ΑΝΑΚΟΙΝΩΣΙΣ ΜΗ ΜΕΛΟΥΣ

ΓΕΩΜΕΤΡΙΑ. – **On a Common Factor Characteristic of Some Plane Coordinate Systems and its Significance in the Analytical Transformations of Equations,** *by C. B. Glavas**. Ἀνεκοινώθη ὑπὸ τοῦ Ἀκαδημαϊκοῦ κ. Ἰωάνν. Ξανθάκη.

Introduction: The purpose of this paper is first to investigate a common factor or characteristic underlying most of the known plane coordinate systems. Second to stress the role of this factor in the transformations among coordinate systems and especially in the use of a possible new system of coordinates for the simplification of equations of curves and for the solution of differential equations.

The procedure in this work consists first from the examination of the Cartesian systems and then from its generalization to other systems. The outcome of the whole discussion leads to the last step, which is the definition of a new possible system and the examination of its advantageous use in the analytical transformations.

1. The formulae of transformation from the Cartesian rectangular system xOy to the oblique x′Oy′ are:

$$x = x'\cos\omega + y'\cos(\omega+\varphi) \qquad (1.1)$$
$$y = x'\sin\omega + y'\sin(\omega+\varphi)$$

Here ω and φ denote the angles xOx′ and x′Oy′ respectively. If Ox′ coincides with Ox, then $\omega = 0$ and the formulae 1.1. become:

$$x = x' + y'\cos\varphi \qquad (1.2)$$
$$y = y'\sin\varphi$$

For purposes of convenience we shall call the axis Ox, which is common to all systems, «basic» axis. The oblique coordinates of any point B (Fig.1) are the segments $OA = x'$ and $AB = y'$. The direction of AB is always parallel to Oy′ making with the basic axis an angle φ equal to the angle xOy′ of the two axes. For each value of φ there corresponds a definite oblique system. It is therefore clear that an infinite number of oblique systems exist, the rectangular corresponding to $\varphi = 90^0$.

* Χ. Β. ΓΚΛΑΒΑ: Ἐπί τινος κοινοῦ χαρακτηριστικοῦ παράγοντος ἐπιπέδων τινῶν συστημάτων συντεταγμένων καὶ τῆς σημασίας του κατὰ τοὺς ἀναλυτικοὺς μετασχηματισμοὺς ἐξισώσεων.

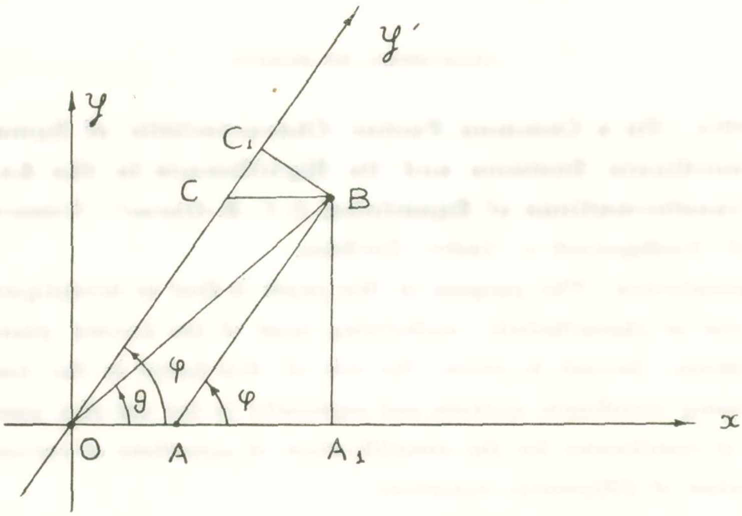

Fig. 1

The angle φ which enters in the formulae 1.2 constitutes an undetermined quantity. In a transformed equation from the rectangular system to the oblique φ and vice versa it may be possible to determine φ to serve a desirable purpose (simplification of the equation, elimination of certain terms etc.). The following examples indicate the importance of the angle φ in various transformations.

