Ἐπειδὴ ἀπὸ δεκαετίας καὶ πλέον λαμβάνω εἰς τὰ Ἡνωμένα Ἔθνη ἐνεργὸν μέρος εἰς τὰ θέματα τῆς ἐφαρμογῆς τῶν περὶ ναρκωτικῶν διεθνῶν συμβάσεων, ἐνόμισα, ὅτι μοὶ ἦτο ἐπιβεβλημένον νὰ ἀνακοινώσω ἐνταῦθα τὰς σκέψεις μου περὶ τῆς σημασίας τοῦ ὅρου «Embargo».

ΒΥΖΑΝΤΙΝΗ ΤΕΧΝΗ.—**Αἱ βυζαντιναὶ τοιχογραφίαι τῆς Τραπέζης τῆς Μονῆς Ἁγ. Ἰωάννου τοῦ Θεολόγου τῆς Πάτμου,** *ὑπὸ Ἀναστ. Κ. Ὀρλάνδου.*

ΑΝΑΚΟΙΝΩΣΕΙΣ ΜΗ ΜΕΛΩΝ

ΜΑΘΗΜΑΤΙΚΑ.— **On certain integrals occuring in diffraction theory,** *by Nicholas Chako** Ἀνεκοινώθη ὑπὸ τοῦ Ἀκαδημαϊκοῦ κ. Ἰωάνν. Ξανθάκη**.

1. INTRODUCTION.

In the theory of diffraction of electromagnetic waves by circular apertures and discs, one is let to evaluate integrals of the type

$$\text{(A)} \qquad \int_0^\infty J_{m+1/2}(at)\, J_{+1/2}(at)\, (t^2-1)^{\pm\frac{1}{2}}\, t^{-m-n}\, dt$$

in order to calculate the diffracted field and the transmission coefficient of a plane wave normally incident on the aperture or disc.

Integrals of this type were obtained by Bouwkamp (1) and by Levine and Schwinger (2), when the unknown field distribution over the aperture or disc is expressed by

$$\text{(a)}\ u(\rho,a)=\sum_0^\infty A_n\, P_{2n+1}\left(\sqrt{1-s^2}\right),\quad \text{(b)}\ u(\rho,a)=\sum_0^\infty B_n(1-s^2)^{n-\frac{1}{2}},\ \left(s=\frac{\rho}{a}\right),$$

where $\rho^2=x^2+y^2$, and A_n , B_n are unknown quantities to be determined from the boundary values of the problem, and a is the radius of the aperture or disc. Integrals of type (A) will be called Bouwkamp - Levine - Schwinger (BLS) integrals. We shall show that the Bouwkamp expansion (a) is more approproate than (b). The coefficients A_n ,B_n are found to satisfy an infinite system of linear equation involving integrals of type (A); their convergence properties have been thoroughly studied by Magnus (3).

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The main object of this paper is to investigate the following integral (*)

$$\text{(B)} \qquad I_{\alpha,\beta,\gamma,\lambda}(\rho,z,a) = \int_0^\infty e^{-z\sqrt{t^2-1}} J_\alpha(\rho t) J_\beta(at)(t^2-1)^\gamma t^{2\lambda}\, dt,$$

where ρ,z,a are real and the parameters α,β,ect., are fixed real or complex quantities. The necessary and sufficient condition for the integral (B) to exist is that the following inequality holds:

$$\text{(a)} \qquad \mathrm{Re}\,(\lambda + \tfrac{1}{2}) > 0.$$

Furthermore, by an appropriete choice of the parameters (B) satifies the so called axial equation of the wave equation in cylindrical coordinates and the boundary conditions $u=0$ on the screen or disc, or $\frac{\partial u}{\partial z} = u_z = 0$, on the screen or on the disc (ρ a), and the Bouwkamp - Meixner edge condition.

The problem of diffraction by a slit or a strip is likewise solved by the integral representation (B) of the wave equation.

