ΦΥΣΙΚΗ.— **Anisotropic Scattering Boltzmann Distributions and Structural Properties,** *by C. Syros**. Ἀνεκοινώθη ὑπὸ τοῦ Ἀκαδημαϊκοῦ Καίσαρος Ἀλεξοπούλου.

1. Introduction.

The structural and spectral properties of the simplest form of the Boltzmann equation with isotropic scattering were studied in a previous work[1] from a completely new point of view. This point of view enabled us to find a general solution expressible directly in terms of elementary functions satisfying the pertinent boundary conditions. In fact it was shown that the solution of the Boltzmann equation amounts to the mere solution of an algebraic system of linear equations. In proving there[1] a number of theorems on the structural properties no assumptions concerning the distributions were made.

The present work extends and generalizes results obtained in (A). In particular anisotropic scattering is considered here and explicit solutions are obtained which satisfy the required boundary conditions. The main result presented here may be summarised as follows: *The Boltzmann distribution with anisotropic scattering kernel of degree L in z is a superposition of distributions of isotropic scattering with coefficients proportional to the L + 1 powers of z.* In conclusion the problem of finding the anisotropic scattering Boltzmann distribution is solved exactly and represented by elementary functions in a very simple way. The present method allows to avoid the Riemann - Hilbert problem concerned with singular integral equations.

2. Distribution properties.

The equation to be studied and solved here is the Boltzmann equation in one space dimension and anisotropic scattering kernel at a given constant energy:

$$z \, . \, \partial_x \psi(x, z) + \psi(x, z) = \int_B K(z, z') \, \psi(x, z') \, dz'. \tag{2.1}$$

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* Κ. ΣΥΡΟΥ, **Κατανομαὶ Boltzmann μὲ ἀνισότροπον σκέδασιν καὶ ἰδιότητες τῆς δομῆς των.**

1. Académie Royale de Belgique, Bulletin de la Classe des Science 6, 1973.

To formulate completely the problem the following definitions are required:

Definition I.

1o $K(z, z') = \sum_{l=0}^{L} a_l z^l z^{e'}$, $(l = 0, 1, 2, \ldots L)$ (2.2)

2o $x \in A \cong [a, b]$; $a, b < \infty$

3o $z \in B^{\cdot} \cong [-\lambda, \lambda]^{\cdot} \equiv B_+ \cup B_-$, where the dot «.» expresses the fact that the point $z = 0 \notin B^{\cdot}$ and $B^{\cdot} = B - \{0\}$.

4o $\varphi^{(\nu)}(x) = \partial_x^\nu \varphi(x) \equiv \partial_x^\nu \int_B \psi(x, z)\, dz$, $(\nu = 0, 1, 2 \ldots)$ (2.3)

5o $\varphi_l(x) = \int_B z^l \psi(x, z)\, dz$, $(l = 0, 1, 2, \ldots L)$ (2.4)

6o $g(x, z) = \sum_{l=0}^{L} a_l z^l \varphi_l(x)$ (2.5)

With the help of the above definitions we state

Theorem I.

Let $\psi(x, z)$ be a solution of Eq. (2.1) with the kernel $K(z, z')$ given by Eq. (2.2). Let further $\psi(x, z)$ satisfy the boundary conditions.

$$\psi_+(a, z) = \psi_a(z),$$

and

$$\psi_-(b, z) = \psi_b(z),$$

where $\psi_a(z)$ and $\psi_b(z)$ are given polynomials or given entire functions. Then:

1o $\varphi_l(x)$ is differentiable $\{(\forall x \mid x \in A) \wedge (l = 0, 1, 2, \ldots L)\}$.

2o $\psi(x, z)$ is uniformly differentiable $\{(\forall x \mid x \in A) \wedge (\forall z \mid z \in B^{\cdot})\}$.

