ΜΑΘΗΜΑΤΙΚΑ.— **The Entry Problem***, *by* ***Demetrios G. Magiros*** *and* ***George Reehl*****. Ἀνεκοινώθη ὑπὸ τοῦ Ἀκαδημαϊκοῦ κ. Ἰ. Ξανθάκη.

I. INTRODUCTION

(a) The Entry Problem.

«An artificial celestial body is moving in an elliptical Keplerian orbit around the earth. A force T is applied to the body, acts for some time, then stops, when the body starts moving in a new Keplerian orbit around the earth. Condition are required under which the new Keplerian orbit intersects the surface of the earth».

The conditions required will be found from the restriction of the perigee distance of the new Keplerian orbit to be smaller than or equal to the radius of the earth, taken spherical.

Use will be made of the results of one of the author's papers in connection with the motion of the body on the non-Keplerian orbit during the action of the general force T.

(b) The Motion of the Body During the Action of the Force.

We state for reference [1(b)] that if the time is split according to: $t = t_0 + \tau$, where t_0 is very small, the reference coordinate system: (P_0; r_0^*, S_0^*, T_0^*), which is in general non-orthogonal and three-dimensional, is defined as shown in Figure 1, the conditions at P_0 are:

$$\left.\begin{array}{ll} \text{(i)} & r(0) = r(t_0) = r_0 = \mathrm{r}_0 \cdot r_0^* \\ \text{(ii)} & \dot{r}(t_0) = \dot{r}(0) + I_0 = V_0 + I_0 = \mathrm{s}_0 \cdot s_0^* \\ \text{(iii)} & I_0 = \int_0^{t_0} T(t)\,dt \end{array}\right\} \quad (1)$$

the motion of the body during the action of the force on the non-Keplerian arc of the orbit is given by the position vector $r(\tau)$ and the velocity $\dot{r}(\tau)$ according to the formulae:

$$r(\tau) = r_0 + r_1(\tau) = a_1(\tau) \cdot r_0^* + a_2(\tau) \cdot s_0^* + a_3(\tau)\, T_0^* \quad (2)$$

$$\dot{r}(\tau) = V(\tau) = \dot{a}_1(\tau)\, r_0^* + \dot{a}_2(\tau) \cdot s_0^* + \dot{a}_3(\tau)\, T_0^* \quad (3)$$

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where:

$$\left.\begin{aligned}
a_1(\tau) &= a_1(o) + \dot{a}_1(o)\tau + \frac{1}{2}\ddot{a}_1(o)\tau^2 + \ldots + \frac{1}{n!}a_1^{(n)}(o)\tau^n + \ldots \\
a_2(\tau) &= a_2(o) + \dot{a}_2(o)\tau + \frac{1}{2}\ddot{a}_2(o)\tau^2 + \ldots + \frac{1}{n!}a_2^{(n)}(o)\tau^n + \ldots \\
a_3(\tau) &= a_3(o) + \dot{a}_3(o)\tau + \frac{1}{2}\ddot{a}_3(o)\tau^2 + \ldots + \frac{1}{n!}a_3^{(n)}(o)\tau^n + \ldots
\end{aligned}\right\} \quad (4)$$

The first two coefficients of these series are given, and the others can be calculated by using:

$$\left.\begin{aligned}
\ddot{a}_1(\tau) &= -\frac{\mu}{r^3(\tau)}a_1(\tau) + T_1, \quad a_1(o) = r_o, \quad \dot{a}_1(o) = 0 \\
\ddot{a}_2(\tau) &= -\frac{\mu}{r^3(\tau)}a_2(\tau) + T_2, \quad a_2(o) = o, \quad \dot{a}_2(o) = s_o \\
\ddot{a}_3(\tau) &= -\frac{\mu}{r^3(\tau)}a_3(\tau) + T_3, \quad a_3(o) = o, \quad \dot{a}_3(o) = 0
\end{aligned}\right\} \quad (4a)$$

T_1, T_2, T_3 are projections of the force T on the coordinate axis, $\mu = K(m_1 + m_2)$, K the gravitational constant, and m_1, m_2 the masses of the earth and the body, respectively.

