ΠΡΑΓΜΑΤΕΙΑΙ ΤΗΣ ΑΚΑΔΗΜΙΑΣ ΑΘΗΝΩΝ
ΤΟΜΟΣ 24 – Αριθ. 1

THE AREAS OF SUNSPOTS IN THE TWO SUN HEMISPHERES

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As it is known most of the statistical relations for the sunspots are expressed in Wolf numbers where the material of observations is abundant.

We propose to study here the areas of the spots, which are given by the observations of Greenwich [1] in relation to the number of synodic rotations of the Sun between the minimum and the next maximum of the solar activities. That is to say, if the minimum and the maximum of the solar activities take place at the rotations R_K and R_L respectively, the number of the solar rotations between the minimum and the next maximum is taken as $R_L - R_{K+1}$.

The data used are given by tables I_a (north hemisphere of the sun) and I_b (south hemisphere of the sun). The first column of these tables gives the ordinary numeration of the cycles of the solar activity. The second and third columns give the rotations of the Sun (numeration of Carrington) where the minimum area of the spots is observed in the corresponding cycle; the same columns also give the mean daily areas of the spots in the same rotation. The fourth and fifth columns give respectively the rotation where the maximum area of the spots is observed and the mean daily areas of the spots $(A_M)_N$, $(A_M)_S$ in this rotation. Finally the sixth and seventh columns give the times of rise (ascension) and descent R_N, R'_N R_S, R'_S expressed in solar synodic rotations. All the given areas of the spots are corrected for foreshortening and are expressed in millionths of the Sun's visible hemisphere.

In two cases the used values for A_M are not the absolutely maximum ones which are observed in the corresponding cycle. Thus the value $(A_M)_N = 1052$ (rotation N⁰ 526) is used for cycle 13 of the north hemisphere instead of the absolute maximum value 1356 (rotation N⁰ 518). Also the value $(A_M)_S = 1085$ (rotation N⁰ 848) is used for cycle 15 of the south hemisphere instead of the maximum value 1204 (rotation N⁰ 889).

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TABLE I_a

Spots North of Equator

Number of Cycles	Date of Minimum		Date of Maximum		Date of Minimum		Rotation Numbers	
	Rotation Number	Area Min.	Rotation Number	Area $(A_M)_N$	Rotation Number	Area Min.	Min-Max R_N	Max-Min R'_N
12	339	0	389	1142	481	0	51	92
13	482	2	546	1052	646	0	65	200
14	647	21	696	2168	802	4	50	106
15	803	12	854	2174	937	34	52	83
16	938	81	1019	2306	1083	17	82	64
17	1084	4	1128	2187	1206	109	45	78
18	1207	96	1242	2721	1342	0.4	36	100

TABLE I_b

Spots South of Equator

Number of Cycles	Date of Minimum		Date of Maximum		Date of Minimum		Rotation Numbers	
	Rotation Number	Area Min.	Rotation Number	Area $(A_M)_S$	Rotation Number	Area Min.	Min-Max R_S	Max-Min R'_S
12	338	1	398	1677	481	59	61	83
13	482	0.3	533	1751	641	0	52	108
14	642	5	714	1682	794	0	73	80
15	795	1	848	1085	933	9	54	85
16	934	3	1003	1196	1072	0	70	69
17	1073	0	1150	2867	1208	1	78	58
18	1209	0	1251	3724	1346	0	43	95

The graphical representation of R_N, $(A_M)_N$, and R_S, $(A_M)_S$, where the values of R_N and R_S are placed on the axis of the abscissas, shows that the corresponding points lie on both sides of the parabolas:

$$(A_M)_N = a_1 (R_N - c_1)^2 + b_1$$
$$(A_M)_S = a_2 (R_S - c_2)^2 + b_2$$

If we trace these parabolas and if we consider the differences of the observed values $(A_M)_N$, $(A_M)_S$ from those of their corresponding parabolas we see that these differences do not occur at random. They are periodic functions of the time and have the form:

$$\pm K \sin (t-1) \frac{2\pi}{8} + P, \qquad \begin{array}{l} t = 0, \; 1, \ldots \; 6 \\ \text{cycles: } 12, \; 13, \ldots \; 18 \end{array}$$