Proof: From Eq. (2.1) follow immediately the expressions

$$\psi_+(x, z) = \psi_a(z)\, e^{-\frac{x-a}{z}} + \int_a^x e^{-\frac{x-x'}{z}} g(x', z) \frac{dx'}{z}, \quad \{(\forall x \mid x \in A) \wedge (\forall z \mid z \in B_+)\}, \tag{2.6}$$

and

$$\psi_-(x, z) = \psi_b(z)\, e^{-\frac{x-a}{z}} - \int_x^b e^{-\frac{x-x'}{z}} g(x', z) \frac{dx'}{z}, \quad \{(\forall x \mid x \in A) \wedge (\forall z \mid z \in B_-)\}. \tag{2.7}$$

By multiplying Eqs. (2. 6) — (2. 7) by z^m and integrating them over B_+ and B_- respectively we get

$$\varphi_{m+}(x) = \chi_{m+}(x) + \sum_{l=0}^{L} a_l \int_a^x E_{l+m+1}(x - x')\varphi_l(x')\,dx' \qquad (2.\ 8)$$

and
$$\varphi_{m-}(x) = \chi_{m-}(x) + \sum_{l=0}^{L} a_l \int_x^b E_{l+m+1}(x' - x)\varphi_l(x')\,dx', \qquad (2.\ 9)$$

where χ_{m+} und $\chi_{m-}(x)$ are given by

$$\chi_{m+}(x) = \int_{B+} z^m e^{-\frac{x-a}{z}} \psi_a(z)\,dz \qquad (2.\ 10)$$

und
$$\chi_{m-}(x) = \int_{B-} z^m e^{-\frac{x-b}{z}} \psi_b(z)\,dz\,. \qquad (2.\ 11)$$

To prove 1° we shall assume (the proof of this assumption has been given in (A)) that $\varphi_l(x)$ is continuous $\{(\forall x \mid x \in A) \wedge (l = 0, 1, 2, \ldots L)\}$. By recalling the relation $\varphi_{m+}(x) + \varphi_{m-}(x) = \varphi_m(x)$ and by omitting the function $\chi_{m+}(x)$ and $\chi_{m-}(x)$ in Eqs. (2. 10) — (2. 11) as being by assumption differentiable we obtain from Eqs. (2. 8) — (2. 9) :

$$\lim_{\Delta x \to +0} \left[\varphi_m(x + \Delta x) - \varphi_m(x)\right](\Delta x)^{-1} = \lim_{\Delta x \to +0} \Big\{\sum_{l=0}^{L} a_l \Big[\int_a^{x+\Delta x}(E_{l+m+1}(x+\Delta x-x')$$
$$- E_{l+m+1}(x - x'))\varphi_l(x')\,dx' + (-)^{l+m}\int_{x+\Delta x}^{b}(E_{l+m+1}(x'-x-\Delta x) -$$
$$- E_{l+m}(x'-x))\varphi_l(x')\,dx'$$
$$+\int_x^{x+\Delta x} E_{l+m+1}(x-x')\varphi_l(x')dx' - (-)^{l+m}\int_x^{\Delta x+x} E_{l+m+1}(x'-x)\varphi_l(x')dx'\Big](\Delta x)^{-1}\Big\} \qquad (2.\ 12)$$

After carrying out the limit operation we get from the first and second sums in Eq. (2. 12) the expression

$$\sum_{l=0}^{L} a_l \left[-\int_a^x E_{l+m}(x - x')\varphi_l(x')\,dx' + (-)^{l+m}\int_x^b E_{l+m}(x'-x)\varphi_l(x')\,dx'\right],$$

whilst the two last sums in Eq. (2. 12) become singular for $l = 0$ at the limit $\Delta x = 0$, when m happens to be also equal to zero. However, the singular terms cancel out mutually and the remaining terms give rise to expressions of the form $\sum_{l=0}^{L} a_l[(1-(-)^{l+m})/(l+m)]\varphi_l(x)$. It is seen therefore that $\varphi_m(x)$ is differentiable so that

$$\varphi_m^{(1)}(x) = \sum_{l=0}^{L} a_l \Big\{ -\int_a^x E_{l+m}(x-x')\varphi_l(x')dx' + (-)^{l+m}\int_x^b E_{l+m}(x'-x)\varphi_l(x')dx' + $$

$$+ \frac{1-(-)^{l+m}}{l+m}\varphi_l(x)\Big\} + \chi_m^{(1)}(\chi), \quad m = 0, 1, 2, , , L), \qquad (2.\ 13)$$

where $\varphi_m^{(1)}(x) = \partial_x \varphi_m(x)$ and $\chi_m^{(1)}(x) = \chi_{m+}^{(1)}(x) + \chi_{m-}^{(1)}(x)$

