)!}\right]\frac{d^{\mu-\nu}u}{dz^{\mu-\nu}}=0 \tag{11}$$

I. If we take $f(z)=z^{\mu+1}-1$, and $\varphi(z)=$constant (e. g. $\varphi(z)=0$), we must have $\eta = -\frac{1}{\mu}$ and the differential equation (11) becomes

$$(1-z^{\mu+1})\frac{d^{\mu}u}{dz^{\mu}} + \sum_{\nu=1}^{\mu} \mathcal{L}^{\nu}_{\mu}\,\Delta^{\nu-1}_{\eta+\mu-\nu+1}\left[(\eta+1)\nu-(\eta+\mu)\right]z^{\mu-\nu+1}\frac{d^{\mu-\nu}u}{dz^{\mu-\nu}}=0 \tag{12}$$

and this latter is satisfied by

$$u=\frac{1}{2\pi i\alpha}\int_{c}\frac{(t^{\mu+1}-1)^{-\frac{1}{\mu}}}{(t-z)^{1-\frac{1}{\mu}}}\,dt. \tag{13}$$

II. By putting $f(z)=z^{\mu}-1$ and $\varphi(z)=0$, the equation (11) becomes

$$(1-z^{\mu})\frac{d^{\mu}u}{dz^{\mu}} + \sum_{\nu=1}^{\mu} \mathcal{L}^{\nu}_{\mu}\,\Delta^{\nu-1}_{\eta+\mu-\nu+1}\left[(\eta+1)\nu-(\eta+\mu)\right]z^{\mu-\nu}\frac{d^{\mu-\nu}u}{dz^{\mu-\nu}}=0 \tag{14}$$

and its solution is now

$$u=\frac{1}{2\pi i\alpha}\int_{c}\frac{(t^{\mu}-1)^{\eta}}{(t-z)^{\eta+1}}\,dt \tag{15}$$

For $\mu=2$ the differential equations (12) and (14) reduce respectively to the equations:

$$(1-z^{3})\frac{d^{2}u}{dz^{2}}-3z^{2}\frac{du}{uz}-\frac{3z}{4}u=0,\ \text{where}\ u=\frac{1}{2\pi i\alpha}\int_{c}\frac{(t^{3}-1)^{-\frac{1}{2}}}{(t-z)^{\frac{1}{2}}}\,dt; \tag{16}$$

$$(17) \quad (1-z^2)\frac{d^2u}{dz^2} - 2z\,\frac{du}{dz} + \eta(\eta+1)\,u = 0, \text{ where } u = \frac{1}{2\pi i a}\int_c \frac{(t^2-1)^{\eta}}{(t-z)^{\eta+1}}\,dt.$$

The latter of them is the differential equation (1) of Legendre.

2° The Riemann's P — equation

$$u = P\begin{Bmatrix} \alpha & \beta & \gamma & \\ \alpha' & \beta' & \gamma' & z \\ \alpha'' & \beta'' & \gamma'' & \end{Bmatrix},$$

that is the linear differential equation of second order

$$(18) \qquad \frac{d^2u}{dz^2} + \left[\frac{1-\alpha'-\alpha''}{z-\alpha} + \frac{1-\beta'-\beta''}{z-\beta} + \frac{1-\gamma'-\gamma''}{z-\gamma}\right]\frac{du}{dz} +$$

$$\left[\frac{\alpha'\alpha''(\alpha-\beta)(\alpha-\gamma)}{z-\alpha} + \frac{\beta'\beta''(\beta-\alpha)(\beta-\gamma)}{z-\beta} + \frac{\gamma'\gamma''(\gamma-\alpha)(\gamma-\beta)}{z-\gamma}\right]\cdot\frac{u}{(z-\alpha)(z-\beta)(z-\gamma)} = 0$$

which has three and only three regular points α, β, γ with exponents (α', α''), (β', β''), (γ', γ''), being subject to the condition $\alpha'+\alpha''+\beta'+\beta''+\gamma'+\gamma''=1$, and which can be satisfied, as it is known, by an integral of the type