where P represents periodic terms of a smaller period and of small amplitude. These terms may therefore be omitted in a first approximation. We are thus finally led to the following relations:

$$(1_a) \quad (A_M)_N = 1083 + 2{,}58 \, (R_N - 65)^2 + 700 \sin (t-1) \frac{2\pi}{8}$$

$$(1_b) \quad (A_M)_S = 1270 + 6{,}0 \, (R_S - 61.5)^2 - 600 \sin (t-1) \frac{2\pi}{8}$$

where:

$$t = 0, \; 1, \ldots \; 6$$
$$\text{cycles: } 12, \; 13, \ldots \; 18$$

The values given by (1_a) and (1_b) for the maximum area $(A_M)_S$ and $(A_M)_S$ differ slightly from the ones given by the observations, the mean square errors being respectively:

$\pm 41{,}8$ for the north hemisphere
$\pm 53{,}8$ » » south »

The fact that the principal periodic term whose period is 8 solar cycles has almost the same amplitude in both hemispheres but with a difference of phase by 180° between the two hemispheres is of particular importance. The presence of this periodic term, which we represent by X, alters sensibly the height of the maximum from one cycle to the other. Thus in the cycles 14, 15 and 16 ($t = 2, 3, 4$), where the periodic term takes positive values equal respectively to 495, 700 and 495 in the north hemisphere and in the south negative ones equal respectively to -424, -600, -424, the heights of maxima are very sensibly higher in the north than in the south hemisphere. On the contrary in the cycles 12 and 18 ($t = 0, 6$), where the periodic term takes negative values in the north hemisphere (-495) and positive ones ($+424$) in the south one, the heights of the maxima are sensibly smaller in the north hemisphere than in the south one. In the cycles 13 and 17 (t=1,5) the periodic term becomes zero. The observed differences in the heights of maxima of the latter two cycles are entirely due to the non-periodic part of (1_a) and (1_b), i.e. to the difference of time of rise.

The above considerations become more evident if we consider the difference $(A_M)_S - (A_M)_N$, which in accordance to (1_a) and (1_b) becomes

$$(1_c) \quad (A_M)_S - (A_M)_N = 163 - 1300 \sin (t-1)\frac{2\pi}{8} + f(R)$$

where:

$$f(R) = 6{,}0\,(R_S - 61.5)^2 - 2.5\,(R_N - 65)^2$$

If the maxima and minima took place during the same rotations of both hemispheres ($R_N = R_S$) and if the corresponding parabolas had the same vertex, then f(R) = 0 and the difference $(A_M)_S - (A_M)_N$ would show a sinuous fluctuation with period of 8 solar cycles. But the above conditions are not entirely fulfilled and for this reason that difference stands away from the sinuous fluctuation especially in the cycles 13 and 17 where f(R) takes its maximum value (cf. table II_a).

If instead of the difference we take the sum, i. e.

$$(1_d) \quad (A_M)_N + (A_M)_S = 2378 + \Phi(R) + 100 \sin (t-1)\frac{2\pi}{8}$$

where:

$$\Phi(R) = 2.5\,(R_N - 65)^2 + 6.0\,(R_S - 61.5)^2$$

the periodic term X almost disappears and the sum in question varies in proportion to the squares $(R_N - 65)^2$, $(R_S - 61.5)^2$ (cf. Table II_b).

TABLE IIa | TABLE IIb

Number of Cycles	f(R)	163 + Term Periodic	$(A_M)_S - (A_M)_N$ obs.	$\Phi(R)$ + 2378	Term Periodic	$(A_M)_S + (A_M)_N$ obs.
12	− 326	+ 1082	+ 535	2870	− 70	2819
13	+ 704	+ 163	+ 700	2920	0	2803
14	+ 393	− 756	− 486	3735	+ 70	3850
15	+ 77	− 1137	− 1089	3139	+ 100	3259
16	− 128	− 756	− 1110	3534	+ 70	3502
17	+ 796	+ 163	+ 680	5012	0	5054
18	+ 54	+ 1082	+ 1003	6474	− 70	6445