This proves 1o. With the help of the continuity property of $\varphi_l(x)$ for $l = 0, 1, 2, \dots L$ the following result can easily be seen:

$$\varphi_m^{(\nu)}(x) = \sum_{l=0}^{L} a_l \Big\{ (-)^{\nu} \int_a^x \widetilde{E}_{l+m+1-\nu}(x-x')\varphi_l(x')dx' + $$

$$+ (-)^{l+m}\int_x^b \widetilde{E}_{l+m+1-\nu}(x'-x)\cdot\varphi_l(x')dx'$$

$$+ \sum_{\mu=1}^{\nu}(-)^{\mu}\frac{1-(-)^{l+m+1-\mu}}{l+m+1-\mu}\varphi_l^{(\mu-1)}(x)\Big\} + \chi_m^{(\nu)}(x) \qquad (2.\ 14)$$

for $\nu = 1, 2, \dots$ In Eq. (2. 14) we have used the definition

$$\widetilde{E}_{l+m+1-\mu}(\xi) = \begin{cases} E_{l+m+1-\mu}(\xi); & \nu \leq l+m \\ \xi^{l+m+1-\mu} & ; \ \nu \geq l+m+1 \end{cases} \qquad (2.\ 15)$$

To prove 2o we only need to observe that interchange of integration and differentiation in Eqs. (2. 6) - (2. 7) is admissible with respect to both variables x and z. By direct differentiations we see that the derivatives are finite and unique and therefore 2o is also proved.

Q. E. D.

Remark I. If the scattering kernel $K(z, z')$ is invariant against the parity transformation, $K(-z, -z') = K(z, z')$, then $\{\varphi_l(x) \mid l = 0, 1, 2, \dots L\}$ satisfy the system of integral equations

$$\varphi_m(x) = \sum_{l=0}^{L} a_l \int_a^b E_{l+m+1}(|x-x'|)\varphi_l(x')dx' + \chi_m(x), \quad (m = 0, 1, \dots L) \qquad (2.\ 16)$$

Corollary I. From the finiteness of $\partial_x \psi(x,z) \{(\forall x \mid x \in A) \wedge (\forall z \mid z \in B)\}$ it follows that

$$\lim_{z \to +0} z \cdot \partial_x \psi(x,z) = 0. \tag{2.17}$$

Corollary II. From Eqs. (2.1) and (2.17) the remarkable result follows:

$$\psi(x,0) = a_0 \int_B \psi(x,z)\, dz, \tag{2.18}$$

which is valid also in the case of isotropic scattering kernel.

Corollary III. From differentiability of $\varphi_l(x)$, $(l = 0, 1, 2, \dots L)$ and from Eq. (2.1) it follows directly that

$$\psi(x,z) = \sum_{\nu=0}^{n-1} (-z)^\nu \sum_{l=0}^{L} a_l z^l \varphi_l^{(\nu)}(x) + (-z)^n \partial_x^n \psi(x,z),$$
$$\{(\forall x \mid x \in A) \wedge (\forall z \mid z \in B^\bullet)\}. \tag{2.19}$$

Corollary IV. It follows by direct differentiation of Eq. (2.1) that

$$(1/n!) \cdot \partial_z^n \psi(x,z)|_{z=0} = \sum_{l+\nu=n} (-)^\nu a_l \varphi_l^{(\nu)}(x), \quad (n = 0, 1, 2, \dots). \tag{2.20}$$

Corollary V.

$$\pm \partial_x^{n+1} \psi(x.z)|_{z=0} = \mp \partial_x^n \partial_z \psi(x,z)|_{z=0} \pm a_l \varphi_l^{(n)}(x) \tag{2.21}$$

The proofs of the Corollaries I to V are very easy and similar to the proofs given in (A).

3. Solutions and Boundary Conditions.

The relations given in the preceding section will now be applied to construct explicitly the general solution of Eq. (2.1). To this end the required definitions are next given.

Definition II.

