ΜΑΘΗΜΑΤΙΚΑ.— **Distributive and complemented hyperlattices,** *by Maria Konstantinidou - Serafimidou**. Ἀνεκοινώθη ὑπὸ τοῦ Ἀκαδημαϊκοῦ κ. Φίλωνος Βασιλείου.

It is known that the notion of hyperlattice [3] is a generalisation of the lattice notion in the classical theory. The difference between hyperlattice and lattice is that in a hyperlattice the union of any two elements is a hyperoperation.

Studying the hyperlattices an important interest is found because many properties hold, arising from the special character of the union, these in addition with the properties of the lattices which also hold [3], [7], [8].

In this paper we study some cathegories of hyperlattices, more specifically the distributive and complemented hyperlatices [3], [7], [8].

1. DISTRIBUTIVE HYPERLATICES

Definition (1. 1) A hyperlattice H [3] is said to be *distributive* when in addition satisfies also the axiom:

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$

for every triple $a, b, c \in H$, that is, when the operation $\wedge$ is distributive with respect to the hyperoperation $\vee$.

Remarks (1. 1) a) It is obvious that a distributive hyperlattice is a modular one.

b) Because the operation $\wedge$ is distributive with respect to the hyperoperation $\vee$ we have:

$$a \vee (b \wedge c) \subseteq (a \vee b) \wedge (a \vee c).$$

In fact, we have:

$$(a \vee b) \wedge (a \vee c) = \bigcup_{x \in a \vee b} [x \wedge (a \vee c)] = \bigcup_{x \in a \vee b} [(x \wedge a) \vee (x \wedge c)] \supseteq \bigcup_{x \in a \vee b} [a \vee (x \wedge c)],$$

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because $a \in (a \vee b) \wedge a$ and $x \wedge a$ runs over the set $(a \vee b) \wedge a$. And since

$$\bigcup_{x \in a \vee b} [a \vee (x \wedge c)] = a \vee [(a \vee b) \wedge c] = a \vee [(a \wedge c) \vee (b \wedge c)] =$$
$$= [a \vee (a \wedge c)] \vee (b \wedge c) \supseteq a \vee (b \wedge c),$$

follows that :

$$a \vee (b \wedge c) \subseteq (a \vee b) \wedge (a \vee c).$$

In case that H is a lattice, the distributivity of the intersection $\wedge$ with respect to the union $\vee$ is equivalent with the distributivity of the union $\vee$ with respect to the intdrseetion $\wedge$ that is :

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c) \Longleftrightarrow a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c).$$

c) If a, b, c are elements of a hyperlattice H and a, b are comparable, that is $a \not\parallel b$ we have:

$$I = [c \wedge (a \vee b)] \cap [c \wedge a) \vee (c \wedge b)] \neq \varnothing.$$

In fact, if $a \leqslant b$, we shall have $b \in a \vee b$, and so $c \wedge b \in [c \wedge (a \vee b)]$. On the other hand the relation $a \wedge c \leqslant b \wedge c$ implies

$$b \wedge c = c \wedge b \in [(c \wedge a) \vee (c \wedge b)].$$

Hence $c \wedge b \in [c \wedge (a \vee b)] \cap [(c \wedge a) \vee (c \wedge b)]$ and consequently $I \neq \varnothing$.

We get the same result too when $b \leqslant \alpha$.

Examples (1.1) a) Easily we can verify that the set $H = \{0, 1\}$ with the hyperoperation $0 \vee 0 = 0$, $0 \vee 1 = 1 \vee 0 = 1$, $1 \vee 1 = H$ and the operation $0 \wedge 0 = 0 \wedge 1 = 1 \wedge 0 = 0$, $1 \wedge 1 = 1$, is a distributive hyperlattice.

b) Also, we can show that the set

$$H^{\vee} = \{(x_1, \ldots, x_n) : x_1, \ldots, x_n \in H\}$$

where $H = \{0, 1\}$ is the hyperlattice of the previous example, is a distributive hyperlattice, where the union and the intersection are defined on it through the corresponding ones of the previous example as follows:

$$(x_1, \ldots, x_n) \vee (y_1, \ldots, y_n) = \{(z_1, \ldots, z_n) : z_i \in c x_i \vee y_i ,$$
$$x_i, y_i \in \{0, 1\}, \quad i=1, \ldots, n\}$$

and $$(x_1, \ldots, x_n) \wedge (y_1, \ldots, y_n) = (x_1 \wedge y_1, \ldots, x_n \wedge y_n).$$

Definition (1.2) A hyperlattice is called *generalised distributive* if in addition satisfies the axiom

$$(\bigvee_{i \in A} a_i) \wedge b = \bigvee_{i \in A} (a_i \wedge b)$$

where A is any set of indices.

In this case the distributivity is called *generalised.*

Remarks (1.2) a) Obviously a generalised distributive hyperlattice H is a distributive one.

b) For every generalised distributive hyperlattice H the following relation is valid

$$(\bigvee_{i \in A} a_i) \wedge (\bigvee_{j \in B} b_j) \subseteq \bigvee_{(i,j) \in A \times B} (a_i \wedge b_j).$$

In fact, since H is generalised distributive we have

$$(\bigvee_{i \in A} a_i) \wedge (\bigvee_{j \in B} b_j) = \bigcup_{\substack{w \in v b_j \\ j \in B}} [(\vee a_i) \wedge w] = \bigcup_{\substack{w \in v b_j \\ j \in B}} [\vee (a_i \wedge w)] \subseteq \bigvee_{i \in A} [a_i \wedge (\bigvee_{j \in B} b_j)] =$$
$$= \bigvee_{i \in A} [\bigvee_{j \in B} a_i \wedge b_j)] = \bigvee_{(i,j) \in A \times B} (a_i \wedge b_j).$$

Relatively to the distributive hyperlattice we have the following propositions:

Let H a distributive hyperlattice.

Proposition (1.1) *If* H *possesses a zero element* o, *which is scalar, then for every pair* $(a, b) \in H \times H$ *such as* $a \wedge b = o$ *we have* $a \vee b = \sup(a, b)$.