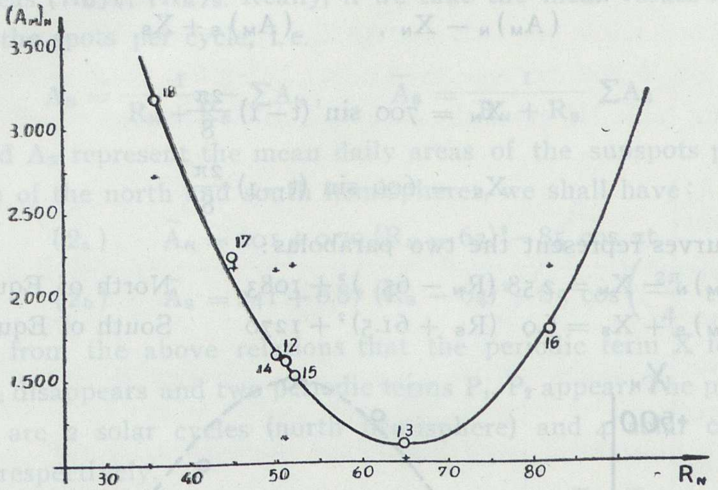


Fig. 1. The crosses represent the observed values of $(A_M)_N$. The open circles represent the values of $(A_M)_N - X_N$ and the curves the parabola:

$$(A_M)_N - X_N = 1083 + 2.58\,(R_N - 65)^2$$

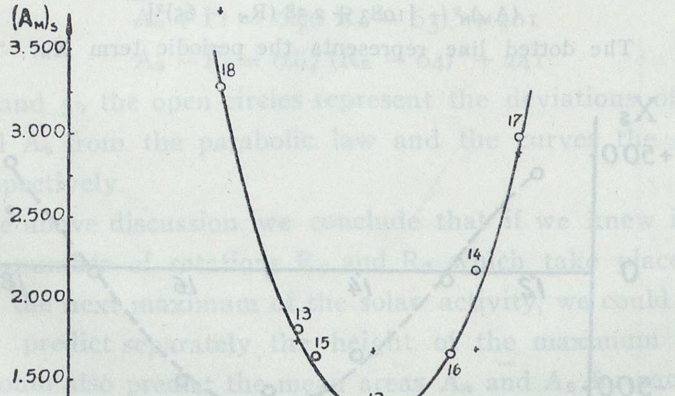


Fig. 2. The crosses represent the observed values of $(A_M)_S$. The open circles represent the values of $(A_M)_S + X_S$ and the curves the parabola:

$$(A_M)_S + X_S = 1270 + 6.0\,(R_S - 61.5)^2$$

In figures 1, 2 the crosses represent the values of $(A_M)_N$ and $(A_M)_S$ which are given by the observations. The open circles represent the quantities:

$$(A_M)_N - X_N \,, \qquad (A_M)_S + X_S$$

where

$$X_N = 700 \sin (t-1)\frac{2\pi}{8}$$

$$X_S - 600 \sin (t-1)\frac{2\pi}{8}$$

The curves represent the two parabolas:

$$(A_M)_N - X_N = 2.58\,(R_N - 65\;)^2 + 1083 \qquad \text{North of Equator}$$

$$(A_M)_S + X_S = 6.0\;\;(R_S + 61.5)^2 + 1270 \qquad \text{South of Equator}$$

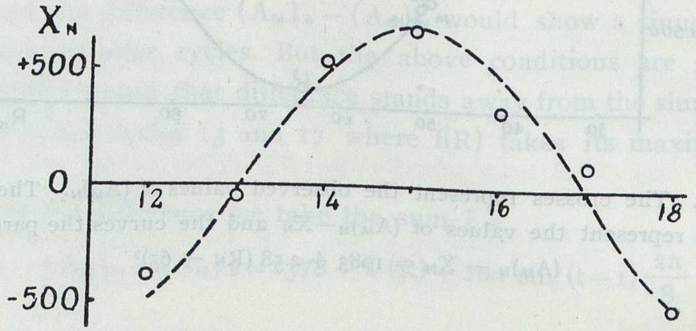


Fig. 3a. The open circles represent the difference:

$$(A_M)_N - [1083 + 2.58\,(R_N - 65)^2]$$

The dotted line represents the periodic term X_N.