$$(19) \quad u=(z-\alpha)^{\alpha''}(z-\beta)^{\beta''}(z-\gamma)^{\gamma''}\int_c (t-\alpha)^{\beta''+\gamma''+\alpha'-1}(t-\beta)^{\gamma''+\alpha''+\beta'-1}(t-\gamma)^{\alpha''+\beta''+\gamma'-1}(t-z)^{-\alpha''-\beta''-\gamma''}\,dt$$

provided that the contour c of integration is suitably chosen, is a particular case of the equation (4) where $\mu=2$; hence the solution (19) can take the form (9). Really, the reader can be easily persuaded about it, directly after some calculations, putting

$$f(z)=(z-\alpha)(z-\beta)(z-\gamma)$$

$$\varphi(z)=(\alpha'=\alpha'')\,l(z-\alpha)+(\beta'-\beta'')\,l(z-\beta)+(\gamma'-\gamma'')\,l(z-\gamma)$$

$$\sigma(z)=2\pi i a\,(z-\alpha)^{\alpha''}(z-\beta)^{\beta''}(z-\gamma)^{\gamma''}$$

$$\eta=-\alpha'-\beta'-\gamma'.$$

4.—When $\eta+\mu=0$, we have to consider instead of the equation (4) the following:

$$(20) \quad A(z)\sigma_1(z)\frac{d^{\mu}u}{dz^{\mu}} + \sum_{\lambda=1}^{\mu}\left[\mathcal{L}^{\lambda}_{\mu}A(z)\sigma_1^{(\lambda)}(z) + \sum_{\nu=1}^{\nu=\lambda}\mathcal{L}^{\lambda-\nu}_{\mu-\nu}\sigma_1^{(\lambda-\nu)}P^{(\nu-1)}(z)(-1)^{\nu-1}\right]\frac{d^{\mu-\lambda}u}{dz^{\mu-\lambda}}=0,$$

$$\sigma_1(z)=\frac{1}{\sigma(z)}$$

where $A(z)$ denotes an arbitrary function and $P(z)$ a polynomial:

$$(21) \qquad P(z)=\alpha_1 z^{\mu-1}+\alpha_2 z^{\mu-2}+\ldots+\alpha_{\mu}$$

at most of $\mu-1$ degree. Its solution is

$$(22) \qquad u = \frac{\sigma(z)}{2\pi i \alpha} \int_c e^{\int \frac{P(t)}{f(t)} dt} \cdot \frac{(t-z)^{\mu-1}}{f(t)} dt$$

where f(t) is an arbitrary continuous function. The equation (20), if $\sigma(z)=1$, becomes

$$(23) \quad A(z)\frac{d^{\mu}u}{dz^{\mu}} + P(z)\frac{d^{\mu-1}u}{dz^{\mu-1}} + \cdots + (-1)^{\lambda-1} \overset{(\lambda-1)}{P(z)} \frac{d^{\mu-\lambda}u}{dz^{\mu-\lambda}} + \cdots + (-1)^{\mu-1} \overset{(\mu-1)}{P(z)}u = 0 .$$

The general solution of this differential equation (23) can be found by means of the formula (22) (where $\sigma(z)=1$). This is

$$(24) \qquad u = K_1\,\omega_1 + K_2\,\omega_2 + \cdots + K_{\mu-1}\,\omega_{\mu-1} +$$

$$+ K_{\mu}\left[\int\left[\int\left[\cdots\left[\int\left[\int B e^{-\int \frac{P(z)}{A(z)}dz} dz\right]\omega_{1,\mu-1}\,dz\right]\cdots\right]\omega_{1,2}\,dz\right]\omega_{1,1}\,dz\right]\omega_1$$