1° $\psi_+(x,z) = \psi(x,z), \{(\forall x \mid x \in A) \wedge (\forall z \mid z \in B_+)\}$

2° $\psi_-(x,z) = \psi(x,z), \{(\forall x \mid x \in A) \wedge (\forall z \mid z \in B_-)\}$

3° $\beta^{ll'}_{kn}(x-a) = \begin{cases} \partial^k_x \int\limits_{B+} \left[S_n(x-a, z) - (-)^n \exp\left(-\frac{x-a}{z}\right)\right] z^l z^{l'} dz \; ; \; n > N \\ 0 \; ; \; n \leqslant N \; ; \; N \text{ is any positive integer} \end{cases}$

4° $V^{ll'}_{kn}(x-a) = \begin{cases} \partial^k_x \int\limits_{B+} S_n(x-a, z) z^l z^{l'} dz \; ; \; n \leqslant N \\ 0 \; ; \; n > N \end{cases}$

5° $\zeta_{kn} = \begin{cases} \frac{[(b-a)/2]^{n-k}}{(n-k)!} \; ; \; k \leqslant n \\ 0 \; ; \; k > n \end{cases}$

6° $\gamma^{ll'}_{kn}(x-b) = \partial^k_x \int\limits_{B-} \left[S_n(x-b, z) - (-)^n \exp\left(-\frac{x-b}{z}\right)\right] z^l z^{l'} dz$

7° The symbols $S_n(\xi, z)$ represent polynomials of n-th degree in the two variables ξ and z defined by

$$S_n(\xi, z) = \sum_{\nu=0}^{n} \frac{\xi^{n-\nu}}{(n-\nu)!} (-z)^\nu$$

and having the properties :

$$\partial_\xi S_n(\xi, z) = - S_{n-1}(\xi, z)$$

$$S_n(\lambda\xi, \lambda z) = \lambda^n \, . \, S_n(\xi, z)$$

$$z \, . \, \partial_\xi S_n(\xi, z) = \frac{\xi^n}{n!} - S_n(\xi, z)$$

$$S_n(\xi, 0) = \frac{\xi^n}{n!}$$

$$S_n(0, z) = (-z)^n$$

8° A few examples of the above polynomials are :

$$S_0(\xi, z) = 1$$

$$S_1(\xi, z) = \xi - z$$

$$S_2(\xi, z) = \frac{\xi^2}{2!} - \xi z + z^2$$

$$S_3(\xi, z) = \frac{\xi^3}{3!} - \frac{\xi^2 z}{2!} + \frac{\xi z^2}{1!} - z^3$$

$$S_4(\xi, z) = \frac{\xi^4}{4!} - \frac{\xi^3 z}{3!} + \frac{\xi^2 z^2}{2!} - \frac{\xi z^3}{1!} + z^4, \quad \text{etc.}$$

With the help of the above definitions we now formulate the following

T h e o r e m II.

The derivatives of any order of $\psi(x,z)\ \{(\forall x \mid x \in A) \wedge (\forall z \mid z \in B)\}$ with respect to both x and z are finite and unique provided this is true for the derivatives of any order of $\varphi_l(x)$; $(l = 0, 1, 2 \dots L)\ (\forall x \mid x \in A)$.

The proof of Theorem II is omitted here because it follows very closely the proof of the corresponding theorem in (A).

The solution of Eq. (2. 1) is characterized completely by

T h e o r e m III.

Let $\psi(x,z) = \psi_+(x,z) + \psi_-(x,z)$ be a solution of Eq. (2. 1) on $A \otimes B$ satisfying the boundary conditions

$$\psi_+(a,z) = \psi_a(z), \quad z \in B_+ \tag{3. 1}$$

and

$$\psi_-(b,z) = 0 \quad , \quad z \in B_- . \tag{3. 2}$$

Let further $\{\psi_l(x,z) \mid l = 0, 1, 2, \dots L\}$ be a set of isotropic scattering solutions satisfying boundary conditions of the same kind. Then:

1° The linear superpositions

$$\psi_+(x,z) = \sum_{l=0}^{L} a_l z^l \psi_{l+}(x,z), \quad \{(\forall x \mid x \in A) \wedge (\forall z \mid z \in B_+)\} \tag{3. 3}$$

and

$$\psi_-(x,z) = \sum_{l=0}^{L} a_l z^l \psi_{l-}(x,z), \quad \{(\forall x \mid x \in A) \wedge (\forall z \mid z \in B_-)\} \tag{3. 4}$$

are solutions of Eq. (2. 1) provided the kernel coefficients $\{a_l \mid l = 0, 1, \dots L\}$ do not belong to the eigenvalue set of the supermatrix defined by

$$D^{ll'} = \left[a_l \left(\beta^{ll'}_{kn} + \sum_{v=0}^{n} \gamma^{ll'}_{kv} \zeta_{vn} \right) - 2^{k-n} \zeta_{kn} \delta_{ll'} \right] . \tag{3. 5}$$

2° The distribution $\psi(x,z) = \psi_+(x,z) + \psi_-(x,z)$ satisfies the boundary condition.