Example 1.1: Let the general equation of the second degree be:

(1.11) $$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$$

In textbooks of Analytic Geometry the usual method for the reduction of 1.11 to the standard form is the application first of a rotation of the rectangular axes by an angle ω and second of a translation of the origin to a new point. However we may transform equation 1.11 to oblique axes φ as follows. Applying formulae 1.2 we get:

$$A(x'^2+y'^2\cos^2\varphi+2x'y'\cos\varphi)+Bx'y'\sin\varphi+By'^2\sin\varphi\cos\varphi+$$
$$+Cy'^2\sin^2\varphi+Dx'+Dy'\cos\varphi+Ey'\sin\varphi+F=0$$

And:

(1.12) $$Ax'^2+(2A\cos\varphi+B\sin\varphi)x'y'+(A\cos^2\varphi+B\sin\varphi\cos\varphi+C\sin^2\varphi)y'^2+$$
$$+Dx'+(D\cos\varphi+E\sin\varphi)y'+F=0$$

Equating the coefficient of $x'y'$ to zero we find:

$$\tan\varphi=-\frac{2A}{B}$$

Eliminating φ between the latter equation and 1.12 we finally find:

$$A(B^2+4A^2)x'^2+A(4AC-B^2)y'^2+D(B^2+4A^2)x'+ \\ +(DB-2AE)\sqrt{B^2+4A^2}y'+F(B^2+4A^2)=0 \tag{1.13}$$

A translation $x'=x''+a, y'=y''+b$ transforms finally the latter equation to the following one of the standard form:

$$\frac{x''^2}{\frac{1}{B^2+4A^2}}+\frac{y''^2}{\frac{1}{4AC-B^2}}=\frac{B^2+4A^2}{4A^2}\left[D^2+\frac{(DB-2AE)^2}{4AC-B^2}-4AF\right]=K$$

It is clear now that the type of the curve this equation represents depends on the signs of $4AC-B^2$ and of the right side K. If for example $4AC-B^2=0$, then 1.13 becomes:

$$A(B^2+4A^2)x'^2+D(B^2+4A^2)x'+(DB-2AE)\sqrt{B^2+4A^2}y'+ \\ +F(B^2+4A^2)=0$$

The translation $x'=x''+a$, $y'=y''+b$ can eliminate the coefficient of x'' and the constant term when the equation takes its standard form $x^2=2py$, if we drop the primes.

Example 1.2: The problem now is to refer the hyperbola $b^2x^2-a^2y^2-a^2b^2=0$ to its asymptotes as axes. From figure 2 it is clear that $2\omega+\varphi=\pi$ and $\varphi'=\omega+\varphi$. But $\varphi'+\omega=\pi$ which gives $\omega=\pi-\varphi'$. If we substitute in 1.1 $\omega+\varphi$ and ω for φ' and $\pi-\varphi'$ respectively we get:

$$x=x'\cos(\pi-\varphi')+y'\cos\varphi'=-x'\cos\varphi'+y'\cos\varphi'=(y'-x')\cos\varphi'$$
$$y=x'\sin(\pi-\varphi')+y'\sin\varphi'=x'\sin\varphi'+y'\sin\varphi'=(x'+y')\sin\varphi'$$

Substituting these values of x,y in the equation of hyperbola we finally get:

$$(b^2\cos^2\varphi'-a^2\sin^2\varphi')(x'^2+y'^2)-2(b^2\cos^2\varphi'+a^2\sin^2\varphi')x'y'-a^2b^2=0$$

Putting the coefficient of $x'^2+y'^2$ equal to zero we find $\tan\varphi'=\pm\frac{b}{a}$ and the equation of the hyperbola referred to the oblique system φ' becomes $x'y'=\text{const}$.

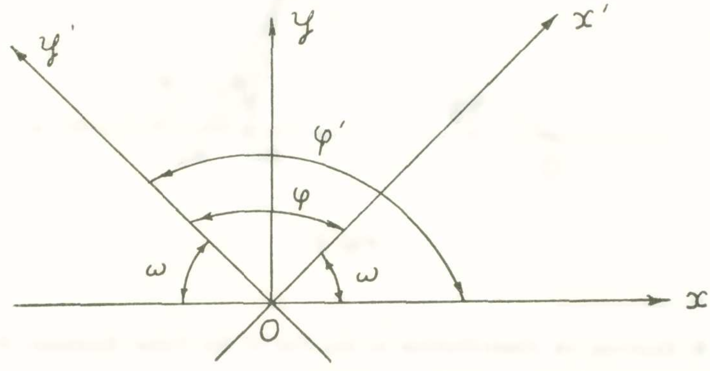

Fig. 2

Note: A special case of the oblique systems φ is the so called perpendicular system where the coordinates of a point B (Fig.1) are the segments $BA_1=y'$ and $BC_1=x'$, BA_1, BC_1 being perpendicular to the oblique axes Ox',Oy' respectively. It is again clear, that the coordinate segments BA_1, BC_1 make an angle φ equal to the angle $x'Oy'$. If $BA=y$, $BC=x$, then $x'=x\sin\varphi$, $y'=y\sin\varphi$. Of course it is easy to express the perpendicular coordinates in terms of the rectangular Cartesian ones. The important thing is the fact that the angle φ enters again in the formulae of transformation as an undetermined factor.

