μιᾶς μάζης M_B δύναται νὰ ἐκφρασθῇ ὑπὸ Γεωμετρίας (α) τοῦ Riemann, εἰς ἣν ἡ μᾶζα ἡ προκαλοῦσα τὸ πεδίον τῆς κινήσεως τῆς μάζης M_B, εἶναι ἡ μᾶζα $\bar{M}$ τῶν ὑπολοίπων μαζῶν, συγκεντρωμένων εἰς τὸ κέντρον βάρους των.

$$g_{11}\, dr^2_B + \frac{M}{\bar{M}}\, r^2_B\, d\phi^2 + g_{44}\, {dx^4}^2 = -c^2 d\tau^2 , \quad dx^4 = icdt \tag{α}$$

$$g^{11}\frac{M^2}{\bar{M}^2} = g_{44} = 1 - \frac{2G\bar{M}^2}{c^2 M r_B}$$

$$M = M_A + M_B + \ldots . M\nu$$

$$\bar{M} = M - M_B$$

$$M_A \bar{r}_A + M_B \bar{r}_B + M\Gamma \bar{r}\Gamma + \ldots M\nu \bar{r}\nu = Mr = 0$$

$$\bar{M}\bar{r} = M r - M_B \bar{r}_B = M_A \bar{r}_A + M\Gamma \bar{r}\Gamma + \ldots . M\nu \bar{r}\nu .$$

Συναλλοιωτικὴ ἰσοδύναμος μᾶζα $= \dfrac{\bar{M}\, M_B}{M}$

Ἀντισυναλλοιωτικὴ » » $= \dfrac{M\, M_B}{\bar{M}}$

$\bar{r}_A$ ἡ ἀπόστασις τῆς μάζης M_A ἐκ τοῦ κέντρου βάρους τοῦ συστήματος.

3) Ἡ Γεωμετρία (α) ἀκολουθεῖ τὰς ἀρχὰς τῆς Μηχανικῆς τῆς ΓΘΣ.

α) Ἀρχὴ Διατηρήσεως τῶν ἀδρανῶν ποσοτήτων κινήσεων - ἐνεργείας (συναλλοιωτικὰ μεγέθη) ἑκάστου σώματος.

β) Ἀρχὴ τῆς Δράσεως καὶ Ἀντιδράσεως τῶν δρωσῶν Ἀντισυναλλοιωτικῶν Δυνάμεων (μὴ λαμβανομένης ὑπ' ὄψιν τῆς μεταθέσεως τοῦ περιηλίου) ἥτις δίδει τὴν Ἀρχὴν τῆς Διατηρήσεως τῶν δρωσῶν ποσοτήτων κινήσεων (Ἀντισυναλλοιωτικῶν μεγεθῶν) ὡς καὶ τὴν θεωρίαν περὶ κέντρου βάρους τῆς Κλασσικῆς Μηχανικῆς.

ΓΕΩΜΕΤΡΙΑ.— **On geometrical equivalence and on a certain group of plane curves,** *by* ***Chr. B. Glavas*** *. Ἀνεκοινώθη ὑπὸ τοῦ κ. Ἰωάνν. Ξανθάκη.

Let $f_1(a_1, b_1) = 0$, $f_2(a_2, b_2) = 0$ be the equations of one and the same plane curve in the two plane coordinate systems (a_1, b_1) and (a_2, b_2) respectively. These two equations will be equivalent if one can transform analytically one to the other. The latter depends upon the possibility to find formulae of analytic transformation between the systems (a_1, b_1) and (a_2, b_2). For example the relations $x^2 + y^2 = a^2$ and $r^2 = a^2$ are analytically equival-

* ΧΡΗΣΓ. Β. ΓΚΛΑΒΑ, Ἐπὶ τῆς γεωμετρικῆς ἰσοδυναμίας καὶ τινος ὁμάδος καμπυλῶν τοῦ ἐπιπέδου.

ent equations. The dual case has already been examined (4 and 5). Let $f(a, b)=0$ be an analytic relation. Given two plane coordinate systems (a_1, b_1) and (a_2, b_2) one gets by substitution in the last relation the two equations $f(a_1, b_1)=0$ and $f(a_2, b_2)=0$. The latter represent two different plane curves which obviously correspond to the same analytic relation $f(a, b)=0$. But here $a=a_1=a_2$ and $b=b_1=b_2$, which means that the corre

