ΜΑΘΗΜΑΤΙΚΑ.— **Global analysis. Morse Theory on Hilbert Manifolds and the Plateau's Problem,** *by T. M. Rassias**. Ἀνεκοινώθη ὑπὸ τοῦ Ἀκαδημαϊκοῦ κ. Φ. Βασιλείου.

1. It is L. Euler who started a new field in Mathematics, now known as Variational Analysis. L. Euler in 1744 deduced the first general rule, now known as Euler's differential equation, for the characterization of the maximizing or minimizing arcs. Much of the terminology of the Variational Analysis was introduced shortly thereafter by J. L. Lagrange. One of the earliest papers along this line was Hilbert's famous address in which he posed some [23] problems. Of these the 19th problem was to prove that the regular solutions, of any analytic regular variational problem are analytic and the 23rd problem was vaguely to develop a theory of variational analysis (cf. [8]).

The Morse theory of critical points of a point function ϕ is concerned with relationships between topological characteristics (Betti or Connectivity numbers) of the graph of ϕ and the numbers of local minima, local maxima, and different kinds of saddle points. The Morse theory goes back to that of H. Poincaré and G. D. Birkhoff, concerning the theory of dynamical systems. Abstract critical point theory can be applied to the theory of functions of a finite number of variables, for example, to harmonic functions of two or three variables, to problems of floating bodies, to problems of celestial mechanics. The theory of critical points has been applied to problems of differential equations in the large by F. Browder, W. B. Gordon, J. Marsden, C. B. Morrey, M. Morse, R. Palais, H. Poincaré, S. Smale, et al. Many questions fromMathematical Physics concerning stable points in a force field with an associated potential are related to the part of Morse theory. Variational analysis has played an important role as a unifying influence in mechanics and as a guide in the mathematical interpretation of many physical phenomena. It is of interest to remark that if the configuration of a system of moving particles is governed by their mutual gravitational attractions, their paths

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* Θ. Μ. ΡΑΣΣΙΑ, **Θεωρία τοῦ Morse ἐπὶ τῶν πολλαπλοτήτων Hilbert καὶ τὸ πρόβλημα τοῦ Plateau.**

will be minimizing curves for the integral with respect to time, of the difference between the Kinetic and the Potential energies of the system. This is known as Hamilton's principle after its discoverer. C. Carathéodory has devised a quite new approach to a considerable portion of the theory of variational analysis. In his theory the extremals appear as so-called «curves of quickest descent».

One of the difficult problems in Global Variational Analysis is Plateau's problem. This is the problem of determining the surfaces of minimum area spanned in a given curve or subject to other boundary conditions. Lagrange raised the question of finding a minimal surface for a given contour and Plateau gave a physical realization by means of physical experiments. Accordingly this problem proposed by Lagrange is now known as Plateau's problem. During the last century Plateau's problem has attracted the attention of mathematicians such as G. D. Darboux, D. Hilbert, B. Riemann, H. A. Schwarz, K. Weierstrass, et al. Among the questions of special interest in the study of Plateu's problem is the one concerning the existence of an unstable extremal. It is remarkable to mention the very interesting work of M. Morse and C. Tompkins [**13**], M. Shiffman [**21**] which independently and almost at the same time studied the question of the existence of an unstable minimal surface.

2. It is well known that R. Palais and S. Smale (cf. [**17**], [**18**], [**22**]) have found an extension of Morse theory of critical points to a certain class of functions on Hilbert manifolds. This theory is applicable to some variational problems and partial defferential equations (systems) on vector bundles. A. Tromba has very recently worked out a modification of the Morse-Palais-Smale theory on Hilbert (or Banach manifolds) to a considerable general nature which allowed him to answer some important questions on the Plateau's problem. It is an outstanding question to answer if the Morse theory on Hilbert (or Banach) manifolds can be applied to Plateau's Problem. This is a problem which has resisted solution up to now (despite some recent results by A. Tromba). It is my intention here to state some of my results concerning the application of Morse theory on Plateau's problem following A. Tromba's approach (cf. [**25**]). The proofs will appear elsewhere.