Proof. Indeed, for every $w \in a \vee b$ it follows that $a \wedge w \in a \wedge (a \vee b)$, which because of the distributivity and the relation $a \wedge b = o$ becomes

$$a \wedge w \in (a \wedge a) \vee (a \wedge b) = a \vee 0 = a.$$

So we have $a \wedge w = a$, that is $a \leqslant w$. Similarly we get also $w \wedge b = b$ that is $b \leqslant w$ and hence it will be

$$a \vee b = \mathscr{X}_a^{a \vee b} = \mathscr{X}_b^{a \vee b}.$$

Consequently we have [8] $a \vee b = \sup(a, b)$.

Proposition (1. 2) *The set* S *of all scalar elements of* H *is a sub-hyperlattice of* H *and also a lattice.*

Proof. It is already known [3] that $S \vee S \subseteq S$. On the other hand, if $a \wedge b = c$, we will have

$$c \vee x = (a \wedge b) \vee x \subseteq (a \vee x) \wedge (b \vee x) = a_1 \wedge b_1 = c_1,$$

that is $c \vee x = c_1$, for every $x \in H$. Hence $S \wedge S \subseteq S$, thus the set S is a sub-hyperlattice of H, and since its elements are scalar it will be a lattice.

Proposition (1. 3) *If* $a, b \in H$, *the interval* $[a, b]$ *is a sub-hyperlattice of* H *if and only if* $a \vee a = a$.

Proof. Let $a \vee a = a$. If $a \leqslant x_1 \leqslant b$ and $a \leqslant x_2 \leqslant b$, we have that $a \leqslant x_1 \wedge x_2 \leqslant b$, that is $x_1 \wedge x_2 \in [a, b]$. On the other hand, for every $w \in x_1 \vee x_2$ it will be $w \leqslant b$ [8] and from the relations $a \leqslant x_1$, $a \leqslant x_2$ we get respectively $a \wedge x_1 = a$, $a \wedge x_2 = a$. Consequently $(a \wedge x_1) \vee (a \wedge x_2) = a \vee a = a$ which becomes $a \wedge (x_1 \vee x_2) = a$, because H is distributive.

Thus for every $w \in x_1 \vee x_2$ it will be $a \wedge w = a$, that is $a \leqslant w$ and hence $x_1 \vee x_2 \subseteq [a, b]$. By consequence the interval $[a, b]$ is a sub-hyper-lattice of H.

Conversely, if the interval $[a, b]$ is a sub-hyperlattice of H and x_1, x_2 two elements of it, we will have $x_1 \vee x_2 \subseteq [a, b]$. Hence for every $z \in x_1 \vee x_2$ it will be $a \leqslant z$, which gives

$$a = a \wedge (x_1 \vee x_2) = (a \wedge x_1) \vee (a \wedge x_2) = a \vee a.$$

Proposition (1. 4) *For every triple* $a, b, c \in H$ *we have*

$$a \vee (b \wedge c) \subseteq [a \vee (b \wedge c)] \vee [(a \vee b) \wedge c].$$

Proof. From the relations

$$a \in a \vee (a \wedge c) \quad \text{and} \quad b \wedge c \in (b \wedge c) \vee (b \wedge c) \quad \text{we get}$$

$$a \vee (b \wedge c) \subseteq [a \vee (a \wedge c)] \vee [(b \wedge c) \vee (b \wedge c)] = [a \vee (b \wedge c)] \vee [(a \wedge c) \vee (b \wedge c)],$$

which, because of the distributivity becomes

$$a \vee (b \wedge c) \subseteq [a \vee (b \wedge c)] \vee [(a \vee b) \wedge c].$$

If H is a lattice, the previous proposition corresponds to the identity

$$a \vee (b \wedge c) = [a \vee (b \wedge c)] \vee [(a \vee b) \wedge c],$$

that is the known inequality

$$(a \vee b) \wedge c \leqslant a \vee (b \wedge c).$$

Proposition (1. 5) *If* a, b, c *are any elements of* H, *then we have the relation*

$$(a \wedge c) \vee (b \wedge c) \vee (a \wedge b) \subseteq (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$$

Proof. In fact, we shall have (1. 1b)

$$(a \vee b) \wedge (b \vee c) \wedge (c \vee a) \supseteq [b \vee (a \wedge c)] \wedge (c \vee a) = \bigcup_{w \in c \vee a} \big[[b \vee (a \wedge c)] \wedge w\big] =$$

$$= \bigcup_{w \in c \vee a} \big[(b \wedge w) \vee [(a \wedge c) \wedge w]\big] \supseteq \bigcup_{w \in c \vee a} \big[(b \wedge w) \vee [(a \wedge c) \wedge x_c^a]\big] =$$

$$= \bigcup_{w \in c \vee a} [(b \wedge w) \vee (a \wedge c)] = \big[\bigcup_{w \in c \vee a} (b \wedge w)\big] \vee (a \wedge c) = [b \wedge (c \vee a)] \vee (a \wedge c) =$$

$$= (b \wedge c) \vee (a \wedge b) \vee (a \wedge c).$$

In case where H is a lattice the proposition (1. 5) is valid as identity

$$(a \vee b) \wedge (b \vee c) \wedge (c \vee a) = (a \wedge c) \vee (b \wedge c) \wedge (a \wedge b)$$

P r o p o s i t i o n (1. 6) *If* a, b, c $\in$ H, *then*

$$\left.\begin{array}{l} a\wedge c = b\wedge c \\ a\vee c = b\vee c \end{array}\right\} \Longrightarrow a = b$$

P r o o f. Since $a\wedge c = b\wedge c$, $a\vee c = b\vee c$ and H is distributive, we have that

$$a \wedge (a\vee c) = a \wedge (b\vee c) = (a\wedge b) \vee (a\wedge c) = (a\wedge b) \vee (b\wedge c) = b\wedge (a\vee c) =$$
$$= b \wedge (b\vee c) = b \vee (b\wedge c) = b\vee (a\wedge c).$$

On the other hand [8], $a\wedge (a\vee c) = a\vee (a\wedge c)$. Hence it will be

$$a\vee (a\wedge c) = b\vee (a\wedge c).$$

Therefore [8] $a = b$, since it is $a\wedge c \leqslant a$ and $a\wedge c = b\wedge c \leqslant b$.

The conditions of this proposition can be considered like a simplification rule.

R e m a r k (1. 3) The conditions of the propositions (1. 4), (1. 5), (1. 6), are not sufficient for the distributivity of the hyperlattice H [3], while in the lattices the above conditions, as it is known, are sufficient and necessary for the distributivity.

In addition, for the distributive hyperlattice we have the following properties.