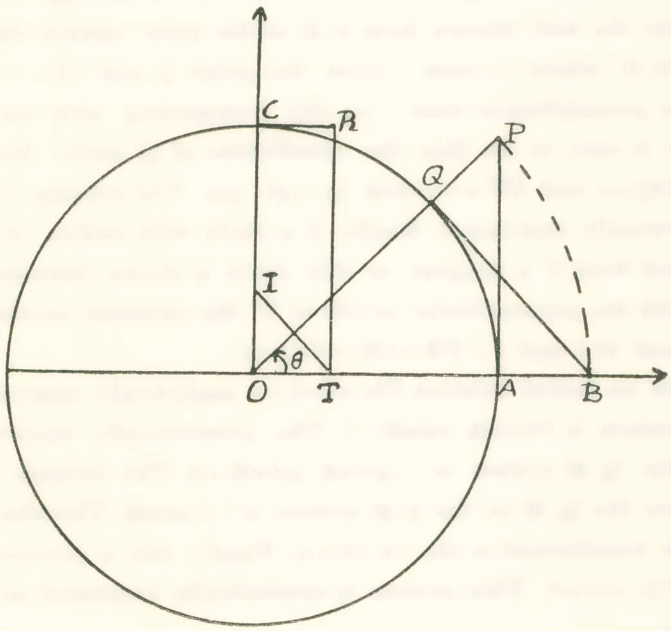

Fig. 1.

sponding coordinates of the two systems must be equal by pairs. This is possible in certain cases as it will be seen immediately.

Take the polar and the cathetic systems. In the latter system the coordinates of a point Q (Fig. 1) are the angle θ and the segment $g=OB$ where B is the intersection of the axis OA with the perpendicular on OQ at Q (3 : 123-27). Let a circle be drawn with center at O and radius OP intersecting with the axis at B. From B draw a perpendicular on OP intersecting with the latter at Q. Then if the points P and Q are defined by the polar and cathetic systems respectively we shall have $r=OP=OB=g$, θ being the same to both systems. It is now clear that the coordinates of the two points are equal by pairs and that one can go geometrically from

P to Q and vice versa. Thus the curves $f(r,\theta)=0$ and $f(g,\theta)=0$ described by the points P and Q respectively and represented by the same analytic relation are geometrically equivalent because $r=g$ and one can go geometrically from one to the other. In conclusion, while analytic equivalence depends upon the existence of formulae of analytic transformation among the coordinate systems in use geometrical equivalence depends upon the existence of a rule for a geometrical transformation among those systems.

Beside the well-known form (r,θ) of the polar system there is another one (r,t), where $t=\tan\theta$. Given the point Q and $OI=1$ on the axis OC draw a perpendicular from I on OQ intersecting with the axis OB at T. Then it is easy to see that the coordinates of Q under the new polar form are $OQ=r$ and $OT=t=\tan\theta$ (3: 146 - 50). The systems (r,t) and (x,y) are geometrically equivalent. Really, if a circle with radius $OQ=r$ is constructed and from C a tangent to this circle is drawn intersecting at the point R with the perpendicular on OB at T, the cartesian coordinate $x=OT$ of R is equal to t and $y=TR=OC=OQ=r$.

Given an initial relation $f(x,y)=0$ its analytically equivalent one in the (r,θ) system is $f(r\cos\theta, r\sin\theta)=0$. The geometrically equivalent of the latter in the (g,θ) system is $f(g\cos\theta, g\sin\theta)=0$. The formula of transformation from the (g,θ) to the (r,θ) system is $r=g\cos\theta$. Therefore the latter equation is transformed to the $f(r,rt)=0$. Finally this is geometrically equivalent to $f(y,xy)=0$. This process is symbolically expressed as follows

$$f(x,y)=0 \overset{a}{\sim} f(r\cos\theta, r\sin\theta)=0 \overset{g}{\sim} f(g\cos\theta, g\sin\theta)=0 \overset{a}{\sim} f(r,rt)=0 \overset{g}{\sim} f(y,xy)=0,$$

where a and g mean analytical and geometrical equivalence respectively.