Consider the function space: $K = \{$all vector functions $q(u, v) \in H^s$ $(D; R^3)$ where D is the open unit disk in R^2 and $s > 2$. q maps the (boundary) ∂D monotonically onto Γ, where Γ is a given Jordan curve in R^3. Moreover $\Gamma = (x, y, z) = (\alpha(\theta), \beta(\theta), \gamma(\theta))$ where $(\alpha(\theta), \beta(\theta), \gamma(\theta))$ is a monotonic increasing representation of Γ, $q|_{\partial D} : \partial D \to \Gamma$ is a homeomorphism, has nowhere vanishing derivative and $\alpha, \beta, \gamma \in H^{s-\frac{1}{2}}(\partial D)$.$\}$

N o t e : The definition of the Sobolev space H^s can be found in J. Marsden [**10**].

It can be easily shown that K is a Hilbert submanifold of the space of maps $H^s(D; R^3)$, $s > 2$. We can also prove that K has the homotopy type of S^1. We define the tangent space of K at $q \in K$, as follows:

$T_q K = \{$all maps $h \in H^s(\bar{D}; TR^3)$, $s > 2$, s a finite real number, such that

1) $\pi \circ h = q$ where $q \in K$ and $\pi : TR^3 \to R^3$

2) $h(\partial D) \subset T\Gamma\}$

It is understood that every $h \in T_q K$ is a six dimensional vector; however we can think of it as a three dimensional vector by considering only the last three coordinates of h. More precisely, every $h \in T_q K$ can be brought to the origin of the space R^3 and thought of as a three dimensional vector, i. e. determined in three coordinates h_1, h_2, h_3.

A weak Riemannian structure can be defined on $T_q K$ by giving a weak inner product on $T_q K$ as follows: Let h, $g \in T_q K$ then

$$< h, g >_q = \iint_D < \nabla h, \nabla g >_{R^6} du dv$$

where $\nabla h = \left(\frac{\partial h}{\partial u}, \frac{\partial h}{\partial v}\right)$, $\nabla g = \left(\frac{\partial g}{\partial u}, \frac{\partial g}{\partial v}\right)$. To be more precise we write

$$< \nabla h, \nabla g >_{R^6} = \frac{\partial h_1}{\partial u}\frac{\partial g_1}{\partial u} + \frac{\partial h_2}{\partial u}\frac{\partial g_2}{\partial u} + \frac{\partial h_3}{\partial u}\frac{\partial g_3}{\partial u} + \frac{\partial h_1}{\partial v}\frac{\partial g_1}{\partial v} + \frac{\partial h_2}{\partial v}\frac{\partial g_2}{\partial v} + \frac{\partial h_3}{\partial v}\frac{\partial g_3}{\partial v}$$

where $h = (h_1, h_2, h_3)$, $g = (g_1, g_2, g_3)$, $\nabla h \in R^6$, $\nabla g \in R^6$.

T h e o r e m 1. *For every* $q \in K$ *there exists a unique vector field* $\bar{V}(q) \in T_q K$ *such that*

(1) $$\iint_D < \nabla q, \nabla h > du dv = \iint_D < \nabla \bar{V}(q), \nabla h > du dv$$

for all $h \in T_q K$, *where by* $\overline{V}(q)$ *we denote the last three coordinates of* $V(q) \in T_q K$.

Remark 1. It is important to notice that $\overline{V}(q)$ is a three dimensional vector, corresponding to the last three coordinates of $V(q) \in T_q K$. We are working with $\overline{V}(q)$ and not with $V(q)$. In other words from now on we can forget $V(q)$ and just work with $\overline{V}(q)$, for which (1) makes sense.

Theorem 2. *For every* $q \in K$ *there exists a unique vector field* $\overline{V}(q) \in T_q K$ *such that* $\nabla^2 \overline{V} = \nabla^2 q$, $\nabla^2 =$ *Laplacian,* $\overline{V}$ *and* q *are three dimensional vectors with boundary conditions*

$$\frac{\partial \overline{V}}{\partial r} - \frac{\partial q}{\partial r} \perp \Gamma$$

and
$$(q, \overline{V})(\partial D) \subset T\Gamma$$

which means for all $(u, v) \in \partial D$, $(q, \overline{V})(u, v) = (q(u, v), \overline{V}(u, v)) =$ *a pair with first element* $q(u, v)$ *and second element* $\overline{V}(u, v)$.