3° Each of the $L+1$ infinite series in Eqs. (3. 3) - (3. 4) converges uniformly on $A \otimes B^{\bullet}$, provided $|\partial^n_z \psi(x,z)|_{z=0}| < c \cdot n!$

P r o o f : Let $\{q_{ln} \mid n = 0, 1, 2, \dots \mid l = 0, 1, 2, \dots L\}$ be the $1 + L$

sets of coefficients determining the $1+L$ isotropic scattering solutions $\{\psi_l(x,z) \mid l=0,1,\ldots L\}$, i.e.,

$$\psi_{l+}(x,z) = \sum_{n=0}^{N} q_{ln} S_n(x-a,z) + \sum_{n=N+1}^{\infty} q_{ln}\left[S_n(x-a,z) - (-z)_n e^{-\frac{x-a}{z}}\right] \quad (3.6)$$

and
$$\psi_{l-}(x,z) = \sum_{n=0}^{\infty} p_{ln}\left[S_n(x-b,z) - (-z)_n e^{-\frac{x-b}{z}}\right], \quad (3.7)$$

where $p_{lk} = \sum_{n=k}^{\infty} q_{ln} \dfrac{(b-a)^{n-k}}{(n-k)!}$; $\{(l=0,1,2,\ldots L) \wedge (k=0,1,\ldots)\}$ (3.8)

The coefficients for $n=0,1,2,\ldots N$ and $l=0,1,2,\ldots L$ will be seen to follow uniquely from the boundary conditions, while the remaining coefficients q_{ln} are to be determined. To do this we have according to (A) from Eqs. (3.3) - (3.4):

$$\sum_{l=0}^{L} a_{l'} z^{l'} \left[z\partial_x \psi_{l+}(x,z) + \psi_{l+}(x,z)\right] =$$
$$= \sum_{l'=0}^{L} a_{l'} \sum_{l=0}^{L} a_l z^{l'} \left\{\sum_{n=0}^{N} V_n^{ll'}(x-a) q_{ln} + \sum_{n=N+1}^{\infty} \beta_n^{ll'}(x-a) q_{ln} + \sum_{n=0}^{\infty} \gamma_n^{ll'}(x-b) p_{ln}\right\} \quad (3.9)$$

The symbols $V_n^{ll'}$, $\beta_n^{ll'}$ and $\gamma_n^{ll'}$ have been given in a slightly more general form in Def. II.

Next we apply the operators $\partial_x^k \big|_{x=\frac{a+b}{z}}$ and $a_l^{-1} \partial_z^l \big|_{z=0}$ on both sides of Eq. (3.9) for all admissible values of k and l. In this way we obtain the following set of linear algebraic equations for the determination of the coefficients $\{q_{ln} \mid l=0,1,2,\ldots L \mid n=N+1, N+2,\ldots\}$:

$$\sum_{l=0}^{L} \sum_{n=N+1}^{\infty} \left[a_l\left(\beta_{kn}^{ll'} + \sum_{v=0}^{n} \gamma_{kv}^{ll'} \zeta_{vn}\right) - 2^{k-n} \zeta_{kn}\right] q_{ln} = C_k^{l'} \quad (3.13)$$

for $l' = 0,1,2,\ldots L$ and $k=0,1,2,\ldots$

The constants C_k^l in Eq. (3.13) are essentially determined from the $(N+1)\times(L+1)$ first coefficients q_{ln} through the relations

$$C_k^{l'} = \sum_{l=0}^{L} \sum_{n=0}^{N} \left[2^{k-n} \zeta_{kn} - a_l\left(V_{kn}^{ll'} + \sum_{v=0}^{n} \gamma_{kv}^{ll'} \zeta_{vn}\right)\right] q_{ln}. \quad (3.14)$$

Since $\{a_l \mid l=0,1,2,\ldots L\}$ does not belong to the set of roots of the determinant involved in Eq. (3.13), assertion 1° of the Theorem is proved.