The convergence of the series (4a) is proved.[1b]

II. THE SOLUTION OF THE ENTRY PROBLEM

By the following procedure we can solve the three-dimensional entry problem.

(a) The Original Keplerian Orbit.

The position vector r_o and the velocity vector $\dot{r}_o = V_o$ at a point P_o, due to the Newtonian force only, determine the Keplerian orbit through P_o with focus the attractive center E of the earth. If these vectors are known, the distance $r_o = EP_o$, the true anomaly d_o, the angle φ_o, and the speed V_o, associated with the point P_o, will be known.

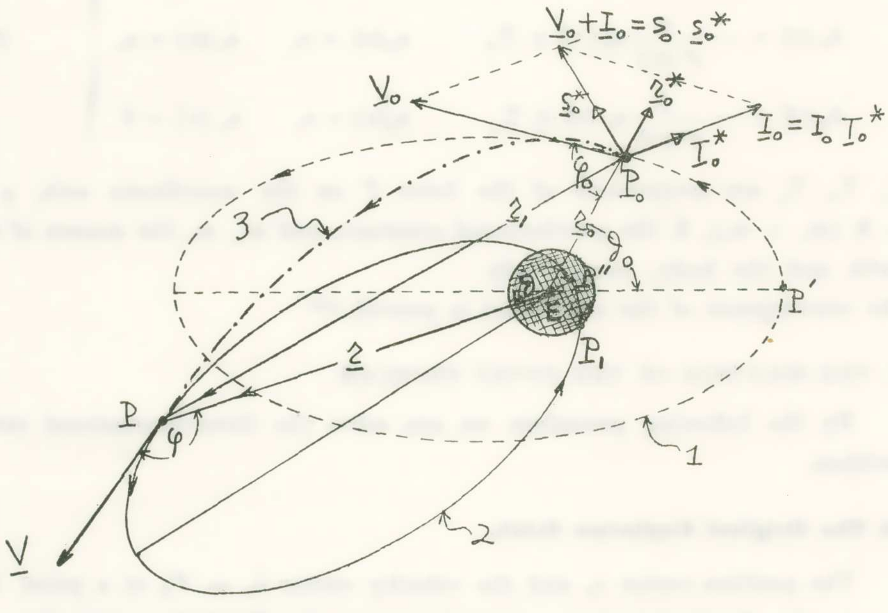
The Keplerian orbit is an ellipse, parabola, or hyperbola if V_o is smaller than, equal to, or bigger than $\sqrt{2\mu/r_o}$, respectively.[1(c)]

The semi-major axis a_o, the eccentricity e_o and the perigee distance p_o of the orbit can be calculated in terms of the known quantities by using appropriate formulae, which in case of elliptic orbit are:[2]

$$\left.\begin{aligned} a_o &= \frac{\mu}{\dfrac{2\mu}{r_o} - V_o}, \qquad & e_o &= \left\{1 - \frac{r_o}{a_o^2}\,(2a_o - r_o)\sin^2\varphi_o\right\} \\ P_o &= a_o\,(1 - e_o), & \sin\varphi_o &= \frac{1 + e_o\cos d_o}{(1 + e_o^2 + 2e_o\cos d_o)^{1/2}} \end{aligned}\right\} \quad (5)$$

(b) The Non - Keplerian Arc of the Orbit.

The three-dimensional entry problem is characterized by the fact that the force T and the unit vector T_o^* are not on the plane (V_o, r_o^*) of the original Keplerian orbit, Figure 1.



1 : Original orbit (Departure). P_o point where the force starts.
2 : New orbit (Destination). P point where the force stops.
3 : Non - Keplerian arc (Transfer). P_o P during the action of the force.