2. Now we are going to extend the previous remarks to systems in which one of their coordinates is an angular measure. In a final analysis the oblique systems φ may be referred to the triangle OAB (Fig.1) and the angle φ which the side AB makes with the basic axis.

In the cathetic system the coordinates of a point B are ϑ and $OA_2=g$, BA_2 being perpendicular to the radius vector OB (Fig.3). We observe here that the angle $\varphi'=\vartheta+\frac{\pi}{2}$, i.e. the angle φ' is a function of ϑ. In the polar system under its new form[1] the coordinates of B are ϑ and $OA_1=r(r=OB)$. From the isosceles triangle OA_1B it is easily found that $\varphi=\frac{\vartheta}{2}+\frac{\pi}{2}$. Therefore φ is again a function of ϑ. It should be noted also that $\varphi'-\varphi=\frac{\vartheta}{2}$.

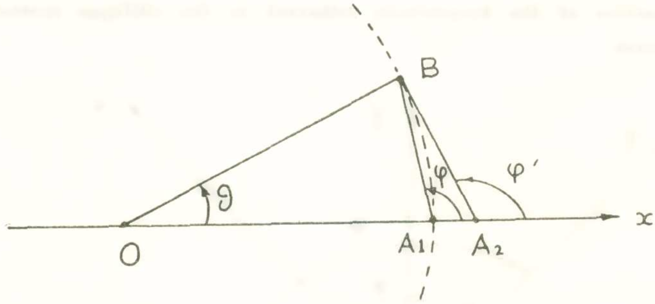

Fig. 3

[1] C. B. GLAVAS, «A Contribution to the Use of the Polar System», Proceedings of the Academy of Athens, 33 (1958), p. 342—353.

When we make the transformation from the rectangular system to the polar or the cathetic no undetermined factor φ enters in the respective analytical relations. It is expected only from the form of these relations to obtain a desired simplification. But if the said relations contain in addition an undetermined factor the chances for simplifications become higher. It is not difficult to see that where φ is a function of ϑ the angle φ is not found in the formulae of transformation. In the oblique systems of case 1 angle φ is constant for each system. Now it is natural to examine what happens in the systems whose one coordinate is the angular measure ϑ if φ remains constant.

If φ is constant the coordinates of a point B are ϑ and $OA=\gamma$ (Fig 4). BA moves always parallel to itself making with the basic axis a constant angle φ. It is clear that for each point B of the plane there corresponds a pair of numbers (γ,ϑ). The converse is also true under the same remarks which accompany the similar case of the polar (or the cathetic) system [2].

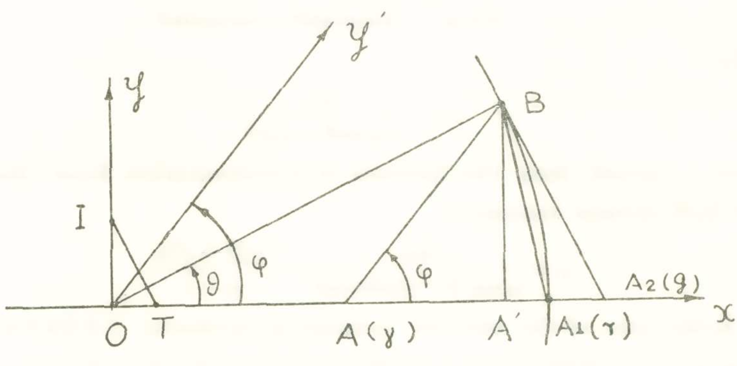

Fig. 4

The rectangular coordinates of B are $OA'=x$, $A'B=y$. We have $y=x\tan\vartheta$, $y=AA'\tan\varphi=(x-\gamma)\tan\varphi$. Hence $x\tan\vartheta=(x-\gamma)\tan\varphi$ from which we get $x=\dfrac{\gamma\tan\varphi}{\tan\varphi-\tan\vartheta}$. Therefore the formulae of transformation from the rectangular system to the (γ,ϑ) system are:

$$x=\frac{\gamma\tan\varphi}{\tan\varphi-\tan\vartheta},\ y=\frac{\gamma\tan\varphi\tan\vartheta}{\tan\varphi-\tan\vartheta} \tag{2.1}$$

[2] Loc. cit.