To prove 2° it is sufficient to see that $\psi_+(x, z)$ and $\psi_-(x, z)$ satisfy the boundary conditions given in Eqs. (3.1) - (3.2). In fact $\psi_+(x,z)$ and $\psi_-(x,z)$ being superpositions of the polynomials given in Def. II, have the same properties concerning their behavior at the boundaries $x = a$ and $x = b$.

Let us first consider $\psi_{l+}(x, z)$ from Eq. (3.6). For $x=a$ the bracket becomes

$$S_n(0, z) - (-z)^n = (-z)^n - (-z)^n = 0$$

and $\psi_{l+}(a, z)$ becomes therefore

$$\psi_a(z) = \sum_{l=0}^{L} a_l z^l \sum_{n=0}^{N} (-z)^n q_{ln}. \qquad (3.15)$$

From Eq. (3.15) we have at once

$$q_{ln} = \frac{(-)^n}{a_l (l+n)!} \partial_z^{l+n} \psi_a(z)\big|_{z=0} \qquad (3.16)$$

for $l = 0, 1, \ldots L$ and $n = 0, 1, \ldots N$. This proves 2° and at the same time gives the $(N+1) \times (L+1)$ first coefficients.

Remark II. It is clear that if $\psi_a(z)$ is a polynomial of z, N is finite.

Now we have to prove the convergence of the series

$$\psi_{l+}(x, z) = \sum_{n=0}^{\infty} q_{ln} \left[S_n(x-a, z) - (-z)^n e^{-\frac{x-a}{z}}\right]. \qquad (3.17)$$

This series converges in fact $\{(\forall x \mid x \in A) \wedge (\forall z \mid z \in B_+)\}$.

Since $S_n(x-a, z) = (-z)^n \cdot e_n\left(-\frac{x-a}{z}\right)$, Eq. (3.17) can be written in the form

$$\psi_{l+}(x, z) = \sum_{n=0}^{\infty} q_{ln}(-z)^n \left[e_n\left(-\frac{x-a}{z}\right) - e^{-\frac{x-a}{z}}\right]. \qquad (3.18)$$

From this equation it becomes clear that

$$\lim_{n \to \infty} \left[e_{n+\varrho}\left(-\frac{x-a}{z}\right) - e^{-\frac{x-a}{z}}\right] = 0 \quad \text{for all } \varrho > 0, \qquad (3.19)$$

where ϱ is any integer, and for all $z \in B_+$, i.e., $z > 0$. On the other hand q_{ln} satisfies the relation

$$q_{ln} = \frac{(-)^n}{a_l (l+n)!} \cdot \frac{\partial^{l+n} \psi_{l+}(a, z)}{\partial z^{l+n}} . \tag{3.20}$$

Consequently, in order that the series in Eq. (3.18) be divergent the derivatives of $\psi_{l+}(a, z)$ of order n should increase with increasing n faster than the factorial n!, but this contradicts the assumption of the theorem. Therefore $\psi_{l+}(x, z)$ as represented in Eq. (3.17) is uniformly convergent on $A \otimes B_+$. Furthermore, since

$$\sum_{n=0}^{\infty} q_{ln} \frac{(x-a)^n}{n!} = \sum_{n=0}^{\infty} p_{ln} \frac{(x-b)^n}{n!}$$

and $\psi_{l+}(x, 0) = \psi_{l-}(x, 0)$ one infers in exactly the same way that the series in Eq. (3.7) is also uniformly convergent on $A \otimes B_-$ and therefore $\psi_l(x, z)$ is uniformly convergent on $A \otimes B^{\bullet}$. This proves 3°. Since $\psi(x, z)$ is the superposition of a finite number of $\psi_l(x, z)$, Theorem III is proved.