P r o p e r t y (1. 1) *Every sub-hyperlattice of a distributive hyperlattice is distributive.*

P r o o f. The proof is obvious.

P r o p e r t y (1. 2) *The homomorph image of a distributive hyperlattice* H *is a distributive hyperlattice.*

P r o o f. f(H), as known [3] is a hyperlattice. Obviously, because of the distributivity of H, for any a, b, c $\in$ H we have

$$a \wedge (b\vee c) = (a\wedge b) \vee (a\wedge c).$$

Therefore, the left side gives

$$f[a \wedge (b \vee c)] = f[\bigcup_{w \in b \vee c} (a \wedge w)] = \bigcup_{w \in b \vee c} f(a \wedge w) = \bigcup_{w \in b \vee c} [f(a) \wedge f(w)] =$$

$$= f(a) \wedge f(b \vee c) = f(a) \wedge [f(b) \vee f(c)],$$

and from the right side we get

$$f[(a \wedge b) \vee (a \wedge c)] = f(a \wedge b) \vee f(a \wedge c) = [f(a) \wedge f(b)] \vee [f(a) \wedge f(c)].$$

Hence $f(a) \wedge [f(b) \vee f(c)] = [f(a) \wedge f(b)] \vee [f(a) \wedge f(c)]$.

Property (1. 3) *The product* H *of a family of distributive hyperlattices* $\{H_i\}_{i \in A}$ *is a distributive hyperlattice. Conversely if the product of a family of hyperlattices* $\{H_i\}_{i \in A}$ *is distributive, then* H_i *for all* $i \in A$ *are distributive hyperlattices.*

Proof. It is known [3] that H is a hyperlattice. Since H_i are distributive, we will have

$$a \wedge (b \vee c) = a \wedge \left\{\{w_i\}_{i \in A} : w_i \in b_i \vee c_i\right\} = \{a_i\}_{i \in A} \wedge \left\{\{w_i\}_{i \in A} : w_i \in b_i \vee c_i\right\} =$$

$$= \left\{\{a_i \wedge w_i\}_{i \in A} : w_i \in b_i \vee c_i\right\} = \left\{\{z_i\}_{i \in A} : a_i \wedge (b_i \vee c_i)\right\} =$$

$$= \left\{\{z_i\}_{i \in A} : z_i \in (a_i \wedge b_i) \vee (a_i \wedge c_i)\right\}.$$

On the other hand we have

$$(a \wedge b) \vee (a \wedge c) = \{a_i \wedge b_i\}_{i \in A} \vee \{a_i \wedge c_i\}_{i \in A} =$$

$$= \left\{\{t_i\}_{i \in A} : t_i \in (a_i \wedge b_i) \vee (a_i \wedge c_i)\right\},$$

hence

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c).$$

Conversely, if H is a distributive hyperlattice, for any $a, b, c \in H$ it is $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$, which can be written

$$\{a_i\}_{i \in A} \wedge \left\{\{b_i\}_{i \in A} \vee \{c_i\}_{i \in A}\right\} = \{a_i \wedge b_i\}_{i \in A} \vee \{a_i \wedge c_i\}_{i \in A}$$

so, $\left\{\{w_i\}_{i \in A} : w_i \in a_i \wedge (b_i \vee c_i)\right\} = \left\{\{z_i\}_{i \in A} : z_i \in (a_i \wedge b_i) \vee (a_i \wedge c_i)\right\}$

and consequently

$$a_i \wedge (b_i \vee c_i) = (a_i \wedge b_i) \vee (a_i \wedge c_i) \quad \text{for all } i \in A.$$

2. COMPLEMENTED AND RELATIVELY COMPLEMENTED HYPERLATTICES

If a hyperlattice H possesses extreme elements zero o and unit element 1, then it is obvious that for every $a \in H$

$$o \wedge a = o, \quad a \in o \vee a$$
$$1 \wedge a = a, \quad 1 \in 1 \vee a$$

Definition (2.1) In a hyperlattice H with zero o and unit 1, the element a′ is said to be *complement of* a, if both the relations $a \wedge a' = 0$, $1 \in a \vee a'$ are satisfied.

The elements a and a″ called complemented (to each other).

Definition (2.2) A hyperlattice H with elements o and 1, o being scalar, is said to be *complemented,* if every element a of it has at least one complemented a′.

Example (2.1) Easily can be verified that the ordered set which is given by the diagram

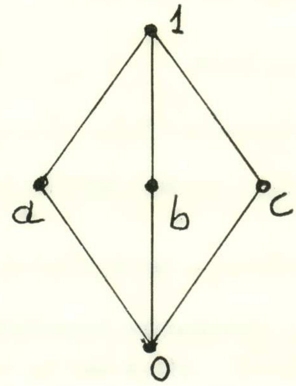

Fig. 1.

is a complemented hyperlattice, when the operation $\wedge$ is defined as $a \wedge b = b \wedge c = c \wedge a = o$ and $x \leqslant y \Longleftrightarrow x \wedge y = x$, and the hyperoperation $\vee$ as follows:

$$o \vee o = o\,,\quad o \vee a = a \vee o = a\,,\quad o \vee b = b \vee o = b\,,\quad o \vee c = c \vee o = c\,,$$

$$o \vee 1 = 1 \vee o = 1\,,\quad a \vee a = \{a, 1\}\,,$$

$$b \vee b = \{b, 1\}\,,\quad c \vee c = \{c, 1\}\,,\quad a \vee b = b \vee a = a \vee c = c \vee a = b \vee c =$$
$$= c \vee b = a \vee 1 = 1 \vee a = b \vee 1 = 1 \vee b = c \vee 1 = 1 \vee c = 1 \vee 1 = 1.$$

R e m a r k s (2. 1) The images f(o), f(1), under an homomorphism of complemented hyperlattices $f : H_1 \to H_2$, are the extreme elements of $f(H_1)$, zero and unit respectively, and the complement of the element f(a) is $[f(a)]' = f(a')$.

Indeed, we have

$f(o) = f(a \wedge o) = f(a) \wedge f(o)\,,\quad f(a) \in f(a \vee o) = f(a) \vee f(0)$ and

$f(1 \wedge a) = f(1) \wedge f(a) = f(a)\,,\quad f(1) \in f(1 \vee a) = f(1) \vee f(a)$

for every $a \in H$. On the other hand, if a' is the complement element of a then it will be

$$f(o) = f(a \wedge a') = f(a) \wedge f(a') \quad \text{and} \quad f(1) \in f(a \vee a') = f(a) \vee f(a')$$

Thus we have $[f(a)]' = f(a')$.

b) The complement of an element is not generally unique, as it can be seen in the previous example, where e. g. the element b and c are complements of a, because $a \wedge b = o$, $1 = a \vee b$ and $a \wedge c = o$, $1 = a \vee c$

c) If $a \neq o, 1$, then $a // a'$.