2. EVALUATION OF THE INTEGRALS.

In evaluating (B) and its derivative I_z, it is found convenient to write it

$$\text{(1)} \quad I_{\alpha,\beta,\sigma} = i\int_0^1 e^{-iz\sqrt{1-t^2}} \frac{J_\alpha(\rho t) J_\beta(at)}{\sqrt{1-t^2}} t^{2\sigma-\alpha-\beta}\, dt - \int_1^\infty e^{-z\sqrt{t^2-1}} \frac{J_\alpha(\rho t) J_\beta(at)}{\sqrt{t^2-1}} t^{2\sigma-\alpha-\beta}\, dt,$$

with $\gamma = -\frac{1}{2}$, $2\lambda = 2\sigma - \alpha - \beta$, since these values cover most of the cases of interest.

With the aid of Cauchy's integral theorem, (1) can be trasformed as an integral (‡)

$$\text{(2)} \quad I_{\alpha,\beta,\sigma} = i\int_{1-i\infty}^1 \left\{ e^{-iz\tau} \frac{J_\alpha(\rho\sqrt{1-\tau^2}) J_\beta(a\sqrt{1-\tau^2})}{(1-\tau^2)^{\frac{\alpha+\beta+1-2\sigma}{2}}} \right\} dt,$$

(‡) This transformation is carried out in the same way as in Watson's «Bessel Functions».

* The intergal form of the axial equation can be easily derived from Whittaker's representation (Math. Ann., (1902), 57, p. 333) in cylindical coordinates.

(3) $J_{\alpha,\beta,\sigma} = I_z = \int_{1-i\infty}^{1} \left\{ » \quad » \quad » \quad » \right\} \tau d\tau.$

For the integrals to make sense, one must impose the following condition:

(b) $\operatorname{Re}(\alpha+\beta+1-2\sigma) > 0.$

Since the analysis of evaluating (2) and (3) is rather long and involved, it is sufficient to give the final forms which are suitable for finding their limiting expressions as $z \to 0$. For this purpose, we write (2) as follows:

(4) $I_{\alpha,\beta,\sigma} = I_1 + I_2 = A_{\alpha,\beta,\sigma} + i B_{\alpha,\beta,\sigma},$

(5) $$I_1 = \frac{\sqrt{\pi}\, e^{i\pi\sigma} \left(\frac{\rho}{2}\right)^{\alpha} \left(\frac{a}{2}\right)^{\beta} \left(\frac{2}{z}\right)^{\sigma}}{2 \sin\pi\sigma\, \Gamma(\alpha+1)} \sum_{0}^{\infty} (-1) \frac{n\Gamma(n+\sigma+\frac{1}{2})}{n!\Gamma(n+\beta+1)} {}_2F_1(-n,-n-\beta;\alpha+1;x^2) J_{n+\sigma}(z),$$

(6) $$I_2 = \frac{\sqrt{\pi}\, e^{i\pi\sigma} \left(\frac{\rho}{2}\right)^{\alpha} \left(\frac{a}{2}\right)^{\beta} \left(\frac{2}{z}\right)^{\sigma}}{2 \sin\pi\sigma\, \Gamma(\alpha+1)} \sum_{0}^{\infty} (-1) \frac{n\Gamma(n+\sigma+\frac{1}{2})}{n!\Gamma(n+\beta+1)} {}_2F_1(-n,-n-\beta;\alpha+1;x^2) J_{-n-\sigma}(z),$$

where $\rho^2 = a^2 x^2$ and σ is not an integer. The function $B_{\alpha,\beta,\sigma}$ is given by

(7) $$B_{\alpha,\beta,\sigma} = -\int_0^1 \cos(z\sqrt{1-t^2}) J_\alpha(\rho t) J_\beta(at) (1-t^2)^{-\frac{1}{2}} t^{2\sigma-\alpha-\beta} dt$$

$$= -\frac{\sqrt{\pi}\left(\frac{\rho}{2}\right)^{\alpha}\left(\frac{a}{2}\right)^{\beta}\left(\frac{2}{z}\right)^{\sigma}}{2\Gamma(\alpha+1)} \sum_{0,0}^{\infty} (-1)^{\frac{1}{2}} \frac{n\Gamma(n+\sigma+\frac{1}{2}) \left(\frac{a}{z}\right)^{2n} \left(\frac{z}{2}\right)^{2m+n+\sigma}}{m!n!\Gamma n+\beta+1)\Gamma(n+m+\sigma+1)}$$