These formulae may be simplified if we put $\tan\varphi = t_o$ and $\tan\vartheta = t$ Then formulae 2.1 become:

$$x = \frac{\gamma t_o}{t_o - t},\quad y = \frac{t_o \gamma t}{t_o - t} \tag{2.2}$$

If we solve 2.2 for γ, t we get:

$$\gamma = x - \frac{y}{t_o},\quad t = \frac{y}{x} \tag{2.3}$$

Instead of the system (γ,ϑ) we can use its tangential form (γ,t), where $t = \tan\vartheta$. If we take on Oy axis (Fig. 4) $OI = 1$ and drop from I a perpendicular on OB, then it is easily found that $OT = \tan\vartheta = t$. The system (γ,t) is the tangential form of (γ,ϑ) and may be used in cases where the formulae of transformation contain only the $\tan\vartheta$ [8].

From triangle OAB we get:

$$\frac{r}{\sin(\pi - \varphi)} = \frac{\gamma}{\sin(\varphi - \vartheta)}$$

Or:

$$\frac{r}{\sin\varphi} = \frac{\gamma}{\sin\varphi\cos\vartheta - \cos\varphi\sin\vartheta}$$

And finally:

$$r = \frac{t_o \gamma}{t_o \cos\vartheta - \sin\vartheta} \tag{2.4}$$

Since $r = g\cos\vartheta$, then the formula of transformation from the cathetic to the (γ,ϑ) system becomes:

$$g = \frac{t_o \gamma}{t_o \cos^2\vartheta - \sin\vartheta\cos\vartheta} = \frac{t_o(1 + t^2)\gamma}{t_o - t} \tag{2.5}$$

As in the case of the oblique systems φ, formulae 2.2,2.3,2.4,2.5 contain the angular undetermined quantity φ due to the fact that the systems (γ,ϑ) correspond to $\varphi = \text{const}$. Before we proceed to a further examination of the (γ,ϑ) system we remark that the most known coordinate systems are contained in the symbol $[(OAB)\ (\varphi)]$, where OAB is the triangle of figure 4 and φ the angle of the side AB with the basic axis, which may be expressed by the general formula $\varphi = a\vartheta + b$. If $a = 0$, then we get the oblique systems as well as the «polar» ones (γ,ϑ). For $b = \frac{\pi}{2}$, $a = 0$ we get the

[8] C. B. GLAVAS, «Plane Coordinate Systems in Mathematics Study». Doctoral Dissertation, New York, Teachers College, Columbia University, 1956, p. 146—152.

rectangular coordinates. For $a=1$, $b=\frac{\pi}{2}$ we take the cathetic and for $a=\frac{1}{2}$ and $b=\frac{\pi}{2}$ the polar system in common use. From all the possible known two systems of $\varphi=a\vartheta+b$ the most important from the point of view of introducing undetermined factors in the transformations are the corresponding to $\varphi=\text{const}$.

It should be noted that for $a=0$ and $b=\frac{\pi}{2}$ we have the (x,ϑ) system [4]. In figure 4, $x=OA'$. Clearly, the formula of transformation from the (γ,ϑ) to the (x,ϑ) systems is the first of 2.2. This shows the close relation which exists between the oblique systems φ and the polar systems φ.

3. For the examination of the meaning of the derivative $\frac{d\gamma}{dt}$ we divide after differentiating formulae 2.2 and we get:

(3.1)
$$\frac{dy}{dx}=\frac{t(t_0-t)\frac{d\gamma}{dt}+t_0\gamma}{(t_0-t)\frac{d\gamma}{dt}+\gamma}$$

For a γ maximum or minimum of a curve we must substitute in 3.1 $\frac{d\gamma}{dt}=0$. Then we take $\frac{dy}{dx}=t_0=\tan\varphi$. But $\frac{dy}{dx}=\tan\omega$ (Fig. 5). Comparing the latter relations we conclude that $\omega=\varphi$, which means that at a point of γ max. or min. the tangent must be parallel to the direction of Oy' axis which is really the case as figure 5 shows.