Indeed, if $a' \leqslant a$, then $a' \wedge a = a'$. But since $a' \wedge a = o$, we will have $a' = o$, so $a = 1$ and this is not true.

Let now a hyperlattice H and consider two eldments $a, b \in H$ such that $a \leqslant b$ and an element $x' \in [a, b]$, that is $a \leqslant x \leqslant b$.

D e f i n i t i o n (2. 3) An element $x' \in [a, b]$ is said *relative complement* of x with respect to a and b if

$$x \wedge x' = a \quad \text{and} \quad b \in x \vee x'$$

x and x' are called *relatively complement* (to each other) with respect to a and b.

R e m a r k (2. 2) Obviously, in a complemented hyperlattice, every complement of x is relative complement with respect to o and 1.

Definition (2.4) A hyperlattice H is called *relatively complemented,* if for every pair $(a,b) \in H \times H$ such that $a \leqslant b$, every element $x \in [a,b]$ has at least one relative complement with respect to a and b.

For the complemented and relatively complemented hyperlattices we have the following properties:

Property (2.1) *Let* H *the product of a family of hyperlattices* $\{H_i\}_{i\in A}$. *If for the elements* $a = \{a_i\}_{i\in A}$, $b = \{b_i\}_{i\in A}$ *and* $x = \{x_i\}_{i\in A}$ *we have* $a \leqslant x \leqslant b$, *then the element* x *has a relative complement with respect to* a *and* b, *if and only if for every* $i \in A$ *the element* $x_i \in [a_i, b_i]$ *has a relatively complement with respect to* a_i *and* b_i.

Proof. In fact, if x_i' is the relative complement of x_i with respect to a_i and b_i, then according to the definition (2.3) we shall have

$$x_i \wedge x_i' = a_i \quad \text{and} \quad b_i \in x_i \vee x_i'$$

for each $i \in A$. From the above relations we take

$$\{x_i\}_{i\in A} \wedge \{x_i'\}_{i\in A} = \{a_i\}_{i\in A}$$

and $$\{x_i\}_{i\in A} \vee \{x_i'\}_{i\in A} = \left\{\{z_i\}_{i\in A} : z_i \in x_i \vee x_i'\right\}$$

But since $$\{b_i\}_{i\in A} \in \left\{\{z_i\}_{i\in A} : z_i \in x_i \vee x_i'\right\}$$

it will be $$\{b_i\}_{i\in A} \in \{x_i\}_{i\in A} \vee \{x_i'\}_{i\in A}.$$

Consequently we shall have

$$x \wedge x' = a \quad \text{and} \quad b \in x \vee x'.$$

Inversely, if $x \wedge x' = a$, then

$$\{x_i\}_{i\in A} \wedge \{x_i'\}_{i\in A} = \{a_i\}_{i\in A} \Longrightarrow x_i \wedge x_i' = a_i$$

for every $i \in A$.

Also, if $b \in x \vee x'$, then

$$\{b_i\}_{i\in A} \in \{x_i\}_{i\in A} \vee \{x_i'\}_{i\in A} = \left\{\{z_i\}_{i\in A} : z_i \in x_i \vee x_i'\right\} \Longrightarrow b_i \in x_i \vee x_i'$$

for all $i \in A$.

The property (2. 1) can be described as follows:

The product of two or more complemented or relatively complemented hyperlattices is a complemented or relatively complemented hyperlattice. Conversely, if the product of two or more hyperlattices is complemented or relatively complemented hyperlattice, then every factor is a complemented or relatively complemented hyperlattice.

Property (2. 2) *The homomorph image of a complemented hyperlattice* H *is a complemented hyperlattice.*

Proof. If $f(a)$, $f(a')$ are the images of the elements a and a' respectively then by the remark (2. 1) we have $f(a) \wedge f(a') = f(o)$ and $f(1) \in f(a) \vee f(a')$.

We examine below some special categories of complemented hyperlattices.

Complemented modular hyperlattices.

Let H a complemented modular hyperlattice.

Proposition (2. 1) *If* x' *the relative complement of* x *with respect to* a *and* b *then* $b = \sup(x, x')$.

Proof. Since $x \leqslant b$, $x' \leqslant b$ and $b \in x \vee x'$, it follows that $b = \sup(x, x')$ [8].

Proposition (2. 2) *If an element* a *of* H, *is covered by the unit* 1, *that is* $a \prec 1$, *then* a' *is an atom of* H, *that is* $o \prec a'$.

Proof. Since $a \prec 1$, $a' \prec 1$ and $a // a'$ [rem. 2. 1c] we have $a \wedge a' \prec a'$, that is $0 \prec a'$ [8].

Proposition (2. 3) *If an element* $a \in H$ *covers* o, *that is* $o \prec a$, *then* a' *is a dual atom of* H, *that is* $a' \prec 1$.

Proof. The relation $o \prec a$ implies that $a \wedge a' \prec a$ and since $1 \in a \vee a'$ we will have [8] $a' \prec 1$.

Proposition (2. 4) *If for the pair* $(a, d) \in H \times H$ *we have* $a \leqslant d$ *and* $d \neq 1$, *then* $a' \nleqq d$.

Proof. We suppose that $a' \leqslant d$. Then from the relation $1 \in a \vee a'$ we get $d \in (a \vee a') \wedge d = a \vee (a' \wedge d) = a \vee a'$, which is not true, because the unique upper bound of a and a', which belongs to the union $a \vee a'$ is the element $1 \neq d$ [8]. So $a' \nleqq d$.

Proposition (2.5) *For every* $a, d \in H$ *with* $a \leqslant d$ *is* $d = \sup(a, a' \wedge d)$.

Proof. From the relation $1 \in a \vee a'$ follows that

$$d \in (a \vee a') \wedge d = a \vee (a' \wedge d).$$

But since $a \leqslant d$ and $a' \wedge d \leqslant d$ we shall have [8] $d = \sup(a, a' \wedge d)$.