Figure 1

Having the impulse I_o at P_o, given by the formula (1,iii) we calculate the vector: $V_o + I_o = s_o \cdot s_o^*$.

During the action of the force T, the body is moving on the non-Keplerian arc of the orbit, which is a curve in space, and the position vector and the velocity vector of the body are given by the formulae (2) and (3).

When the force T stops acting, say at the point P, formulae (2) and (3) give the position vector $r = EP$ and the velocity vector $\dot{r} = V$ at P, that is the magnitudes r = EP and V, as well as the angles which determine the direction of the vectors r and V.

We notice that the plane (r_1, V) in general does not contain the point E.

(c) The New Keplerian Orbit.

To determine the new Keplerian orbit, which corresponds to the body at P, Figure 1, where the force T stops, one must know the position vector $r = EP$ and the velocity vector V at P, that is, one must know the magnitudes r and V, and the angle φ on the plane (E, V) of the new orbit, which are now known.

The new orbit is an ellipse, parabola or hyperbola, if V is smaller than, equal to, or bigger than $\sqrt{2\mu/r}$, respectively.

By using the appropriate formulae the Keplerian orbit can be calculated. In case of elliptic new Keplerian orbit formulae (5) can be used, if, instead of the quantities r_o, V_o, a_o, φ_o, e_o, p_o, d_o, the quantities r, V, a, φ, e, p, d, corresponding to the new Keplerian orbit, are used.

(b) The Condition for Entry.

The new Keplerian orbit, constructed as above, intersects the surface of the earth, (when one has a realization of the entry problem), if its perigee distance is smaller than or equal to the radius ρ of the earth, that is, in the case of elliptical orbit, if the condition:

$$EP'' = a\,(1 - e) \leqq \rho \qquad (6)$$

is satisfied, Figure 1. The quantities a and e are dependent, as we saw, upon known and calculated angles and lengths, and upon the time t' of the duration of the force T.

III. THE SOLUTION IN RESTRICTED PROBLEMS

By restricting the force T, that is by imposing restrictions to its magnitude and direction, which are valid during its action on the body, a variety of entry problems may occur, when the above general procedure for their solution is simplified.

We consider two restricted cases of the entry problem of special interest.

(a) Force T Constant.

If the magnitude and the direction of T are constant, the problem is simplified. As an example, in case $T_1 = O$, $T_2 = O$, $T_3 = T$, and large r_o and small τ, formulae (2) and (3) become: [1(b)]

$$r(\tau) = \left(r_o - \frac{\mu\tau^2}{2r_o^2}\right) r_o^* + \left(s_o\tau - \frac{\mu s_o \tau^3}{6\, r_o^3}\right) s_o^* + \frac{1}{2}\, T\tau^2 T_o^* \qquad (7)$$

$$\dot{r}(\tau) = V = -\frac{\mu\tau}{r_o^2}\, r_o^* + \left(s_o - \frac{\mu s_o \tau^2}{2r_o^3}\right) s_o^* + T\tau T_o^* \qquad (8)$$

(b) Two-dimensional Entry Problem.

The two-dimensional entry problem occurs in case the force T is on the plane of the original orbit. In this case the non-Keplerian arc and the new orbit are on the plane of the original orbit, and the coordinate system (P_o; r_o^*, s_o^*) is suggested. All calculations of the general procedure of the previous section are simplified, especially in case of large r_o and small τ, when the following formulae can be used:

$$r(\tau) = \left(r_o - \frac{\mu\tau^2}{2r_o^2}\right) r_o^* + \left(s_o\tau - \frac{T\tau^2}{2} - \frac{\mu s_o \tau^3}{6\, r_o^3}\right) s_o^* \qquad (9)$$

$$\dot{r}(\tau) = V = -\frac{\mu\,\tau}{r_o^2}\, r_o^* + \left(s_o - T\tau - \frac{\mu s_o \tau^2}{2\, r_o^3}\right) s_o^* \qquad (10)$$

REMARK. The present mathematical discussion of the entry problem can be taken as basic discussion of «physical entry problems».