If we apply the same process to the relation $f(y,xy)=0$ we shall get:

$$f(y,xy)=0 \overset{a}{\sim} f(r\sin\theta, r^2\sin\theta\cos\theta)=0 \overset{g}{\sim} f(g\sin\theta, g^2\sin\theta\cos\theta)=$$

$$=0 \overset{a}{\sim} f(rt, r^2t)=0 \overset{g}{\sim} f(xy, xy^2)=0$$

It is now clear that each relation $f=0$ is produced from its preceding one if one substitutes in the latter y for x and xy for y. The corresponding geometrical picture of this process is seen in Fig. 1. The point $P(x,y)$ of the initial curve $f(x,y)=0$ is the same with $P(r=OP,\theta)$. But the geometrically equivalent point of the latter is the point $Q(g=OB,\theta)$ which is the same with $Q(r=OQ, t=OT)$. Finally $Q(r,t)$ has as its geometrically equivalent the point $R(y=TR, x=OT)$.

$$f(x, y)=0$$

$f(y, xy)=0$	$f\left(\frac{y}{x}, x\right)=0$
$f(xy, xy^2)=0$	$f\left(\frac{x^2}{y}, \frac{y}{x}\right)=0$
$f(xy^2, x^2y^3)=0$	$f\left(\frac{y^2}{x^3}, \frac{x^2}{y}\right)=0$
$f(x^2y^3, x^3y^5)=0$	$f\left(\frac{x^5}{y^3}, \frac{y^2}{x^3}\right)=0$
$f(x^3y^5, x^5y^8)=0$	$f\left(\frac{y^5}{x^8}, \frac{x^5}{y^3}\right)=0$
———	———
———	———
$f(x^{U_n}y^{U_{n+1}}, x^{U_{n+1}}y^{U_{n+2}})=0$	$f(x^{(-1)^n U_{n+2}}y^{(-1)^{n+1}U_{n+1}}, x^{(-1)^{n+1}U_{n+1}} y^{(-1)^n U_n})=0$
———	———
———	———

DIAGRAM I

The outcome of the previously described process is the left column of Diagram I. If this process will be reversed then we shall get:

$$f(x, y)=0 \overset{g}{\sim} f(t, r)=0 \overset{a}{\sim} f(\tan\theta, g\cos\theta)=0 \overset{g}{\sim} f(\tan\theta, r\cos\theta)=0 \overset{a}{\sim} f\left(\frac{y}{x}, x\right)=0$$

Taking $f(y/x, x)=0$ and repeating this same process we get $f(x^2/y, y/x)=0$. Continuing in this way we produce the relations of the right column of Diagram I. Each relation of this column is produced by dividing the first and second terms inside the parenthesis of the two preceding relations.

It is now necessary to find the general expression for the relations of both columns of Diagram I. But if the general term of column I will be found it will be easy to write the corresponding one of the right column by looking at the way each relation ot the right column is produced from its corresponding one of the left column.

Looking at the exponents of the variable x of the first term inside the parenthesis in the relations of the left column we see that they form the sequence 0,1,1,2,3,5,8,.... Here each term of the sequence is equal to the sum of the two preceding ones. Then the general term (1:364) is

$$U_n = \frac{1}{\sqrt{5}}\left(a^{n-1} - b^{n-1}\right), \tag{1}$$

where $a=(1+\sqrt{5})/2$ and $b=(1-\sqrt{5})/2$, i.e. a and b are the roots of the equation $z^2-z-1=0$.

The exponent of y of the first term inside the parenthesis of each relation of the left column is the immediate successor to the exponent of x in the sequence. Therefore the exponent of y will be U_{n+1}. Then the exponent of x of the second term inside the parenthesis is equal to U_{n+1} and the exponent of y of the same term is the immediate successor to the exponent of x of the same term in the sequence and is equal to U_{n+2}. Finally the general expression of the relations of the right column is easily found.

We define the following law of composition symbolized by a small circle for any two elements (relations) of the set of Diagram I. Given any two elements $f(A,B)=0$, $f(C,D)=0$ of this set, where A,B,C,D are functions of x and y, $[f(A,B)=0] \circ [f(C,D)=0]$ means to substitute C and D for x and y respectively in A and B. For example:

$$[f(y,xy)=0] \circ [f(xy^2,x^2y^3)=0]=[f(x^2y^3,x^3y^5)=0]$$

In the above example the combination of two elements under the defined law of composition gives a new element of the set in question. It must be shown however that this holds for any two elements of the same set.