Proposition (2.6) *If* x' *is the complement of an element* $x \in [a, b] \subseteq H$, *then it will exist at least one relative complement of* x *with respect to* a *and* b.

Proof. In fact, if x' is the complement of the element x then, because of the modularity, the relation $a \leqslant x \leqslant b$ implies that

$$a \vee (x' \wedge b) = (a \vee x') \wedge b.$$

By getting the right hand side of this equality it follows that

$$[(a \vee x') \wedge b] \wedge x = (a \vee x') \wedge (b \wedge x) = (a \vee x') \wedge x = a \vee (x \wedge x') = a \vee o = a$$

and
$$[(a \vee x') \wedge b] \vee x = [a \vee (x' \wedge b)] \vee x =$$
$$= (a \vee x) \vee (x' \wedge b) \supseteq x \vee (x' \wedge b) = (x \vee x') \wedge b.$$

But the relation $1 \in x \vee x'$ implies that

$$1 \wedge b = b \in (x \vee x') \wedge b \quad \text{so} \quad b \in [(a \vee x') \wedge b] \vee x.$$

From all the above we have that will exist an element $z \in (a \vee x') \wedge b$ such that the relations $z \wedge x = a$ and $b \in z \vee x$ are satisfied. On the other hand, since $z \in (a \vee x') \wedge b$ will exist an element $w \in a \vee x'$ such that $z = w \wedge b$, that is $z \leqslant b$ and from $z \wedge x = a$ follows that $a \leqslant z$ and so $z \in [a, b]$.

Consequently the element z is the relative complement of x with respect to a and b.

Thus we have the proposition :

P r o p o s i t i o n (2. 7) *Every complemented modular hyperlattice is relatively complemented.*

Consequently we have the following proposition :

P r o p o s i t i o n (2. 8) *The homomorph image of a complemented modular hyperlattice is relatively complemented hyperlattice.*

C o m p l e m e n t e d d i s t r i b u t i v e h y p e r l a t t i c e s.

Let H be a distributive hyperlattice.

P r o p o s i t i o n (2. 9) *For every* $a \in H$ *its complement, if it exists, is unique.*

P r o o f. In fact, if a' is a complement, of the element a, then $a \wedge a' = o$ and $1 \in a \vee a'$. Suppose that there exists and another element a'' sush that $a \wedge a'' = o$ and $1 \in a \vee a''$. Then the relation $1 \in a \vee a'$ implies $a'' \in (a \vee a') \wedge a'' = (a \wedge a'') \vee (a' \wedge a'') = o \vee (a' \wedge a'') = a' \wedge a''$, that is $a'' = a' \wedge a''$. Similarly we will have $a' = a'' \wedge a'$. So $a' = a''$.

P r o p o s i t i o n (2. 10) *For each* $x \in [a, b] \subseteq H$ *the relative complement of it with respect to* a *and* b*, if it exists, is unique.*

P r o o f. Let x' the relative complement of the element $x \in [a, b] \subseteq H$ witht respect to a and b, then $x \wedge x' = a$ and $b \in x \vee x'$. Let assume that there exists and another element x'' sush that $a \leqslant x'' \leqslant b$, $x \wedge x'' = a$ and $b \in b \vee x''$. From the relation $b \in x \vee x'$ we have $b \wedge x'' = x'' \in (x \vee x') \wedge x'' = (x \wedge x'') \vee (x' \wedge x'') = a \vee (x' \wedge x'')$.

Also since $b \in x \vee x''$ it will be

$$b \wedge x' = x' \in (x \vee x'') \wedge x' = (x \wedge x') \vee (x'' \wedge x') = a \vee (x' \wedge x'').$$

As H is distributive and $a \leqslant x'$, $x' \wedge x'' \leqslant x'$ we will have $z \leqslant x'$ for all $z \in a \vee (x' \wedge x'')$ [8], (rem. 11a) and consequently $x'' \leqslant x'$ since $x'' \in a \vee (x'' \wedge x')$. Similarly it is proved that $x' \leqslant x''$, thus $x' = x''$.

Going on we consider that the hyperlattice H is complemented and distributive.

P r o p o s i t i o n (2. 11) *If* $a, b \in H$ *and* a' *the complement of the element* a *then*

$$a \wedge b = o \iff b \leqslant a'$$

P r o o f. Let assume that $a \wedge b = o$. From the relation $1 \in a \vee a'$ follows that

$$1 \wedge b = b \in (a \vee a') \vee b = (a \wedge b) \vee (a' \wedge b) = o \vee (a' \wedge b) = a' \wedge b, \text{ so } b \leqslant a'$$

Conversely, if $b \leqslant a'$, that is $a' \wedge b = b$, then we have

$$(a' \wedge b) \wedge a = b \wedge a \Rightarrow a \wedge (a' \wedge b) = b \wedge a \Rightarrow (a \wedge a') \wedge b = b \wedge a \Rightarrow o =$$
$$= b \wedge a, \text{ that is } a \wedge b = o.$$

From the above proposition we conclude that the complement a' of an element a in a complemented distributive hyperlattice is the maximum element of all the elements $x \in H$, for which the relation $a \wedge x = o$ is valid.

P r o p o s i t i o n (2. 12) *If* $a, b \in H$ *and* a' *is the complement of of the element* a *then*

$$1 \in a \vee b \iff a' \leqslant b$$

P r o o f. Indeed, if $1 \in a \vee b$, then $1 \wedge a' = a' \in (a \vee b) \wedge a' =$
$= (a \wedge a') \vee (b \wedge a') = b \wedge a'$, that is $b \wedge a' = a'$. Thus $a' = b$.

Conversely, if $a' \leqslant b$, that is $a' \wedge b = a'$, then

$$1 \in a \vee a' = a \vee (a' \wedge b) \subseteq (a \vee a') \wedge (a \vee b) \text{ [rem. 1. 1b]}$$

Consequently there are $x \in a \vee a'$ and $y \in a \vee b$ such that $1 = x \wedge y$, from which we obtain $x = y = 1$. Hence $1 \in a \vee b$.

As consequence of the above proposition we have that in a complemented distributive hyperlattice the complement a' of an element a is the minimum of all the elements $x \in H$, for which we have $1 \in a \vee x$.