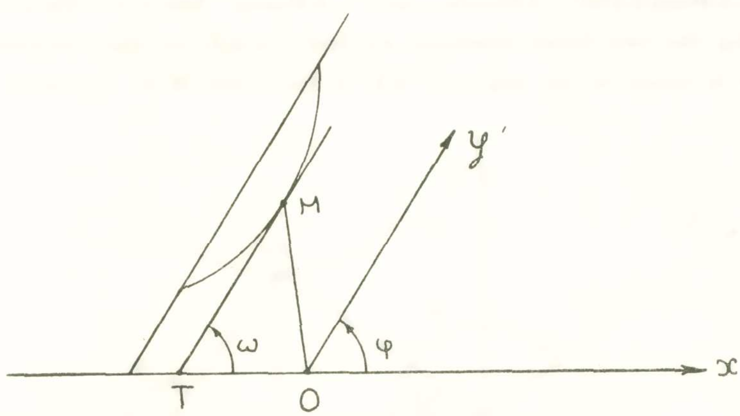

Fig. 5

[4] Op. cit., p. 106.

If we have a y max. or min. point then we substitute in 3.1 $\frac{dy}{dx}=0$. Hence we get:

$$t\,(t_o-t)\frac{d\gamma}{dt}+t_o\gamma=0$$

Solving for $\frac{d\gamma}{dt}$:

$$\frac{d\gamma}{dt}=\frac{t_o\gamma}{t(t-t_o)}$$

And:

$$\frac{d\gamma}{\gamma}=\frac{t_o dt}{t(t-t_o)}$$

Integrating both sides we find:

$$\gamma=C\frac{t-t_o}{t}$$

Transforming the latter equation to rectangular coordinates (formulae 2.3) we finally get:

$$(t_o x-y)(y+Ct_o)=0$$

Hence there must be either $t_o x-y=0$ or $y+Ct_o=0$. If $t_o x-y=0$, then $\frac{y}{x}=t_o=\tan\varphi$. But $\frac{y}{x}=\tan\vartheta$ (Fig.4). Therefore we must have $\vartheta=\varphi$ which is impossible because $\varphi>\vartheta$. The other possibility is $y+Ct_o=0$ or $y=-Ct_o$. This means that $y=$const., which is really the case for a y max. or min. On the other hand from figure 6 it is easy to see that max. $y=MA=AB\tan(ABM)=AB\tan(\pi-\varphi)=-AB\tan\varphi$. But $y=-Ct_o=-C\tan\varphi$. Comparing the two latter relations we find $C=AB$, i.e. the constant of integration is equal to the segment AB if the point M is a y max. or min.

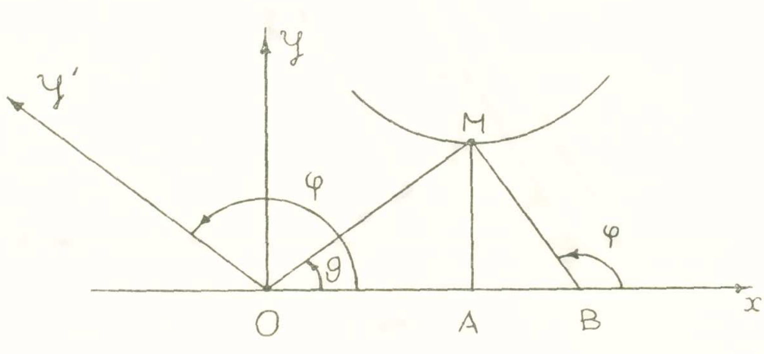

Fig. 6

The following examples demonstrate the value of the undetermined factor of the (γ,t) systems in various situations.

Example 3.1: Let be given the equation:

(3.11) $$y^2(x+ay)-y^3-2xy+x^2=0$$

Transforming 3.11 to the system (γ,t) (formulae 2.2) we get after the necessary manipulations:

$$t_o\gamma t^2+t_o(a-1)\gamma t^3-(t_o-t)(2t-1)=0$$

Now we put $t_o.(a-1)=-1$. Then:

$$t_o\gamma t^2-\gamma t^3-(t_o-t)(2t-1)=0$$

And:

$$\gamma t^2(t_o-t)-(t_o-t)(2t-1)=0$$

Or:

$$(t_o-t)(\gamma t^2-2t+1)=0$$

Since $t \neq t_o(\vartheta \langle \varphi)$, equation 3.11 finally takes the much simpler form:

(3.12) $$\gamma t^2-2t+1=0$$

It is easier to study the properties of the curve 3.11 under the simplified expression 3.12. The result is due to the choice of a (γ,t) system such that $t_o=\frac{1}{1-a}$ or $\tan\varphi=\frac{1}{1-a}$.