Q. E. D.

4. Conclusions.

The most fundamental result of this work is that the operator $(z \cdot \partial_x + 1)$ acting on the function $\psi(x, z)$ of the variables x and z makes it depend only on x in the case $K(z, z') = a_0$. In the more general case $K(z, z') = \sum_{l=0}^{L} a_l (z \cdot z')^l$ the application of the operator $(z\partial_x + 1)$ on $\psi(x, z)$ makes it depend on z exactly in the same way like $\int_B \sum_{l=0}^{L} a_l (zz')^l \psi(x, z') dz'$. This property allows the exact algebraization of the Boltzmann equation. This result in turn puts the problem of studying the spectral properties of the equation in question on a very simple algebraic basis. A second important result is that the method presented here leads directly to such simple results that the problem of solving singular integral equations — like in the Case theory — is completely avoided.

Finally the present method is directly applicable also to the problems of the time and energy dependent Boltzmann equation on which further work is in preparation.

Π Ε Ρ Ι Λ Η Ψ Ι Σ

Αἱ χαρακτηριστικαὶ ἰδιότητες τῆς γραμμικῆς ἐξισώσεως τοῦ Boltzmann μετ᾽ ἀνισοτρόπου πυρῆνος σκεδάσεως ἐμελετήθησαν καὶ ἐχρησιμοποιήθησαν διὰ τὴν ρητὴν κατασκευὴν τῆς γενικῆς λύσεως διὰ σύστημα πεπερασμένης ἐκτάσεως ($a \leqslant x \leqslant b$). Αἱ εὑρεθεῖσαι κατανομαὶ ἐκφράζονται τῇ βοηθείᾳ στοιχειωδῶν συναρτήσεων καὶ ἱκανοποιοῦν αὐστηρῶς τὰς ὁρικὰς συνθήκας. Τὰ ἀποδειχθέντα θεωρήματα ἀφοροῦν εἰς: (i) τὸ διαφορίσιμον τῆς $\varphi(x)$, (ii) τὴν ὕπαρξιν παραγώγων $\psi(x, z)$ καὶ (iii) τὴν ὕπαρξιν ἀναπαραστάσεων τοῦ $\psi(x, z)$ διὰ κατανομῶν ἰσοτρόπου σκεδάσεως.

*

Ὁ Ἀκαδημαϊκὸς κ. **Κ. Ἀλεξόπουλος** κατὰ τὴν ἀνακοίνωσιν τῆς ἀνωτέρω ἐργασίας εἶπε τὰ κάτωθι:

Κύριε Πρόεδρε,

Μέγα μέρος τῶν φαινομένων τοῦ φυσικοῦ κόσμου συνίσταται εἰς τὴν κίνησιν τῆς ὕλης. Ὅταν αὕτη ἀποτελῆται ἀπὸ πολλὰ σωμάτια, ἡ περιγραφὴ τῆς συμπεριφορᾶς τοῦ συνόλου δίδεται ὑπὸ μιᾶς ἐξισώσεως, καλουμένης ἐξισώσεως Boltzmann, ἡ ὁποία περιγράφει τὴν ταχύτητα μὲ τὴν ὁποίαν μεταβάλλεται ἡ πιθανότης, ἓν τῶν σωματίων νὰ εὑρίσκεται εἰς ὡρισμένην θέσιν καὶ νὰ κινῆται μὲ ὡρισμένην ταχύτητα πρὸς ὡρισμένην διεύθυνσιν.

Μολονότι ἡ ἐξίσωσις Boltzmann εἶναι ἀπὸ αἰῶνος γνωστὴ ἐξακολουθεῖ νὰ εἶναι τὸ ἀντικείμενον μελέτης διὰ εἰδικὰς περιπτώσεις.

Σήμερον ἔχω τὴν τιμὴν νὰ παρουσιάσω εἰς τὴν Ἀκαδημίαν ἐργασίαν τοῦ κ. Κ. Σύρου, ὅστις ἐργαζόμενος εἰς τὴν Εὐρωπαϊκὴν Κοινότητα Ἀτομικῆς Ἐνεργείας εἰς τὸ Βέλγιον ἐξεπόνησεν ἐργασίαν περὶ τῆς ἐξισώσεως Boltzmann ἐντὸς ἀνισοτρόπων ὑλικῶν. Ἡ περίπτωσις αὕτη εὑρίσκει ἐφαρμογὴν εἰς τὰ νετρόνια, σωμάτια ἔχοντα μεγάλην σημασίαν διὰ τοὺς πυρηνικοὺς ἀντιδραστῆρας καὶ τὴν ἀκτινοθεραπευτικήν.