Consider first the combination of any two elements each belonging to each column of Diagram I. Then we get:

$$[f(x^{U_n}y^{U_{n+1}}, x^{U_{n+1}}y^{U_{n+2}})=0] \circ [f(x^{(-1)^m U_{m+1}}y^{(-1)^{m+1}U_{m+1}}, x^{(-1)^{m+1}U_{m+1}}$$

$$y^{(-1)^m U_m})=0] = [f[(x^{(-1)^m U_{m+2}}y^{(-1)^{m+1}U_{m+1}})^{U_n}(x^{(-1)^{m+1}U_{m+1}}y^{(-1)^m U_m})^{U_{n+1}},$$

$$(x^{(-1)^m U_{m+2}}y^{(-1)^{m+1}U_{m+1}})^{U_{n+1}}(x^{(-1)^{m+1}U_{m+1}}y^{(-1)^m U_m})^{U_{n+2}}]=0]= \tag{2}$$

$$=[f[(x^{(-1)^m(U_{m+2}U_n-U_{m+1}U_{n+1})}y^{(-1)^m(U_mU_{n+1}-U_nU_{m+1})},$$

$$(x^{(-1)^m(U_{m+2}U_{n+1}-U_{m+1}U_{n+2}}y^{(-1)^m(U_mU_{n+2}-U_{m+1}U_{n+1})})]=0]$$

We shall first show that any of the exponents of the powers of x and y of the last expression is of the standard form (1). Taking the first exponent of x and making the necessary substitutions:

$$(-1)^m(U_{m+2}U_n-U_{m+1}U_{n+1})=\frac{(-1)^m}{5}\Big[(a^{m+1}-b^{m+1})(a^{n-1}-b^{n-1})-$$

$$-(a^m-b^m)(a^n-b^n)\Big]=$$

$$=\frac{(-1)^m}{5}\left[a^{m+n}-a^{n-1}b^{m+1}-a^{m+1}b^{n-1}+b^{m+n}-a^{m+n}+a^nb^m+a^mb^n-b^{m+n}\right]=$$

$$=\frac{(-1)^m}{5}\left[a^n b^m\left(1-\frac{b}{a}\right)+a^m b^n\left(1-\frac{a}{b}\right)\right]=$$

$$=\frac{(-1)^m}{5}\left[a^n b^m\left(\frac{a-b}{a}\right)-a^m b^n\left(\frac{a-b}{b}\right)\right]=\frac{(-1)^m(a-b)}{5}(a^{n-1}b^m-a^mb^{n-1})=$$

$$=\frac{(-1)^m}{\sqrt{5}}(a^{n-1}b^m-a^mb^{n-1})$$

Suppose now that $n>m$. Then $n\geq m+1$ or $n-1\geq m$. Therefore we can put $n-1=m+k$ where $k\geq 0$. Substituting $m+k$ for $n-1$ in the last expression we get:

$$\frac{(-1)^m}{\sqrt{5}}(a^{m+k}b^m-a^mb^{m+k})=\frac{(-1)^m(ab)^m}{\sqrt{5}}(a^k-b^k)=\frac{(-1)^{2m}}{\sqrt{5}}(a^k-b^k)$$

$$=\frac{1}{\sqrt{5}}(a^k-b^k)=\frac{1}{\sqrt{5}}(a^{n-m-1}-b^{n-m-1})=U_{n-m}$$

It now becomes clear that the exponent in question is of the standard form (1). One can prove likewise that the three remaining exponents of (2) are of the same form. Thus the exponent of y in the first term inside the parenthesis of (2) is U_{n-m+1} and U_{n-m+2} respectively. Therefore (2) becomes:

$$[f(x^{U_{n-m}}y^{U_{n-m-1}},\ x^{U_{n-m+1}}y^{U_{n-m+2}})=0]$$

But this relation is the $(n-m)$th element of the left column of Diagrsm I. Similarly, it is proven that if $m>n$ (2) gives the $(m-n)$th element of the right column. In case both elements belong to the same column one finds in a similar way that the final result is the $(n+m)$th element of that column. We have thus proved that the combination of any two elements of the set of Diagram I under the defined law of composition gives a new element (composite) belonging to the same set.

Theorem I: The defined law of composition for the set of Diagram I is commutative.