R E F E R E N C E S

1. D.G. Magiros : «Proceedings of International Astronautical Congress», (a) XIV 1963 (Paris), (b) XV 1964 (Warsaw), (c) XVI 1965 (Athens).

2. A.E. Roy : «The foundations of Astrodynamics», The MacMillan Company, New York, (1965), pg. 80, 83.

Π Ε Ρ Ι Λ Η Ψ Ι Σ

Τὸ πρόβλημα τῆς εἰσόδου ἑνὸς τεχνητοῦ δορυφόρου ἐξετάζεται εἰς τὴν παροῦσαν ἐργασίαν ὡς ἐφαρμογὴ γενικωτέρας ἐρεύνης δημοσιευθείσης ὑφ᾽ ἑνὸς τῶν συγγραφέων. Ἡ καμπύλη μεταφορᾶς λαμβάνεται συμφώνως πρὸς τὴν ἐργασίαν 1 (b). Ἡ τροχιὰ προορισμοῦ πρέπει νὰ πληροῖ τὴν συνθήκην (6). Δίδονται δύο εἰδικαὶ περιπτώσεις.

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Ὁ Ἀκαδημαϊκὸς κ. **Ἰω. Ξανθάκης** κατὰ τὴν ἀνακοίνωσιν τῆς ἀνωτέρω ἐργασίας εἶπε τὰ κάτωθι:

Εἰς τὴν παροῦσαν ἐργασίαν ἐξετάζεται τὸ «Πρόβλημα τῆς Εἰσόδου» ὑπὸ τὴν ἀκόλουθον μορφήν:

«Τεχνητὸς δορυφόρος τῆς Γῆς κινεῖται πέριξ αὐτῆς εἰς ἐλλειπτικὴν τροχιάν. Μία γενικὴ δύναμις ἐφαρμόζεται ἐπὶ τοῦ δορυφόρου, δρᾷ ἐπὶ χρονικόν τι διάστημα καὶ μετὰ παύει δρῶσα, ὁπότε ὁ δορυφόρος ἀρχίζει νὰ κινῆται ἐπὶ νέας Κεπλερείου τροχιᾶς. Ζητοῦνται αἱ συνθῆκαι καὶ οἱ περιορισμοὶ ὑπὸ τοὺς ὁποίους ἡ νέα τροχιὰ δύναται νὰ συναντήσῃ τὴν ἐπιφάνειαν τῆς Γῆς».

Ἡ λύσις τοῦ παρόντος προβλήματος ἐπιτυγχάνεται δι᾽ ἐφαρμογῆς τῶν πορισμάτων προηγηθεισῶν ἐργασιῶν ἑνὸς ἐκ τῶν συγγραφέων τῆς παρούσης ἐρεύνης. Οἱ ζητούμενοι περιορισμοὶ προκύπτουν ἐκ τοῦ γεγονότος ὅτι «ἡ ἐκ τοῦ περιγείου ἀπόστασις τοῦ δορυφόρου τῆς νέας Κεπλερείου τροχιᾶς δέον νὰ εἶναι μικροτέρα ἢ ἴση τῆς ἀκτῖνος τῆς Γῆς, θεωρουμένης σφαιρικῆς».

Εἶτα ἐξετάζεται τὸ πρόβλημα τοῦτο τῆς εἰσόδου εἰς τὰς μερικὰς περιπτώσεις κατὰ τὰς ὁποίας:

α) Ἡ δρῶσα δύναμις ἔχει σταθερὸν μέγεθος καὶ διεύθυνσιν καὶ

β) Αὕτη κεῖται ἐπὶ τοῦ ἐπιπέδου τῆς ἀρχικῆς τροχιᾶς.