Remark 2. It is easy to prove each of Theorems 1 and 2 from the other. Consider the function space:

$\Sigma = \{$Emb $(\partial D; R^3)$ be the open submanifold of $H^k(S^1; R^3)$ which consists of embeddings of S^1 into $R^3\}$

Set $G_\alpha = \{$the component of $H^k(S^1; \Gamma)$, the Hilbert manifold of H^k maps from S^1 to Γ determined by the embedding α, where α is a C^∞ embedding of S^1 in R^3 whose image determines $\Gamma\}$.

It follows that G_α is an open submanifold of $H^k(S^1; \Gamma)$ since G_α is a component. For every $\hat{u} \in H^k(S^1; \Gamma) \subset H^k(S^1; R^3)$ we can extend $\hat{u} = (\hat{u}_1, \hat{u}_2, \hat{u}_3)$ harmonically to $u : D \to R^3$ such that $u|_{S^1} = \hat{u}$. This is a consequence of the Dirichlet Principle. Define now the space:

$\hat{G}_\alpha = \{$all the harmonic extensions of elements of G_α to D, i.e., $u : D \to R^3$ and $u|_{S^1} = \hat{u}\}$.

Define the C^∞-energy functional

$$E_\alpha : \hat{G}_\alpha \to R$$

by $E_\alpha(u) = \frac{1}{2}\iint\limits_D \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right] dxdy = \frac{1}{2}\iint\limits_D \|\nabla u\|^2 dxdy$

where $\nabla u = \left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right)$, $u \in \hat{G}_\alpha$.

Theorem 3. $dE_{\alpha_q}(\bar{V}(q)) \geqq 0$ *and equals zero only if* q *is simultaneously a critical point of* E_α *(the energy functional) and a zero of* $\bar{V}$. *By* $dE_{\alpha_q}(\bar{V}(q))$ *we mean the differential of the energy functional* E_α *at* q *in the direction of* $\bar{V}(q)$.

Remark 3. It is understood that

$$dE_{\alpha_q}(h) = \iint\limits_D < \nabla\bar{V}(q), \nabla h > dudv$$

for $q \in K$, $h \in T_q K$.

By looking at Theorem 2 and using the regularity of the Laplacian, we get $\bar{V}(q) \in H^s$ whenever $q \in H^s$. By making use of the open mapping theorem we get that the operator

$$q \rightarrow \bar{V}(q)$$

is bounded.

Theorem 4. *The Condition (C) of Palais - Smale is not applicable for Plateau's problem formulated on Sobolev spaces* H^s, $s > 2$.

(Note: If M is a C^1 Finsler manifold and $f : M \rightarrow R$ a C^1 map, then f satisfies condition (C) if given any sequence $\{s_n\}$ in M on which f in bounded but on which $\|df\|$ is not bounded away from zero there is a subsequence $\{s_{n_j}\}$ which converges.)

Theorem 5. *Given* $\{P_i\}$ *a bounded sequence in* $\hat{G}_\alpha$ *(in the given Finsler metric) and* $\|\bar{V}(P_i)\| \rightarrow 0$ *then there exists a subsequence* $\{P_{i_j}\}$ *which converges to a zero* q *of* $\bar{V}$.

(Note: This proves Tromba's condition (CV).)

Conjecture. Although I do not have a complete proof I believe that for any $p \in \hat{G}_\alpha$, the trajectory σ_p of $\bar{V}$ through p has a maximal domain

$$(\bar{\alpha}, \bar{\beta}) \subset R.$$

Then as $t \to \bar{\alpha}$, $\| \bar{V}(\sigma_p(t)) \| \to 0$ and $\sigma_p(\bar{\alpha}, 0]$ is bounded in the Sobolev sence, provided that $E_\alpha(\sigma_p(t))$ is a positive real number.

(Note: This will prove Tromba's condition (G2).)