Example 3.2: Let be given the equation:

(3.21) $$x^2-axy-y=0$$

Transforming as before to the (γ,t) system:

$$t_o\gamma(1-at)-t(t_o-t)=0$$

Or:

$$at_o\gamma(\frac{1}{a}-t)-t(t_o-t)=0$$

For the simplification it is clear that a system (γ,t) corresponding to $t_o=\frac{1}{a}$ must be chosen. Then:

$$at_o\gamma(t_o-t)-t(t_o-t)=0$$

And:

$$(t_o-t)(\gamma-t)=0$$

Since $t_o \neq t$ we finally get the very simple equation:

$$\gamma=t$$

Example 3.3: Let be given the partial differential equation:

(3.31) $$(p+q)(px+qy)=1$$

From formulae 2.3 we take:

(3.32) $$\frac{\partial\gamma}{\partial x}=1, \frac{\partial\gamma}{\partial y}=-\frac{1}{t_o}, \frac{\partial t}{\partial x}=-\frac{y}{x^2}=-\frac{t}{x}=-t\frac{t_o-t}{\gamma t_o}=\frac{t(t-t_o)}{\gamma t_o}, \frac{\partial t}{\partial y}=\frac{1}{x}=\frac{t_o-t}{\gamma t_o}$$

And:

$$P=\frac{\partial z}{\partial x}=\frac{\partial z}{\partial\gamma}\frac{\partial\gamma}{\partial x}+\frac{\partial z}{\partial t}\frac{\partial t}{\partial x}, q=\frac{\partial z}{\partial y}=\frac{\partial z}{\partial\gamma}\frac{\partial\gamma}{\partial y}+\frac{\partial z}{\partial t}\frac{\partial t}{\partial y}$$

Putting $\frac{\partial z}{\partial\gamma}=P, \frac{\partial z}{\partial t}=Q$ and substituting the values of $\frac{\partial\gamma}{\partial x}, \frac{\partial\gamma}{\partial y}, \frac{\partial t}{\partial x}, \frac{\partial t}{\partial y}$ from 3.2 we get:

(3.33) $$p=P+Q\frac{t(t-t_o)}{\gamma t_o}, q=-\frac{P}{t_o}+Q\frac{t_o-t}{\gamma t_o}$$

Substituting p,q from 3.33 and x,y from 2.2 in 3.31 we finally take:

$$P\gamma\left[P(1-\frac{1}{t_o})+Q\frac{t-t_o}{\gamma t_o}(t-1)\right]=1$$

Clearly, the simplification of this partial equation requires to put $1-\frac{1}{t_o}=0$ or $t_o=1$ Hence $\tan\varphi=1$ or $\varphi=\frac{\pi}{4}$. Then the equation becomes:

(3.34) $$PQ(t-1)^2=1$$

Here the dependent variable is absent and there is a quasi separation of variables[5]. Therefore we may put $Q(t-1)^2=a, \frac{1}{P}=a$, or $Q=\frac{a}{(1-t)^2}$, $P=\frac{1}{a}$. But we have:

$$dz=\frac{\partial z}{\partial\gamma}d\gamma+\frac{\partial z}{\partial t}dt$$

Or:

$$dz=Pd\gamma+Qdt$$

Substituting in the latter equation P and Q for $\frac{1}{a}$ and $\frac{a}{(t-1)^2}$, respectively we take:

$$dz=\frac{1}{a}d\gamma+\frac{a}{(t-1)^2}dt$$

This total differential is evidently exact. Integrating we get:

(3.35) $$z=\frac{\gamma}{a}-\frac{a}{t-1}+b$$

[5] A. Cohen, An Elementary Treatise on Differential Equations, Boston, D.C. Heath and Co., c 1933, p. 270-71.

If we want to return to the variables x,y we substitute in 3.35 the equals of γ and t from 2.3. where $t_0 = 1$. Then we get:

$$a(x-y)(z-b)=(x-y)^2+a^2x \tag{3.36}$$

These examples demonstrate the suitability for transformations of the oblique systems φ and the other systems φ which may be named «parapolar systems». Of course further research is necessary to reveal some more characteristics of all these systems which may be called parametric. The important thing in such transformations is that the simplified relation may be considered under the new system. There is not merely a change of variables since the latter refer to a definite coordinate system. For example equation 3.35 is referred to the system (γ,t,z) and its investigation may be more desirable through this system than through the rectangular (x,y,z).