P r o p o s i t i o n (2. 13) *For* $a, b \in H$ *we have that*

$$a \leqslant b \iff b' \leqslant a'$$

P r o o f. In fact, from $a \leqslant b$, that is $b \in a \vee b$, we get

$$b \wedge b' \in (a \vee b) \wedge b' = (a \wedge b') \vee (b \wedge b') = (a \wedge b') \vee o = a \wedge b',$$

that is $a \wedge b' = o$. Consequently according to the proposition [2. 11] it will be $b' \leqslant a'$.

Similarly we can prove that $b' \leqslant a' \Rightarrow a \leqslant b$.

P r o p o s i t i o n (2. 14) *If* $a, b \in H$ *then*

$$b \in (a \wedge b') \vee (a' \wedge b) \Longleftrightarrow a = o.$$

P r o o f. Indeed, if $b \in (a \wedge b') \vee (a' \wedge b)$, then $b \wedge b =$ $= b \in [(a \wedge b') \vee (a' \wedge b)] \wedge b = [(a \wedge b') \wedge b] \vee [(a' \wedge b) \wedge b] = o \vee (a' \wedge b) =$ $= a' \wedge b$, that is $a' \in b \vee a'$, from which $a \wedge a' = o \in a \wedge (b \vee a') =$ $= (a \wedge b) \vee (a \wedge a') = a \wedge b$ that is $a \wedge b = o$.

Also $b \in (a \wedge b') \vee (a' \wedge b) \Rightarrow b \wedge b' = o \in [(a \wedge b') \vee (a' \wedge b)] \wedge b' =$ $= [(a \wedge b') \wedge b'] \vee [(a' \wedge b) \wedge b'] = (a \wedge b') \vee o = a \wedge b'$ that is $a \wedge b' = o$. Consequently we will have

$$(a \wedge b) \vee (a \wedge b') = a \wedge (b \vee b') = o.$$

Since $1 \in b \vee b'$, it will be $a \wedge 1 = o$, so $a = o$

Conversely, if $a = o$, then $a' = 1$, and so

$$(a \wedge b') \vee (a' \wedge b) = o \vee (1 \wedge b) = b$$

In case of a lattice the proposition (2. 14) corresponds to the law of Poretzky.

$$b = (a \wedge b') \vee (a' \wedge b) \Longleftrightarrow a = o$$

P r o p o s i t i o n (2. 15) *For all* $a \in H$, *the set*

$A = \{x : x \leqslant a\}$ *is a sub-hyperlattice and even more is a complemented distributive hyperlattice with unit the element* a.

P r o o f. Obviously the element a is the maximum of the set A, that is, its unit. If now consider $x_1, x_2 \in A$, we will have $x_1 \wedge x_2 \leqslant a$ that is $x_1 \wedge x_2 \in A$ and for all $w \in x_1 \vee x_2$ it will be [8] $w \leqslant a$, and consequently $x_1 \vee x_2 \subseteq A$. Hence the set A is a sub-hyperlattice of H and so it is distributive [pr. (1. 1)]. On the other hand, if $x \in A$ and x' is its complement into H, then $x' \wedge a \leqslant a$, and consequently $x' \wedge a \in A$. Also $x' \wedge x = o \Longrightarrow (x' \wedge x) \wedge a = (x' \wedge a) \wedge x = o$ and $1 \in x \vee x' \Longrightarrow a \in (x \vee x') \wedge a = (x \wedge a) \vee (x' \wedge a) = x \vee (a \wedge x')$. So the element $x' \wedge a$ is the complement of x into the hypperlattice A.

P r o p o s i t i o n (2. 16) *If* $a, b \in H$, *the union* $a \vee b$ *will contain an element* x *such that* $x \leqslant a$ *and* $x \leqslant b$, *if and only if* $a = b$.

P r o o f. In fact, if $a = b$, then we will have [8] $x \leqslant a$ for every $x \in a \vee b = a \vee a$.

Conversely, we suppose that there exists an element $x \in a \vee b$ such that $x \leqslant a$ and $x \leqslant b$. Then we shall have [2. 11] $x \wedge a' = o$ and $x \wedge b'$, from which relations we have

$$(x \wedge a') \vee (x \wedge b') = x \wedge (a' \vee b') = o$$

Also the relation $x \in a \vee b$ results in

$$x \wedge (a' \vee b') = o \in (a \vee b) \wedge (a' \vee b') \subseteq (a \wedge a') \vee (a \wedge b') \vee (a' \wedge b) \vee (b \wedge b') = \\ = (a \wedge b') \vee (a' \wedge b),$$

which gives on one hand

$$o \wedge b' = o \in [(a \wedge b') \vee (a' \wedge b)] \wedge b' = [(a \wedge b') \wedge b'] \vee [(a' \wedge b) \wedge b'] = \\ = (a \wedge b') \vee o = a \wedge b', \text{ that is } a \wedge b' = o$$

and on the other hand

$$o \wedge a' = o \in [(a \wedge b') \vee (a' \wedge b)] \wedge a' = [(a \wedge b') \wedge a'] \vee [(a' \wedge b) \wedge a'] = \\ = o \vee (a' \wedge b) = a' \wedge b, \text{ that is } a' \wedge b = o.$$

So by the proposition (2. 11) we will have $a \leqslant b$ and $b \leqslant a$, that is $a = b$.

R e m a r k s (2. 3) a) $o \notin a \vee a'$, because if it was $o \in a \vee a'$, then it should be $a = a'$, [pr. (2. 16)] which is not true [rem. (2. 1c)].

b) The zero element will belong to unions of equal elements, because if $o \in a \vee b$ then $a = b$ [pr. (2. 16)].

P r o p o s i t i o n (2. 17) *In a complemented distributive hyperlattice for every pair* $(a, b) \in H \times H$ *there exists the sup* (a, b) *and it is*

$$\sup(a, b) = (a' \wedge b')'.$$

P r o o f. As it is known, we have the relations $a' \wedge b' \leqslant a'$ and $a' \wedge b' \leqslant b'$ from which by the proposition (2. 13) we have $a \leqslant (a' \wedge b')'$ and $b \leqslant (a' \wedge b')'$. Hence the element $(a' \wedge b')'$ is an upper bound of a and b. If now there exists x such that $a \leqslant x$ and $b \leqslant x$, then we shall have $x' \leqslant a'$ and $x' \leqslant b'$, and then $x' \leqslant a' \wedge b'$ and finally $(a' \wedge b')' \leqslant x$. Thus $(a' \wedge b')' = \sup(a, b)$.

The next proposition gives a condition, which when is fulfilled then $\sup(a, b) \in a \vee b$.