If the above conjecture is true then a complete proof of the famous problem, concerning the application of Morse theory on Hilbert manifolds to the Plateau's problem is given. This will follow from the recent paper by A. Tromba [25]. A consequence of the above investigation will be a global analytic proof of the Morse-Tompkins [13] and Shiffman [21] theorem which states that the existence of two relative minima for the energy functional E_α implies the existence of a third unstable minimal surface.

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ΠΕΡΙΛΗΨΙΣ

Εἰς τὴν παροῦσαν ἀνακοίνωσιν ἀποδεικνύεται ὅτι ἡ θεωρία τῶν Smale καὶ Palais δὲν δύναται νὰ ἐφαρμοσθῇ εἰς τὸ πρόβλημα τοῦ Plateau καὶ διατυποῦται μία σειρὰ συμπερασμάτων τοῦ συγγραφέως, τὰ ὁποῖα ὁδηγοῦν εἰς τὴν γενικὴν λύσιν τοῦ προβλήματος. Ἕνα κύριον χαρακτηριστικὸν τῆς παρούσης ἀνακαλύψεως εἶναι ἡ εἰσαγωγὴ τοῦ συναρτησιακοῦ χώρου K μὲ στοιχεῖα ἀπεικονίσεις εἰς τὸν χῶρον τοῦ Sobolev. H^s, $s > 2$. Τοιουτοτρόπως ὁ K ἔχει τὴν δομὴν μιᾶς πολλαπλότητος τοῦ Hilbert. Ἀποδεικνύεται ἡ ὕπαρξις μοναδικοῦ διανυσματικοῦ πεδίου $\bar{V}(q)$ ἐπὶ τοῦ $T_q K$, ἐφαπτομένου χώρου τοῦ K εἰς τὸ $q \in K$, μὲ τὴν ἰδιότητα ὅτι τὰ σημεῖα τοῦ K ὅπου $\bar{V}(q) = 0$ εἶναι ἀκριβῶς ἐκεῖνα, ἔνθα ἡ πρώτη παράγωγος τοῦ συναρτησιακοῦ τοῦ Dirichlet μηδενίζεται. Τοιουτοτρόπως τὸ πρόβλημα ἀνάγεται εἰς τὸ πεδίον τῆς Τοπολογίας τῶν διανυσματικῶν πε-

δίων. Ἡ καταφατικὴ ἀπόδειξις τῆς εἰκασίας (Conjecture), ὅπου τίθεται εἰς τὸ τέλος τῆς ἐργασίας, ἐπιλύει τὸ πρόβλημα ἐφαρμογῆς τῆς θεωρίας τοῦ Morse ἐπὶ τῶν πολλαπλοτήτων τοῦ Hilbert εἰς τὸ πρόβλημα τοῦ Plateau.

Ἐφαρμογαὶ αὐτοῦ δύνανται νὰ δοθοῦν εἰς τὴν Γενικὴν Θεωρίαν τῆς Σχετικότητος τοῦ A. Einstein.

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Ὁ Ἀκαδημαϊκὸς κ. **Φ. Βασιλείου** παρουσιάζων τὴν ἀνωτέρω ἀνακοίνωσιν εἶπε τὰ ἑξῆς :

Ἡ ἐργασία, τὴν ὁποίαν ἔχω τὴν τιμὴν νὰ παρουσιάσω εἰς τὴν Ἀκαδημίαν Ἀθηνῶν ἔχει τὸν τίτλον «Ἡ θεωρία τοῦ Marston Morse ἐπὶ τῶν πολλαπλοτήτων Hilbert καὶ τὸ πρόβλημα τοῦ Plateau». Ἡ ἐργασία αὐτὴ εἶναι συντεταγμένη ἀγγλιστί. Ὁ συγγραφεύς της εἶναι Postdoctoral Research Fellow καὶ μέλος τῆς Faculty τοῦ μαθηματικοῦ τμήματος τοῦ Πανεπιστημίου τῆς Καλλιφορνίας εἰς Berkeley Ἡνωμ. Πολιτειῶν Ἀμερικῆς, ἀφιερώνει δὲ τὴν ἐργασίαν του εἰς μνήμην τοῦ πρὸ ἔτους ἀποθανόντος διαπρεποῦς μαθηματικοῦ, διατελέσαντος καὶ ἀντεπιστέλλοντος μέλους τῆς Ἀκαδημίας Ἀθηνῶν, Χρήστου Παπακυριακοπούλου.