Proof: We shall restrict ourselves only to two elements of the left column of Diagram I, the proof being the same for any other elements. Using the result of (2):

$$[f(x^{U_n}y^{U_{n+1}},\ x^{U_{n+1}}y^{U_{n+2}})=0]\ o\ [f(x^{U_m}y^{U_{m+1}},\ x^{U_{m+1}}y^{U_{m+2}})=0]=$$

$$=[f(x^{U_{n+m}}y^{U_{n+m+1}},\ x^{U_{n+m+1}}y^{U_{n+m+2}})=0]$$

And:

$$[f(x^{U_m} y^{U_{m+1}}, x^{U_{m+1}} y^{U_{m+2}})=0] \circ [f(x^{U_n} y^{U_{n+1}}, x^{U_{n+1}} y^{U_{n+2}})=0]=$$

$$=[f(y^{U_{m+n}} y^{U_{m+n+1}}, x^{U_{m+n+1}} y^{U_{m+n+2}})=0]$$

Comparing the two results we see that they are the same.

Theorem II: The defined law of composition for the set of Diagram I is associative.

Proof: We omit the proof which is a matter of routine substitutions and computations.

Theorem III: The defined law of composition for the set of Diagram I has a neutral (identity) element.

Proof: This element is the initial relation $f(x, y)=0$.

$$[f(x^{U_n} y^{U_{n+1}}, x^{U_{n+1}} y^{U_{n+2}})=0] \circ [f(x, y)=0]=[f(x^{U_n} y^{U_{n+1}}, x^{U_{n+1}} y^{U_{n+2}})=0]=$$

$$=[f(x, y)=0] \circ [f(x^{U_n} y^{U_{n+1}}, x^{U_{n+1}} y^{U_{n+2}})=0]$$

Since the set in question is closed under the defined law of composition that is commutative, associative and has a neutral element it then constitutes a commutative or Abelian monoid (2:4-5).

Theorem IV: The set of Diagram I is an Abelian group.

Proof: Since the set of Diagram I is a commutative monoid it is enough to show that each element of the set has its inverse in the same set. Taking two corresponding elements of the two columns of Diagram I it suffices to put $m=n$ in (2). Then:

$$[f(x^{U_n} y^{U_{n+1}}, x^{U_{n+1}} y^{U_{n+2}})=0] \circ [f(x^{(-1)^n U_{n+2}} y^{(-1)^{n+1)} U_{n+1}},$$

$$x^{(-1)^{n+1} U_{n+1}} y^{(-1)^n U_n})=0]=[f(x^{U_0} y^{U_1}, x^{U_1} y^{U_2})=0]$$

But from (1) we take:

$$U_0=\frac{1}{\sqrt{5}}\left(\frac{1}{a}-\frac{1}{b}\right)=\frac{1}{\sqrt{5}} \frac{b-a}{ab}=\frac{a-b}{\sqrt{5}}=1,\ U_1=0, \text{ and } U_2=\frac{a^2-b^2}{\sqrt{5}}=1$$

Therefore we shall have as the composite element of the two corresponding expressions the element $f(x, y)=0$, i.e. the neutral element. Thus every element of the set in question has its inverse and the set of curves of Diagram I constitutes an Abelian group.

ΠΕΡΙΛΗΨΙΣ

Εἶναι γνωστόν, ὅτι μία καμπύλη εἶναι δυνατὸν νὰ παρίσταται ὑπὸ δύο διαφόρων ἀναλυτικῶν σχέσεων, ἐκπεφρασμένων εἰς δύο διακεκριμένα συστήματα συντεταγμένων. Αἱ σχέσεις αὗται εἶναι «ἀναλυτικῶς ἰσοδύναμοι ἢ μετατρέψιμοι» ἡ μία εἰς τὴν ἄλλην, ὅταν ὑπάρχουν ἀναλυτικοὶ τύποι μετασχηματισμοῦ ἐκ τοῦ ἑνὸς συστήματος εἰς τὸ ἄλλο. Μία ἀναλυτικὴ σχέσις εἶναι δυνατὸν νὰ παριστᾷ δύο διακεκριμένας καμπύλας, ὁριζομένας ὑπὸ δύο διαφόρων συστημάτων συντεταγμένων. Αἱ καμπύλαι αὗται εἶναι «γεωμετρικῶς ἰσοδύναμοι ἢ μετατρέψιμοι» ἡ μία εἰς τὴν ἄλλην, ὅταν ὑπάρχῃ γεωμετρικὸς τρόπος μεταβάσεως ἐξ ἑνὸς σημείου, ὁριζομένου ὑπὸ τοῦ ἑνὸς συστήματος, εἰς ἕνα ἄλλο ὁριζόμενον ὑπὸ τοῦ ἑτέρου τῶν συστημάτων.