P r o p o s i t i o n (2. 18) *If* $a' \wedge b' \prec a'$ *and* $b \nleqq a$ *then*

$$(a' \wedge b')' \in a \vee b.$$

P r o o f. From the relation $a' \wedge b' \prec a'$ follows that $a \prec (a' \wedge b')'$, because if there was x such that $a < x \leqslant (a' \wedge b')'$, we would have $a' \wedge b' < x' < a'$, which is not true. On the other hand according to the proposition (2. 17) we have $b < (a' \wedge b')'$. Consequently $(a' \wedge b')' \in a \vee b$ [8].

P r o p o s i t i o n (2. 19) *For all* $a, b \in H$ *we have*

$$(a \wedge b') \vee (a' \wedge b) = \sup(a \wedge b', a' \wedge b).$$

P r o o f. Indeed, if $w \in (a \wedge b') \vee (a' \wedge b)$ then

$$w \wedge w' = o \in [(a \wedge b') \vee (a' \wedge b)] \wedge w' = [(a \wedge b') \wedge w'] \vee [(a' \wedge b) \wedge w'].$$

Hence [pr. (2.16)] we have $(a \wedge b') \wedge w' = (a' \wedge b) \wedge w'$, from which we get $[(a \wedge b') \wedge w'] \wedge a = [(a' \wedge b) \wedge w'] \wedge a = 0$, namely $(a \wedge b') \wedge w' = 0$. Thus [pr. (2.11)] it will be $a \wedge b' \leqslant w$. Similarly, we will have $a' \wedge b \leqslant w$. So

$$\mathscr{X}_{a \wedge b'}^{(a \wedge b') \vee (a' \wedge b)} = \mathscr{X}_{a' \wedge b}^{(a \wedge b') \vee (a' \wedge b)} = (a \wedge b') \vee (a' \wedge b) = \sup (a \wedge b', a' \wedge b) \ [\mathbf{8}].$$

P r o p o s i t i o n (2.20) *For all* $a, b \in H$ *is*

$$a \vee b \subseteq \left[[(a \wedge b') \wedge (a' \wedge b)]', (a' \wedge b')'\right].$$

P r o o f. If x any element of the union $a \vee b$, then from $x \in a \vee b$ we get

$$x \wedge x' = o \in (a \vee b) \wedge x' = (a \wedge x') \vee (b \wedge x').$$

Consequently we will have

$$a \wedge x' = b \wedge x',$$

and this results in

$$(a \wedge x') \wedge b' = x' \wedge (a \wedge b') = o \quad \text{and} \quad (b \wedge x') \wedge a' = x' \wedge (a' \wedge b) = o,$$

that is $a \wedge b' \leqslant x$ and $a' \wedge b \leqslant x$ respectively. The element x as an upper bound of $a \wedge b'$ and $a' \wedge b$ will be greater or equal to

$$\sup\{(a \wedge b'), (a' \wedge b)\} = [c \wedge b')' \wedge (a' \wedge b)']'.$$

In conclusion the elements of the union $a \vee b$ belong to the interval $[[(a \wedge b')' \wedge (a' \wedge b)]' (a' \wedge b')']$.

P r o p o s i t i o n (2.21) *For all* $a, b \in H$ *we have*

$$\sup (a, b) = [b \vee (a \wedge b')] \cap [a \vee (a' \wedge b)].$$

P r o o f. Let $x \in [b \vee (a \wedge b')] \cap [a \vee (a' \wedge b)]$, that is

$$x \in b \vee (a \wedge b') \quad \text{and} \quad x \in a \vee (a' \wedge b).$$

From these last two relations we have respectively

$$x \wedge b \in [b \vee (a \wedge b')] \wedge b = b \vee [(a \wedge b') \wedge b] = b$$

that is $x \wedge b = b$ and $x \wedge a \in [a \vee (a' \wedge b)] \wedge a = a \vee [(a' \wedge b) \wedge a) = a$ that is $x \wedge a = a$. Consequently it will be $b \leqslant x$ and $a \leqslant x$. On the other hand

$$x \wedge b' \in [b \vee (a \wedge b')] \wedge b' = (b \wedge b') \vee [(a \wedge b') \wedge b'] = o \vee (a \wedge b') = a \wedge b',$$

that is $x \wedge b' = a \wedge b'$, from which

$$(x \wedge b') \wedge a' = x \wedge (a' \wedge b') = o.$$

Thus by the proposition (2.11) we shall have $x \leqslant (a' \wedge b')'$ and since $(a' \wedge b')' = \sup(a, b)$ it will be $x \leqslant \sup(a, b)$, therefore $x = \sup(a, b)$.

From all the above we conclude that

$$\sup(a, b) = \sup(a, a' \wedge b) = \sup(b, a \wedge b'),$$

because $\sup(a, b) = (a' \wedge b')'$ is an upper bound of a, $a' \wedge b$, b, $a \wedge b'$ and belongs to the union $a \vee (a' \wedge b)$, $b \vee (a \wedge b')$ [**8**].

In the case of a lattice we have

$$a \vee (b \wedge a') = b \vee (a \wedge b') = a \vee b \quad \text{that is} \quad (a' \wedge b')' = a \vee b$$

well known as the type of de Morgan of a Boolean algebra.

C o r o l l a r y (2.1) For every pair $(a, b) \in H\ H$ the product $(a \vee b) \wedge (a \wedge b)'$ is a set with only one element.

P r o o f. Indeed,

$$(a \vee b) \wedge (a \wedge b)' =$$
$$= [a \wedge (a \wedge b)'] \vee [b \wedge (a \wedge b)'] \subseteq [a \wedge [a' \vee (b' \wedge a)]] \vee [b \wedge [b' \vee (a' \wedge b)]] =$$
$$= (a \wedge a') \vee (b' \wedge a) \vee (b \wedge b') \vee (a' \wedge b) = (b' \wedge a) \vee (a' \wedge b) =$$
$$= (a \wedge b') \vee (a' \wedge b).$$

But by the proposition (2.19) we have that

$$(a \wedge b') \vee (a' \wedge b) = \sup(a \wedge b', a' \wedge b),$$

$$\text{hence} \quad (a \vee b) \wedge (a \wedge b)' = \sup(a \wedge b', a' \wedge b).$$

Note. In the classical theory, a distributive complemented lattice is called *Boolean algebra* and as it is known, in every such an algebra $(B, \vee, \wedge)$ corresponds bijectively a ring $(B, +, .)$ called *Boolean ring.* As we know, the operations of these structures are related to each other by

$$a \vee b = a + b + ab, \quad a + b = (a \wedge b') \vee (a' \wedge b) \quad ab = a \wedge b, \quad \text{for all} \quad a, b \in B.$$

Therefore it is natural of course, and in the case of the hypercomposed structures, to name *Boolean hyperalgebra* every distributive complemented hyperlattice.