$${}_1F_2(n+\sigma+\tfrac{1}{2}, n+m+\sigma+1; \alpha+1; -x^2)$$

valid for $0 < z < \rho$, $0 < \rho \leq a$. Another form of (7) is

(8) $$B_{\alpha,\beta,\sigma} = \frac{\sqrt{\pi}}{2}\Gamma\left(\sigma+\frac{1}{2}\right) \sum_{0}^{\infty} (-1)^{m+1} \frac{\left(\frac{z}{2}\right)^{2m}}{m!\Gamma(m+\frac{1}{2})} \sum_{0,0}^{\infty} \frac{\left(\frac{\rho}{2}\right)^{k}\left(\frac{a}{2}\right)^{n}\Gamma\left(k+n+m+\frac{1}{2}\right)}{k!n!\Gamma(k+m+n+\sigma+1)}$$

$$J_{k+\alpha}(\rho) J_{n+\beta}(a).$$

On the other hand $A_{\alpha,\beta,\sigma}$ is equal to the real part of I_1 plns I_2. The expession for I_2, suitable for taking the limit $z=0$, is of the form

(9) $$I_2 = \frac{1}{2}\left(\frac{\rho}{2}\right)^{\alpha}\left(\frac{a}{2}\right)^{\beta}\left(\frac{2}{z}\right)^{2\sigma} \sum_{0,0}^{\infty} \frac{(-u)^k(-v)\sigma + \frac{1}{2}\left(\frac{z}{2}\right)^{2m}\Gamma\left(m+\sigma+\frac{1}{2}\right)\Gamma\left(-m-\frac{1}{2}\right)}{m!n!\Gamma(n+\alpha+1)\Gamma\left(\beta+\frac{1}{2}-\sigma-n\right)}$$

$$ {}_2F_1\left(m+\sigma+\frac{1}{2}, n+\sigma-\beta+1; m+\frac{3}{2}; v\right) $$

$$ +\sum_{0,0}^{\infty} \frac{\left(\frac{\rho}{2}\right)^{2n}\left(-v\right)^{\sigma+n-m}\Gamma\left(n+\sigma-m\right)\Gamma\left(m+\frac{1}{2}\right)}{m!n!\Gamma(n+\alpha+1)\Gamma\left(\beta+\frac{1}{2}-\sigma-n\right)} {}_2F_1\left(n+\sigma-m, n+\sigma-\beta-m; \frac{1}{2}-m; v\right), $$

where u and v stand for ρ^2/a^2 and z^2/a^2 respectively. This expression is valid for $0 \leqq z < a$, $0 < \rho \leqq a$. The limiting value of I_2 as $z \to 0$ is

$$ (10) \lim_{z\to 0} I_2 = I_2^{\circ} = \frac{1}{\sqrt{\pi}} \frac{\sin\pi(\sigma-\beta)}{2\pi\Gamma(\alpha+1)} \left(\frac{1}{2}\right)^{\beta-2\sigma} \left(\frac{a}{2}\right)^{\alpha} \sum_{0}^{\infty} (-1)^n \frac{\left(\frac{\rho}{2}\right)^{2n}\Gamma\left(n+\frac{1}{2}\right)\Gamma(n-\sigma)\Gamma(\sigma-\beta-n)}{n!} $$

$$ {}_2F_1\left(n+\sigma+\frac{1}{2}, n-\sigma+m; \alpha+1; -\frac{\rho^2}{a^2}\right). $$

On ther hand the limiting valne of $B_{\alpha,\beta,\sigma}$ is easily derived from (7-8). This is

$$ (11)\ B_0 = \frac{\sqrt{\pi}}{2}\left(\frac{\rho}{2}\right)^{\alpha}\left(\frac{a}{2}\right)^{\beta} \sum_{0}^{\infty} (-1)^n \frac{\left(\frac{a}{2}\right)^{2n}\Gamma\left(n+\sigma+\frac{1}{2}\right)}{(n!\Gamma n+\beta+1)\Gamma(n+\sigma+1)} {}_1F_2\left(n+\sigma+\frac{1}{2}, n+\sigma+1; \alpha+1; -\frac{\rho^2}{a^2}\right) $$

$$ = \frac{\sqrt{\pi}}{2}\left(\frac{\rho}{2}\right)^{\alpha}\left(\frac{a}{2}\right)^{\beta}\Gamma\left(\sigma+\frac{1}{2}\right) \sum_{0,0}^{\infty} \frac{\left(\frac{\rho}{2}\right)^{k}\left(\frac{a}{2}\right)^{n}\Gamma\left(n+\sigma+\frac{1}{2}\right)}{k!n!\Gamma(n+k+\sigma+1)} J_{k+\alpha}(\rho) J_{n+\beta}(a) $$

valid for $z=0$, $0<\rho \leqq a$.