Τὸ τελευταῖον συνιστᾷ τὴν ἀρχὴν τῆς κληθείσης γεωμετρικῆς ἰσοδυναμίας, τῆς ὁποίας ἔγινεν ἤδη ἀνάπτυξις καὶ χρῆσις εἰς προηγουμένας πραγματείας (βλ. *Πρακτικὰ τῆς Ἀκαδημίας Ἀθηνῶν*, 32 (1957), σ. 122-131 καὶ σ. 507-518). Ἀπεδείχθη μάλιστα καὶ ἓν γενικὸν θεώρημα κατανομῆς τῶν καμπυλῶν τοῦ ἐπιπέδου εἰς ὁμάδας βάσει ὡρισμένου νόμου συνθέσεως (*Πρακτ. Ἀκαδ.*, τόμ. 34 (1959), σ. 413-422).

Κατὰ τὰς ὡς ἄνω ἐργασίας ἐχρησιμοποιήθησαν δύο συστήματα συντεταγμένων. Τὸ χαρακτηριστικὸν τῆς παρούσης πραγματείας εἶναι ἡ χρησιμοποίησις περισσοτέρων τῶν δύο συστημάτων συντεταγμένων, συγκεκριμένως τεσσάρων. Πλὴν τῶν γνωστῶν ὡς ἀναλυτικῶς καὶ γεωμετρικῶς ἰσοδυνάμων συστημάτων, τῶν (r, θ) καὶ (g, θ), εἰσάγονται καὶ δεικνύονται ὁμοίως ἰσοδύναμα τὰ συστήματα (r, t) καὶ (χ, y).

Λαμβάνεται ἐν ἀρχῇ μία τυχοῦσα καμπύλη ἐκπεφρασμένη εἰς καρτεσιανὰς συντεταγμένας. Ἡ ἐξίσωσις ταύτης μετατρέπεται εἰς πολικὰς συντεταγμένας. Ἡ τελευταία ἐξίσωσις μετατρέπεται εἰς τὴν γεωμετρικῶς ἰσοδύναμόν της εἰς τὸ σύστημα (g, θ). Ἐκ ταύτης εὑρίσκεται ἡ ἀναλυτικῶς ἰσοδύναμός της τοῦ συστήματος (r, t). Τέλος ἡ ἐξίσωσις αὕτη μετασχηματίζεται εἰς τὴν γεωμετρικῶς ἰσοδύναμόν της τοῦ καρτεσιανοῦ συστήματος (χ, y), λόγῳ τῆς ὑφισταμένης γεωμετρικῆς μετατρεψιμότητος μεταξὺ τῶν δύο συστημάτων. Οὕτω ἐκ τῆς ἀρχικῆς καμπύλης, ἐκπεφρασμένης εἰς τὸ σύστημα (χ, y), παράγεται τελικῶς μία ἄλλη ἐκπεφρασμένη καὶ αὕτη εἰς τὸ ἴδιον σύστημα.

Ἡ ἐργασία αὕτη ἐπαναλαμβάνεται ἀκριβῶς ὁμοίως ὡς καὶ προηγουμένως ἀλλὰ μὲ ἀρχικὴν ἑκάστοτε καμπύλην τὴν τελικῶς εὑρισκομένην τῆς προηγουμένης περιπτώσεως. Οὕτω παράγεται ἓν σύνολον καμπυλῶν εἰς τὸ σύστημα (χ, y). Ἐὰν τώρα ἡ πορεία τῆς ἐργασίας ἀντιστραφῇ καὶ ἡ ἰδία ἀρχικὴ καμπύλη εἰς τὸ σύστημα (χ, y) μετατραπῇ εἰς τὴν γεωμετρικῶς ἰσοδύναμόν της τοῦ συστήματος (r, t), ἐν συνεχείᾳ ἡ τελευταία εἰς τὴν ἀναλυτικῶς ἰσοδύναμόν της τοῦ (g, θ), ἔπειτα εἰς τὴν γεωμετρικῶς ἰσοδύναμόν της τοῦ (r, θ) καὶ τέλος εἰς τὴν ἀναλυτικῶς ἰσοδύναμον τοῦ (χ, ψ), εἶναι φανερὸν ὅτι διὰ τῆς συνεχοῦς ἐπαναλήψεως τῆς μεθόδου ταύτης θὰ παραχθῇ ἕτερον σύνολον καμπυλῶν, ἐκπεφρασμένων εἰς τὸ ἀρχικὸν σύστημα (χ, y).