Εἰς τὴν παροῦσαν ἀνακοίνωσιν ὁ συγγραφεὺς ἐκθέτει ὡρισμένα πρόσφατα ἐξαγόμενά του ἀναφερόμενα εἰς τὴν ἐπίλυσιν ἑνὸς τῶν θεμελιωδῶν προβλημάτων τοῦ Λογισμοῦ τῶν Μεταβολῶν, προβλήματος ἐκφραζομένου εἰς πεδίον τῆς Διαφορικῆς Τοπολογίας τῶν πολλαπλοτήτων τοῦ Hilbert ἢ τοῦ Banach. Τὸ πρόβλημα αὐτὸ ἠμπορεῖ νὰ διατυπωθῇ ὡς ἑξῆς : «Ἡ λεγομένη θεωρία τοῦ Marston

Morse, διατυπουμένη εἰς πολλαπλότητας Hilbert ἢ Banach, εἶναι δυνατὸν νὰ ἐφαρμοσθῆ εἰς τὸ πρόβλημα τοῦ Plateau;» Τὸ πρόβλημα Plateau ἀφορᾶ εἰς τὴν εὕρεσιν ἐπιφανειῶν ἐλαχίστου ἐμβαδοῦ ἐκτεινομένων ἀπὸ δοθεῖσαν κλειστὴν γραμμὴν καὶ ὑποκειμένων εἰς ὡρισμένας συνοριακὰς συνθήκας. Τὸ πρόβλημα Plateau ἐτέθη τὸ πρῶτον ἀπὸ τὸν Lagrange. Ἂν ὅμως τοῦτο φέρει σήμερον τὸ ὄνομα τοῦ Βέλγου φυσικοῦ Plateau, ὀφείλεται εἰς τὸ γεγονός, ὅτι πρῶτος ὁ φυσικὸς αὐτὸς ἔδωσε πειραματικὴν λύσιν τοῦ προβλήματος. Τὸν προηγούμενον καὶ τὸν παρόντα αἰῶνα, μερικοὶ τῶν διασημοτέρων μαθηματικῶν, ὅπως οἱ B. Riemann, K. Weierstrass, H. Schwarz, C. Carathéodory καὶ D. Hilbert, ἠσχολήθησαν μὲ διαφόρους περιπτώσεις μαθηματικῆς ἐπιλύσεως τοῦ προβλήματος Plateau.

Ὁ σύγχρονος Ἀμερικανὸς μαθηματικὸς Marston Morse, ἐπεκτείνας τὴν θεωρίαν τῶν λεγομένων «κριτικῶν σημείων» (critical points) τῶν H. Poincaré καὶ G. D. Birkhoff εἰς τοὺς συναρτησιακοὺς χώρους, ἠδυνήθη νὰ ἐπιλύση πολλὰ τῶν προβλημάτων τοῦ Λογισμοῦ τῶν Μεταβολῶν. Παρέμενεν ὅμως ἐντελῶς ἄλυτον τὸ πρόβλημα, κατὰ πόσον ἡ θεωρία τοῦ Morse εἰς τὰς πολλαπλότητας Hilbert ἢ Banach ἠμπορεῖ νὰ ἐφαρμοσθῆ καὶ εἰς τὸ πρόβλημα Plateau.