ΠΕΡΙΛΗΨΙΣ

Εἰς ἕνα πλαγιογώνιον σύστημα Καρτεσιανῶν συντεταγμένων $\chi'O\psi'$ ὁ ἄξων $O\psi'$ σχηματίζει γωνίαν φ μετὰ τοῦ ἄξονος $O\chi'$. Ἐὰν ὁ ἄξων $O\chi'$ συμπίπτῃ μὲ τὸν ἄξονα $O\chi$ τοῦ ὀρθογωνίου συστήματος $\chi O\psi$, τότε κατὰ τοὺς μετασχηματισμοὺς ἐκ τοῦ συστήματος $\chi O\psi$ εἰς τὸ $\chi'O\psi'$ καὶ ἀντιστρόφως εἰσέρχεται ὡς ἀπροσδιόριστος παράγων ἡ ἐν λόγῳ γωνία φ. Ἐφόσον εἰς ἑκάστην τιμὴν τοῦ φ ἀντιστοιχεῖ ἓν σύστημα $\chi'O\psi'$, εἰς τὴν πραγματικότητα ὁρίζεται ἀπειρία τοιούτων συστημάτων, τῶν ὁποίων τὸ ὀρθογώνιον $\chi O\psi$ εἶναι μερικὴ περίπτωσις ($\varphi = 90°$).

Εἰς τὴν ἀνακοίνωσιν ταύτην ἐξετάζεται κατὰ πρῶτον ὁ ρόλος τοῦ παράγοντος φ κατὰ τοὺς μετασχηματισμοὺς τῶν πλαγιογωνίων συστημάτων καὶ διὰ συγκεκριμένων παραδειγμάτων δεικνύεται ἡ δυνατότης ἁπλουστεύσεως ἐξισώσεων καμπυλῶν, διὰ τὰς ὁποίας ἀκολουθοῦνται συνήθως εἰς κείμενα τῆς Ἀναλυτικῆς Γεωμετρίας ἄλλαι μέθοδοι. Ἐν συνεχείᾳ ἐπεκτείνεται ἡ ἔρευνα αὕτη εἰς συστήματα τῶν ὁποίων ἡ μία συντεταγμένη εἶναι ἡ πολικὴ γωνία ϑ. Διαπιστοῦται, ὅτι τὸ Καρτεσιανὸν ὅσον καὶ τὰ ἄλλα συστήματα ἀνάγονται εἰς ἓν βασικὸν τρίγωνον OAB, τοῦ ὁποίου ἡ πλευρὰ AB σχηματίζει μὲ τὸν ἄξονα OA(ἢ$O\chi$) γωνίαν φ. Ἂν ἡ γωνία φ εἶναι συνάρτησις τῆς πολικῆς γωνίας ϑ (πολικόν, καθετικὸν σύστημα), τότε δὲν εἰσέρχεται ὁ παράγων φ εἰς τοὺς τύπους μετασχηματισμοῦ ἀπὸ τῶν συστημάτων τούτων εἰς ἄλλα καὶ ἀντιστρόφως.

Ἐὰν ἡ γωνία φ εἶναι σταθερὰ καὶ ἀνεξάρτητος τῆς ϑ, ὁρίζεται μία οἰκογένεια συστημάτων, τὰ ὁποῖα ἐκλήθησαν «παραπολικά». Ὅπως καὶ εἰς τὰ πλαγιογώνια συστήματα, οὕτω καὶ εἰς τὰ παραπολικὰ οἱ τύποι μετασχηματισμοῦ περιέχουν τὸν ἀπροσδιόριστον παράγοντα φ· Τῶν συστημάτων τούτων ἐξετάζονται διάφορα χαρακτηριστικὰ καὶ διατυποῦνται οἱ τύποι μετασχηματισμοῦ εἰς ἄλλα συστήματα. Ἡ ἀξία τῶν παραπολικῶν τούτων συστημάτων φαίνεται ἐκ τῶν σχετικῶν παραδειγμάτων, ὅπου διὰ καταλλήλου προσδιορισμοῦ τῆς γωνίας φ ἐπιτυγχάνεται ὁ μετασχηματισμὸς τῶν διδομένων σχέσεων εἰς Καρτεσιανὰς συντεταγμένας εἰς κατὰ πολὺ ἁπλουστέρας παραπολικά. Μεταξὺ τῶν παραδειγμάτων τούτων περιλαμβάνεται ἡ διὰ τῆς ὡς ἄνω μεθόδου ἐπίλυσις διαφορικῆς ἐξισώσεως μὲ μερικὰς παραγώγους.