Combining (10) and (11) and noticing that $A_{\alpha,\beta,\sigma}$ is equal to I_2 plus $e^{\iota\sigma\pi}$ $(\sin\sigma\pi)^{-1}$ $B_{\alpha,\beta,\sigma}$, we get the final limiting value of $I_{\alpha,\beta,\sigma}$,

$$ (12)\quad I_{\alpha,\beta,\sigma}(\rho,a,z=0) = I^{\circ}(\rho,a) = \frac{e^{i\sigma\pi}}{\sin\pi\sigma} B_0 + I_2^1 $$

for $0<\rho \leqq a$, provided σ is not an integer. Similarly, one can derive expressions for I_2 and $B_{\alpha,\beta,\sigma}$ valid in the region $a<\rho$ oo Since $B_{\alpha,\beta,\sigma}$ is an even function of z, we have only to interchange α with β and ρ with a in the above formulas. To derive expressions for $A_{\alpha,\beta,\sigma}$ requires rather lengthy manipulations, which we shall not discuss here. The result foa I_2° is

$$ (13)\ I_2^{\circ} = \frac{\sin\pi(\sigma-\alpha)}{2\sqrt[3]{\pi}\Gamma(\beta+1)} \left(\frac{\rho}{2}\right)^{\alpha-2\sigma}\left(\frac{a}{2}\right)^{\beta} \sum_{0}^{\infty} (-1)^n \frac{\left(\frac{\rho}{2}\right)^{2n}\Gamma\left(n+\frac{1}{2}\right)\Gamma(\sigma-n)\Gamma(\sigma-\alpha-n)}{n!} $$

$$ {}_2F_1\left(\sigma-n, \sigma-\alpha-n; \beta+1; \frac{a^2}{\rho^2}\right) $$

To obtain the limit of J as $z\to 0$, we either differentiate I and let $z\to 0$, or evaluate J and afterwards take the limit as $z\to 0$. The result is

$$(14)\quad J_0(\rho,a,z=0)=\left(\frac{\partial I}{\partial z}\right)_{z=o}=-\frac{\cos\pi(\sigma-\beta)}{2\pi}\left(\frac{\rho}{2}\right)^{\alpha}\left(\frac{2}{a}\right)^{2\sigma+1-\beta}\frac{\Gamma\left(\sigma+\frac{1}{2}\right)\Gamma\left(\sigma+\frac{1}{2}-\beta\right)}{\Gamma(\alpha+1)}$$

$$ {}_2F_1\left(\sigma+\frac{1}{2}-\beta,\sigma+\frac{1}{2};\alpha+1;\frac{\rho^2}{a^2}\right),\ (0<1\leq a).$$

3 EXTENSTION FOR INTEGRAL VALUES OF σ.

In order to derive expressions for $I_{\alpha,\beta,\sigma}$ and $J_{\alpha,\beta,\sigma}$ when σ in an integer, we replace z by $\sqrt{r^2-\rho^2}$ in (5) and (6). After some rather heavy and involved calculations, we have found the following expressions:

$$(15)\quad I_{\alpha,\beta,\sigma}(r,\rho,a)=-\frac{\pi}{2}\left(\frac{\rho}{2}\right)^{\alpha}\left(\frac{a}{2}\right)^{\beta}\left(\frac{2}{r}\right)^{\sigma}\sum_{o,o}^{\infty}(-1)^{-m-n}\frac{\Gamma\left(m+\sigma+\frac{1}{2}\right)\Gamma\left(m+\alpha+\frac{1}{2}-\sigma-n\right)}{\Gamma(m+\alpha+1)\Gamma(n+\beta+1)\Gamma\left(\alpha+\frac{1}{2}-\sigma-n\right)m!n!}$$