In our paper together with professor J. Mittas, titled «Introduction à l'hyperalgèbre de Boole» [7] we deal exactly with the construction of this structure. Indeed, in this paper we prove that a hyperring $(H, +, .)$ of special form, called *Boolean strong hyperring* [2], namely a Boolean hyperring in which for all $a, b \in H$ the condition

$$a + b = \{w \in H : a'w' = b'w' = a'b'\}$$

is satisfied or equivalently the condition

$$a + b = \{w \in H : \sup(a, w) = \sup(b, w) = \sup(a, b)\},$$

which relates the hyperoperation of the hyperring with the order relation of H, is at the same time a Boolean hyperalgebra and even more a *strong Boolean hyperalgebra,* because satisfies one more axiom:

For all $a, b \in H$ we have

$$a \vee b = \{w \in H : a'w' = b'w' = a'b'\}$$

or equivalently

$$a \vee b = \{w \in H : \sup(a, w) = \sup(b, w) = \sup(a, b)\}$$

which relates the hyperoperation of the hyperlattice H with the order relation of H. We prove also in this paper that, and conversely every strong Boolean hyperalgebra $(H, \vee, \wedge)$ is also a powerful Boolean hyperring (H, +, .) and the operations and hyperoperations of their structures are related to each other as follow:

$$a+b = a \vee b \qquad ab = a \wedge b.$$

We note also that these structures have o as unique scalar element.

ΠΕΡΙΛΗΨΗ

Στὴ θεωρία τῶν ὑπερσυνθετικῶν δομῶν ἡ θεώρηση τῶν ὑπερδακτυλίων τοῦ Boole ὁδήγησε, ὡς γνωστόν, στὴν εἰσαγωγὴ τῆς ἰσχυρῆς ὑπεράλγεβρας τοῦ Boole [2] [7] καὶ μέσω αὐτῆς στὴ γενικὴ θεωρία τῶν ὑπερδικτυωτῶν [3]. Ἀπὸ τὰ τελευταῖα αὐτὰ στὴν παροῦσα ἐργασία πραγματεύομαι τὶς εἰδικὲς κατηγορίες τῶν ἐπιμεριστικῶν, συμπληρωματωμένων καὶ σχετικῶς συμπληρωματωμένων ὑπερδικτυωτῶν.

Τὰ ἐπιμεριστικὰ ὑπερδικτυωτὰ ὁρίζονται ὅπως ἀκριβῶς καὶ τὰ ἐπιμεριστικὰ δικτυωτά.

Χαρακτηριστικὸ ὅμως στὴν περίπτωση τῶν ὑπερδικτυωτῶν αὐτῶν εἶναι, ὅτι δὲν ἰσχύει ἐν γένει σ' αὐτὰ ἡ ἐπιμεριστικότητα τῆς ἑνώσεως ὡς πρὸς τὴν τομή, πρᾶγμα ποὺ ἰσχύει γιὰ τὰ συνήθη δικτυωτά, καθὼς ἐπίσης ὅτι ὁρισμένες προτάσεις, ποὺ εἶναι ἀναγκαῖες καὶ ἱκανὲς συνθῆκες γιὰ τὴν ἐπιμεριστικότητα τῶν δικτυωτῶν, στὴν περίπτωση τῶν γνήσιων ὑπερδικτυωτῶν εἶναι μόνο ἀναγκαῖες ὅπως αὐτὸ προκύπτει ἀπὸ σχετικὰ παραδείγματα.

Στὴ συνέχεια δίνεται ὁ ὁρισμὸς τοῦ συμπληρώματος ἑνὸς στοιχείου καὶ τοῦ σχετικοῦ συμπληρώματός του ἀναφορικὰ πρὸς δύο στοιχεῖα του καὶ ἀμέσως μετὰ καὶ κατὰ τρόπο ἀνάλογο πρὸς τοὺς ἀντιστοίχους ὁρισμοὺς τῆς κλασσικῆς θεωρίας οἱ ὁρισμοὶ τῶν συμπληρωματωμένων καὶ σχετικῶς συμπληρωματωμένων ὑπερδικτυωτῶν, τὰ ὁποῖα καὶ μελετῶνται διεξοδικά. Μελετῶνται ἐπίσης οἱ ἀκόμη πιὸ εἰδικὲς κατηγορίες τῶν τροπικῶν συμπληρωματωμένων καὶ τῶν ἐπιμεριστικῶν συμπληρωματωμένων ὑπερδικτυωτῶν. Τὰ τελευταῖα αὐτὰ ἀποτελοῦν τὴ γενικὴ μορφὴ ὑπεραλγεβρῶν τοῦ Boole, οἱ ὁποῖες ὡς δομὲς ὑπερσυνθετικὲς εἶναι ἀντίστοιχες τῶν ἀλγεβρῶν τοῦ Boole τῆς κλασσικῆς θεωρίας, εἰδικὴ μορφὴ τῶν ὁποίων εἶναι οἱ παραπάνω ἀναφερθεῖσες ἰσχυρὲς ὑπεράλγεβρες τοῦ Boole. Ἀποδεικνύεται ὅτι στὶς ὑπεράλγεβρες τοῦ Boole ὑπάρχει πάντα τὸ supremum δύο στοιχείων (πρᾶγμα ποὺ δὲν ἰσχύει, ὅπως εἶναι γνωστό, σ' ἕνα ὁποιοδήποτε γνήσιο ὑπερδικτυωτὸ [3]).

Γενικὰ παρατηροῦμε ὅτι ἰσχύουν γιὰ τὰ ὑπερδικτυωτὰ τῶν πιὸ πάνω κατηγοριῶν πολλὲς προτάσεις τῶν δικτυωτῶν, προκύπτουν ὅμως καὶ πολλὲς νέες, ποὺ ὀφείλονται στὸ ὅτι ἡ ἕνωση σ' αὐτὰ εἶναι ὑπερπράξη.

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