Εἰς τὸ σημεῖον αὐτὸ πρέπει παρεμπιπτόντως νὰ σημειωθῆ, ὅτι ὁ σύγχρονος μαθηματικὸς κλάδος τῆς «Ἀναλύσεως ἐπὶ τῶν πολλαπλοτήτων» (Analysis on Manifolds), λεγόμενος καὶ Global Analysis, μελετᾶ προβλήματα τῆς μὴ γραμμικῆς ἀναλύσεως, τὰ ὁποῖα ἐπιλύονται μὲ μεθόδους τῆς Τοπολογίας. Διὰ τὸν λόγον ἀκριβῶς αὐτὸν ὁ κλάδος αὐτὸς φέρεται εἰς τὴν μαθηματικὴν βιβλιογραφίαν καὶ ὡς «Τοπολογικὴ Ἀνάλυσις». Εἰς τὸν κλάδον αὐτὸν ὑπάρχουν προβλήματα προερχόμενα ἀπὸ τὴν (μὴ γραμμικὴν) Ἀνάλυσιν, μὴ δυνάμενα ὅμως νὰ ἐπιλυθοῦν, τουλάχιστον μέχρι σήμερον, μὲ μεθόδους τῆς Ἀναλύσεως αὐτῆς. Ἡ Ἀνάλυσις ἐπὶ τῶν πολλαπλοτήτων παρέχει τρόπους ἐκφράσεως προβλήματος τῆς Ἀναλύσεως εἰς τὸ ἀντίστοιχόν του πρόβλημα διατυπούμενον εἰς τὴν γλῶσσαν τῆς Τοπολογίας, ὡς καὶ τρόπους ἐπιλύσεως τοῦ τελευταίου μὲ τοπολογικὰς μεθόδους. Εἶναι γεγονός, ὅτι προβλήματα τοιούτου εἴδους ἀπαιτοῦν γνώσεις ἀπὸ ἀρκετοὺς κλάδους τῆς μαθηματικῆς ἐπιστήμης, ἰδιαιτέρως τῆς θεωρίας τῶν πολλαπλοτήτων Hilbert (ἢ Banach).

Ἐπανερχόμενοι εἰς τὸ πρόβλημα τῆς ἐφαρμογῆς τῆς θεωρίας Morse ἀναφορικῶς μὲ τὰς πολλαπλότητας Hilbert ἢ Banach εἰς τὸ πρόβλημα τοῦ Plateau, παρατηροῦμεν ὅτι τοῦτο ἀπετέλεσε θέμα πολλῶν καὶ ἐντατικῶν ἐρευνῶν. Τὰ ἔτη 1963 - 64 οἱ μαθηματικοὶ S. Smale καὶ R. Palais ἐπενόησαν ἐπέκτασιν τῆς θεωρίας Morse εἰς πολλαπλότητας ἀπείρων διαστάσεων διανοίξαντες οὕτω τὸν δρόμον διὰ τὴν ἐπίλυσιν ὡρισμένων προβλημάτων τοῦ Λογισμοῦ τῶν Μεταβολῶν, τῆς μὴ γραμμικῆς συναρτησιακῆς Ἀναλύσεως καθὼς καὶ τῶν Μερικῶν Διαφορικῶν Ἐξισώσεων.

Ἐκεῖνο ποὺ ἐπιτυγχάνεται ἀπὸ τὸν συγγραφέα εἰς τὴν παροῦσαν ἀνακοίνωσιν εἶναι κατ' ἀρχὰς ἡ ἀπόδειξις τοῦ ὅτι ἡ θεωρία αὐτὴ τῶν Smale - Palais δὲν ἠμπορεῖ νὰ ἐφαρμοσθῇ εἰς τὸ πρόβλημα τοῦ Plateau. Κατόπιν ὁ συγγραφεὺς διατυπώνει σειρὰν συμπερασμάτων του διὰ τὴν ἐπίλυσιν τοῦ ἀνωτέρω μνημονευομένου προβλήματος δηλαδὴ τῆς δυνατότητος ἐφαρμογῆς τῆς θεωρίας τοῦ Morse, διατυπουμένης εἰς πολλαπλότητας τοῦ Hilbert ἢ Banach, — ἐφαρμογῆς εἰς τὸ πρόβλημα τοῦ Plateau. Τὸ πρόβλημα τοῦτο ἀνάγει ὁ συγγραφεὺς εἰς μίαν εἰκασίαν (conjecture) ἡ ὁποία, ἂν ἀποδειχθῇ ἀληθής, θὰ δώσῃ πλήρη ἀπόδειξιν τοῦ περιφήμου ἐκείνου προβλήματος. Ἐφαρμογαὶ αὐτοῦ δύνανται, κατὰ τὸν συγγραφέα, νὰ γίνουν εἰς τὴν Γενικευμένην θεωρίαν τῆς Σχετικότητος.