$$\times\left(\frac{\rho}{r}\right)^{2m}\left(\frac{a}{r}\right)^{2n}\left(\frac{r}{2}\right)^{m+n}H^{(2)}_{m+n+\sigma}(r),$$

$$(16)\quad J_{\alpha,\beta,\sigma}(r,\rho,a)=\frac{\pi}{2}\left(\frac{\rho}{2}\right)^{\alpha}\left(\frac{a}{2}\right)^{\beta}\left(\frac{2}{r}\right)^{\sigma+1}\left(\frac{r^2-\rho^2}{4}\right)^{\frac{1}{2}}$$

$$\sum_{o,o}^{\infty}(-1)^{m+n}\frac{\left(\frac{\rho}{r}\right)^{2m}\left(\frac{a}{r}\right)^{2n}\left(\frac{r}{2}\right)^{m+n}\Gamma(n+\alpha+\frac{1}{2}-)\Gamma\left(m+\alpha+\frac{1}{2}-\sigma-n\right)}{m!n!\Gamma(m+\alpha+1)\Gamma(n+\beta+1)\Gamma\left(\alpha+\frac{1}{2}-\sigma-n\right)}H^{(2)}_{m+n+\sigma+1}(r),$$

valid for $0<\rho\leq r$, $0<a<r$, and furthermore (15) is valid for $z\to0$, since it is continuous across the plane $z=0$, whereas (16) is valid for $z>0$, as J is discontinuous.

In a similar manner one obtains expressions for I and J valid in the region $0<\rho\leq r'$, $0<a<r'$ if we take $r'=\sqrt{\rho^2+z^2}$, where ρ satisfies the inequality, $0<\rho\leq a$.

The limiting values of (15) and (16) are obtained by setting $r=o$ The result is

$$(17)\quad I_{\alpha,\beta,\sigma}(r=\rho,\rho,a)=i\frac{\sqrt{\pi}}{2}\left(\frac{\rho}{2}\right)^{\alpha-\sigma}\left(\frac{a}{2}\right)^{\beta}\sum_{o,o}^{\infty}(-1)^{m+n}$$

$$\frac{\left(\frac{\rho}{2}\right)^{m}\left(\frac{a}{2}\right)^{n}\Gamma\left(n+\sigma+\frac{1}{2}\right)\Gamma\left(m+\alpha+\frac{1}{2}-\sigma-n\right)}{m!n!\Gamma(m+\alpha+1)\Gamma(m+\beta+1)\Gamma\left(\alpha+\frac{1}{2}-\sigma-n\right)}H^{(2)}_{m+n+\sigma}(\rho),$$

$$(18)\quad J_{\alpha,\beta,\sigma}(r=\rho,\rho,a)=J_o\ (\rho,a)=\left(\frac{a}{2}\right)^{\beta}\left(\frac{2}{\rho}\right)^{2\sigma+1-\alpha}\sum_{0}^{\infty}(-1)^n\frac{\left(\frac{a}{\rho}\right)^{2n}\Gamma\left(n+\sigma+\frac{1}{2}\right)}{n!\Gamma(n+\beta+1)\Gamma\left(\alpha+\frac{1}{2}-\sigma-n\right)}$$

for $z=0$ and $0<a\leqq\rho$. For $0<\rho\leqq a$, we interchange the roles of α and β, ρ and a in (18).

As a final result we evaluate $I_{\alpha,\beta,\sigma}$ for $\rho=a$. This is obtained directly from (17), e.i.

$$(19)\quad I_{\alpha,\beta,\sigma}\ (a)=i\,\frac{\pi}{2}\left(\frac{a}{2}\right)^{\alpha+\beta-\sigma}\sum_{0}^{\infty}(-1)^{m+n}$$

$$\frac{\Gamma\left(m+\alpha+\frac{1}{2}-\sigma-n\right)\Gamma\left(n+\sigma+\frac{1}{2}\right)}{m!n!\Gamma(m+\alpha+1)\Gamma(n+\beta+1)\Gamma\left(\alpha+\frac{1}{2}-\sigma-n\right)}\,H^{(2)}_{m+n+\sigma}\quad(a).$$

This expression is a generalization of integrals of Bouwkamp - Levine - Schwinger type (A).