Ὅθεν διὰ τῆς ἀνακοινώσεως ταύτης γίνεται πρῶτον ἀναγωγὴ τῶν πλέον γνωστῶν ἐπιπέδων συστημάτων συντεταγμένων εἰς τὸν αὐτὸν χαρακτηριστικὸν παράγοντα φ, εἰς διαφόρους τιμὰς τοῦ ὁποίου ἀντιστοιχεῖ ἕκαστον σύστημα. Ἀπὸ τῆς ἀπόψεως τῶν μετασχηματισμῶν μετ' ἀπροσδιορίστου παράγοντος δεικνύεται ἡ σπουδαιότης τῶν συστημάτων τῶν ἀντιστοιχούντων εἰς σταθερὸν φ. Τέλος διὰ γενικεύσεως τοῦ πλαγιογωνίου συστήματος φ ἐπιτυγχάνεται ὁ ὁρισμὸς τῶν παραπολικῶν συστημάτων φ, τῶν ὁποίων ἐπισημαίνεται ἡ ἀξία κατὰ τοὺς ἀναλυτικοὺς μετασχηματισμοὺς ἐξισώσεων.

BIBLIOGRAPHY

CAMPBELL, A.D. Advanced Analytic Geometry. New York, John Wiley and Sons, Inc., 1938.

COHEN, A. An Elementary Treatise on Differential Equations. BOSTON, D.C. Heath and Co., c 1933.

GLAVAS, C. B. «Plane Coordinate Systems in Mathematics Study». Doctoral Dissertation. New York, Teachers College, Col. Uni., 1956.

SMITH, P. E. and OTHERS. New Analytic Geometry. Rev. Ed Boston, Ginn and Co., c1928.

Ὁ Ἀκαδημαϊκὸς κ. **Ἰωάνν. Ξανθάκης** ἀνακοίνωσε τὴν ἀνωτέρω μελέτην διὰ τῶν κάτωθι.

Εἰς τὴν παροῦσαν ἀνακοίνωσιν ὁ κ. Χ. Γκλαβᾶς ἐξετάζει πρῶτον τὸν ρόλον τὸν ὁποῖον διαδραματίζει ἡ γωνία φ τῶν ἀξόνων εἰς ἓν πλαγιογώνιον σύστημα συντεταγμένων κατὰ τοὺς διαφόρους μετασχηματισμοὺς καὶ δεικνύει τὴν δυνατότητα ἁπλουστεύσεως τῶν χρησιμοποιουμένων συνήθως σχέσεων εἰς τὰ κλασσικὰ συγγράμματα τῆς Ἀναλυτικῆς Γεωμετρίας. Ἐν συνεχείᾳ ἐπεκτείνει τὴν ἔρευναν ταύτην εἰς συστήματα τῶν ὁποίων ἡ μία συντεταγμένη εἶναι πολικὴ γωνία. Εἰς τὴν περίπτωσιν ταύτην διαπιστοῦται, ὅτι τόσον τὸ Καρτεσιανὸν σύστημα ὡς καὶ τὰ ἄλλα τοιαῦτα ἀνάγονται εἰς ἓν βασικὸν τρίγωνον, τοῦ ὁποίου ἡ μία πλευρὰ σχηματίζει μὲ τὸν ἄξονα τῶν τετμημένων γωνίαν φ. Ἀποδεικνύεται δέ, ὅτι ἡ γωνία αὕτη φ δὲν ὑπεισέρχεται εἰς τοὺς τύπους μετασχηματισμοῦ ἀπὸ τῶν συστημάτων τούτων εἰς ἄλλα, ὅταν αὕτη εἶναι συνάρτησις τῆς πολικῆς γωνίας.

Τέλος ἀποδεικνύεται, ὅτι, ὅταν ἡ γωνία φ εἶναι σταθερὰ καὶ ἀνεξάρτητος τῆς πολικῆς γωνίας, δυνάμεθα νὰ ὁρίσωμεν οἰκογένειαν συστημάτων, τὰ ὁποῖα ὁ συγ-

γραφεὺς καλεῖ παραπολικά, εἰς τὰ ὁποῖα οἱ τύποι μετασχηματισμοῦ περιέχουν τὸν ἀπροσδιόριστον παράγοντα φ ὡς καὶ εἰς τὰ πλαγιογώνια συστήματα. Ἡ ἀξία τῶν παραπολικῶν τούτων συστημάτων φαίνεται ἐκ τῶν παρατιθεμένων παραδειγμάτων, ὅπου διὰ καταλλήλου προσδιορισμοῦ τῆς γωνίας φ ἐπιτυγχάνεται ὁ μετασχηματισμὸς τῶν διδομένων σχέσεων εἰς Καρτεσιανὰς συντεταγμένας εἰς κατὰ πολὺ ἁπλουστέρας παραπολικὰς συντεταγμένας.