Τὸ σύνολον τῶν οὕτω πως παραγομένων καμπυλῶν ἀποδεικνύεται ὅτι συνιστᾷ ὁμάδα ὑπὸ ὡρισμένον νόμον συνθέσεως. Πρὸς τοῦτο εὑρίσκεται ἡ παράστασις τοῦ γενικοῦ ὅρου ἑκάστου τῶν ὑποσυνόλων τῆς προηγουμένης παραγράφου καὶ ἐν συνεχείᾳ

διαπιστοῦται ἡ πλήρωσις ὅλων τῶν συνθηκῶν δυνάμει τῶν ὁποίων τὸ ἐν λόγῳ σύνολον συνιστᾷ Ἀβελιανὴν ὁμάδα.

Δύο ἀξιοσημείωτα πορίσματα τῆς ἐργασίας ταύτης εἶναι πρῶτον, ὅτι δὲν ὑφίσταται σχέσις ἰσοδυναμίας μεταξὺ δύο σημείων τοῦ ἐπιπέδου, ὁριζομένων διὰ τῆς περιγραφείσης ἤδη μεθόδου τῶν τεσσάρων συστημάτων συντεταγμένων. Ἕπεται, ὅτι δὲν καθίσταται δυνατὴ ἡ κατανομὴ τῶν καμπυλῶν τοῦ ἐπιπέδου εἰς διακεκριμένα σύνολα - ὁμάδας, ὅπως συμβαίνει διὰ δύο ἀναλυτικῶς καὶ γεωμετρικῶς ἰσοδύναμα συστήματα. Τοῦτο ὀφείλεται εἰς τὸ γεγονός, ὅτι τὰ χρησιμοποιούμενα ἐδῶ τέσσαρα συστήματα δὲν εἶναι ὅλα μεταξύ των γεωμετρικῶς καὶ ἀναλυτικῶς ἰσοδύναμα, ἀλλὰ μόνον τινὰ τούτων. Δεύτερον, ἡ σημασία τῆς ἐφαρμοζομένης μεθόδου ἔγκειται εἰς τὴν δυνατότητα, ὅπως μία δοθεῖσα ἐξίσωσις καμπύλης εἰς καρτεσιανὰς συντεταγμένας ἐκφρασθῇ τελικῶς δι' ἄλλης εἰς τὸ ἴδιον σύστημα, ἀλλ' ἢ ἀσυγκρίτως ἁπλουστέρας ἢ καὶ γνωστῆς. Τότε δι' ἀντιστρόφου πορείας ἐπιτυγχάνεται ἡ ἐπάνοδος καὶ ἑπομένως ἡ κατασκευὴ τῆς ἀρχικῆς καμπύλης.

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ΑΝΑΛΥΤΙΚΗ ΧΗΜΕΙΑ — Χρωματογραφία ἐπὶ χάρτου. Μέθοδος διαχωρισμοῦ καὶ ἀνιχνεύσεως τῶν στοιχείων τῆς ὁμάδος τοῦ λευκοχρύσου καὶ τοῦ χρυσοῦ [1], *ὑπὸ* ***Κωνστ. Β. Βασιλειάδου*** *καὶ* ***Γεωργ. Μανουσάκη****. Ἀνεκοινώθη ὑπὸ τοῦ κ. Λεων. Ζέρβα.

Περιγράφεται μέθοδος διαχωρισμοῦ καὶ ἀνιχνεύσεως ἰχνῶν στοιχείων τῆς ὁμάδος τοῦ λευκοχρύσου διὰ τῆς χρωματογραφίας ἐπὶ χάρτου διὰ τῆς χρησιμοποιήσεως, ὡς εἰδικοῦ ἀντιδραστηρίου ἐμφανίσεως, ὑδατικοῦ διαλύματος ὑδροχλωρικῆς διανισιδίνης.

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[1] *Ἐκ τοῦ Ἐργαστηρίου Ἀνοργάνου Χημείας τοῦ Πανεπιστημίου Θεσσαλονίκης.*

* **B. VASSILIADIS and G. MANOUSSAKIS, Chromatographie sur papier. Séparation et détection des élements de la famille de platine et de l'or.**