4. APPLICATIONS TO DIFFRACTION THEORY.

The connection of our integral with diffraction problems has already been alluded in section 1. If the parameters take the values $\alpha=m,\beta=m+2n+\frac{3}{2}$, $\lambda=\frac{1}{4}$ or $2\sigma=2m+2n+1$, then $I_{\alpha,\beta,\sigma}$ satisfies the axial wave equation, the boundary condition $I_z = J_o = 0$ on the screen $z=0$, Sommerfeld's condition and Bouwkamp - Meixner condition. With these values of the parametets J_o is of the form

$$(22)\quad J_o=-\frac{k}{2a}\int_0^{\infty}J_m\,(xu)J_{m+2n+\frac{3}{2}}\,(u)u^{-\frac{1}{2}}du=-\sqrt{\frac{k}{2a}}\,\frac{\Gamma(m+n+1)x^m}{\Gamma(m+1)\Gamma\left(n+\frac{3}{2}\right)}$$

$$ {}_2F_1\left(m+n+1,\,-n-\frac{1}{2};m+1;x^2\right),$$

where $\rho=ax$. The integral above is of well known, namely, Sonine - Schafheitlin type, which reduces in our case to a hypergeometric polynomial, or a Gegenbauer polynomial. Using the well known transformation of hypergeometric functions, (22) is also given by

$$(23)\quad J_o(\rho,a)=-\sqrt{\frac{k}{2a}}\,\frac{\Gamma(n+1)}{2^m\Gamma\left(m+n+\frac{1}{2}\right)}\,P^m_{m+2n+1}\quad\left(\sqrt{1-x^2}\right),\ \text{for}\ 0<\rho\leqq a.$$

$$=0\ \text{for}\ \rho>a.$$

If the field distribution over the aperture is expressed in terms of J_o, in the form

$$(24)\quad u(\rho,\varphi,a)=\sum_{o,o}^{\infty} A_{n}^{m} \cos m\varphi + B_{n}^{m} \sin m\varphi) P_{m+2n+1}^{m} (\sqrt{1-x^2}), \left(x=\frac{\rho}{a}\right),$$

(case of oblique incidence), then A_n^m, B_n^m satisfy infinite systems of linear equations with coefficients involving integrals of BLS type. For normal incidence, we have m=0, so (23) reduces to the Bouwkamp expansion (a) of section 1. Therefore, Bouwkamp's expansion is obtained directly form our integral representation of the wave equation.

On the other hand, if $\alpha=m$, $\beta=n+1, 2\sigma=n-m$, $a=1$, we get the Zernike Polynomials (4)

$$(25)\qquad Z_n^m(\rho) = (-1)^{\frac{n-m}{2}} J^{0}_{m,n+1,\sigma}(\rho)$$

which are orthogonal over a unit circle. These polynomials have been used extensively in the diffraction theory of aberrations.

As a final observation, one can derive as special cases of the general formulas given here other functions of mathematical physics, especially the important functions of Lommel, Struve, Lambda and other functions entering in various branches of physics.

ΠΕΡΙΛΗΨΙΣ

Τὸ πρόβλημα τῆς περιθλάσεως ὑπὸ κυκλικοῦ ἀνοίγματος ἢ δίσκου δύναται νὰ λυθῇ διὰ γενικοῦ ὁλοκληρώματος λύσεως τῆς κυματικῆς ἐξισώσεως καὶ ἱκανοποιοῦντος καταλλήλους συνθήκας. Δίδονται ἐνταῦθα ἐκπεφρασμέναι μορφαί, ἐπίσης δεικνύεται, ὅτι τὸ ἀνάπτυγμα τοῦ BOUWKAMP διὰ τὸ πεδίον πέραν τοῦ ἀνοίγματος εἶναι ἄμεσος συνέπεια τῆς ὁριακῆς τιμῆς, ἡ ὁποία ἱκανοποιεῖται ὑπὸ τῆς ὁλοκληρωτικῆς παραστάσεως τῆς λύσεως τῆς κυματικῆς ἐξισώσεως.

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