where $\omega_{\nu} = \mathcal{L}^{\nu}_{\mu-1}\alpha_1 z^{\mu-\nu} + (-1)^{\mu-\nu+1}\alpha_{\mu-\nu+1}$, $\omega_{1,0} = \omega_1$, $\left(\frac{\omega_{i,\nu-1}}{\omega_{1,\nu-1}}\right)' = \omega_{i-1,\nu}$,

$$(i=2, 3, \ldots, \mu-\nu+1), \qquad (\nu=1, 2, \ldots, \mu-1),$$

$B^{-1} = \omega_{1,0}^{\mu}\,\omega_{1,1}^{\mu-1} \cdots \omega_{1,\mu-1}^{2}$ and $K_1, K_2, \ldots, K_{\mu}$ arbitrary constants. Consequently the product of the right member of (24) by $\sigma(z)$ is the general solution of the linear differential equation (30).

ΠΕΡΙΛΗΨΙΣ

Ὁ συγγραφεὺς γενικεύων τὴν διαφορικὴν ἐξίσωσιν (1) τοῦ Legendre διατυπώνει τὴν ἑπομένην πρότασιν.

Ἡ γραμμικὴ διαφορικὴ ἐξίσωσις (4), μ τάξεως, ἔνθα ἡ ἔννοια τῶν συμβόλων $\mathcal{L}^{\lambda}_{\mu}$, $\Delta^{\varrho}_{\eta+1}$ ὁρίζεται ὑπὸ τῶν τύπων (5) καὶ αἱ συναρτήσεις f(z), φ(z) ὑπὸ τῶν τύπων (6), (7), (8) εἰς τοὺς ὁποίους τὰ $\alpha_0, \alpha_1, \ldots, \alpha_{\mu+1}, a_1, a_2, \ldots, a_{\mu}$, η εἶναι οἱοιδήποτε δοθέντες ἀριθμοί, $(\mu+\eta \neq 0)$, καὶ ἡ συνάρτησις $\sigma_1(z) = 1$: $\sigma(z)$ αὐθαίρετος, ἐπαληθεύεται ὑπὸ τοῦ ὁλοκληρώματος (9), εἰς ὃ ἡ καμπύλη c τῆς ὁλοκληρώσεως εἶναι εἴτε μία κλειστὴ καμπύλη τοῦ t— ἐπιπέδου, τοιαύτη ὥστε ἡ συνάρτησις Φ(t), ἡ (10), νὰ λαμβάνῃ ἐκ νέου τὴν ἀρχικὴν αὐτῆς τιμήν, ἀφοῦ τὸ t γράψῃ ὁλόκληρον ταύτην, εἴτε ἄλλως μία ἁπλῆ καμπύλη τοιαύτη, ὥστε ἡ Φ(t) νὰ λαμβάνῃ τὴν αὐτὴν τιμὴν εἰς τὰ ἄκρα ταύτης, (τῆς c).

Ὁ γνωστὴ διαφορικὴ ἐξίσωσις P—Riemann, δηλαδὴ ἡ (18), προκύπτει τώρα ὡς μερικὴ περίπτωσις τῆς (4) διὰ $\mu=2$ καὶ συνεπῶς ἡ λύσις αὐτῆς (19) δύναται νὰ τεθῇ ὑπὸ τὴν ἁπλουστέραν μορφὴν (9).

Τέλος ἡ περίπτωσις καθ' ἣν $\mu+\eta=0$, ἥτις ἐκφεύγει τῆς ἀνωτέρω γενικῆς προτάσεως, ἄγει εἰς τὴν διαφορικὴν ἐξίσωσιν (20), ἤ, ἀκόμη ἁπλούστερον, (διὰ $\sigma(z)=1$), εἰς τὴν (23), ἔνθα A(z) εἶναι τυχοῦσα συνάρτησις καὶ P(z) πολυώνυμόν τι τὸ πολὺ $\mu-1$ βαθμοῦ. Ταύτης ἡ γενικὴ λύσις παρέχεται ὑπὸ τοῦ τύπου (24), ἐξ οὗ προκύπτει ἀμέσως καὶ ἡ γενικὴ λύσις τῆς (20).