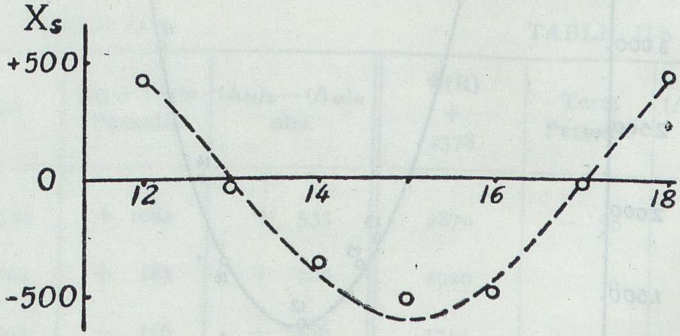


Fig. 3b. The open circles represent the difference:

$$(A_M)_S - [1270 + 6.0\,(R_S - 61.5)^2]$$

The dotted line represents the periodic term X_S.

In figures 3_a and 3_b the open circles represent the deviations of the observed values $(A_M)_N$ and $(A_M)_S$ from the parabolic law, i.e. the differences:

$$(A_M)_N - [2.58\,(R_N - 65)^2 + 1083] \,, \qquad (A_M)_S - [6.0\,(R_S - 61.5)^2 + 1270]$$

and the dotted lines the periodic terms X_N, X_S.

Besides it should be noted that the periodic term X appears only in the maximum areas $(A_M)_N$, $(A_M)_S$. Really, if we take the mean values $\overline{A}_N$ and $\overline{A}_S$ of the areas of the spots per cycle, i. e.

$$\overline{A}_N = \frac{1}{R_N + R_S} \Sigma A_N , \qquad \overline{A}_S = \frac{1}{R_N + R_S} \Sigma A_S$$

where A_N and A_S represent the mean daily areas of the sunspots per each calendar month of the north and south hemispheres, we shall have:

$$(2_a) \qquad \overline{A}_N = 201 + 0.70\,(R_N - 63)^2 - 85 \cos \pi t$$

$$(2_b) \qquad \overline{A}_S = 241 + 0.87\,(R_S - 64)^2 - 85 \cos\left(\frac{2\pi}{4}\,t + \frac{\pi}{4}\right)$$

We see from the above relations that the periodic term X for the mean values $\overline{A}_N$, $\overline{A}_S$ disappears and two periodic terms P_1, P_2 appear. The periods of the latter terms are 2 solar cycles (north hemisphere) and 4 solar cycles (south hemisphere) respectively.

In figures 4_a, 4_b the crosses represent the values of $\overline{A}_N$, $\overline{A}_S$ which are given by the observations. The open circles represent the quantities:

$$\overline{A}_N + P_1, \qquad \overline{A}_S - P_2$$

where

$$P_1 = 85 \cos \pi t, \quad P_2 = 85 \cos\left(\frac{2\pi}{4}\,t + \frac{\pi}{4}\right)$$

The curves represent the two parabolas:

$$\overline{A}_N + P_1 = 0.70\,R_N - 63)^2 + 201$$

$$\overline{A}_S - P_2 = 0.87\,(R_S - 64)^2 + 241$$

In figures 5_a and 5_b the open circles represent the deviations of the observed values $\overline{A}_N$ and $\overline{A}_S$ from the parabolic law and the curves the periodic terms P_1 and P_2 respectively.

From the above discussion we conclude that if we knew in advance for each cycle the number of rotations R_N and R_S which take place between the minimum and the next maximum of the solar activity, we could with satisfactory accuracy predict separately the height of the maximum in both hemispheres. We could also predict the mean areas $\overline{A}_N$ and $\overline{A}_S$ for each cycle.

From the given data of the observations not only we do not see any relation between the values of R_N, R_S and the time but even the determination of the values of the two above quantities is not absolutely certain due to the fluctuations of the values of the areas in the vicinity of both the minimum and the maximum.

A notable relation however seems to exist between the alternate cycles and the difference $R_S - R_N$. Really, this difference for the hitherto known cycles is given with satisfactory accuracy from the relation:

$$(4) \qquad R_S - R_N = 6.65 - 24.1 \sin\frac{2\pi}{3}\, t + 4.8 \sin\left(\frac{2\pi}{5}\, t + 98^\circ\right)$$

$$t = 0, \quad 1, \ldots \quad 6$$

$$\text{cycles: } 12, \; 13, \ldots \; 18$$

The following table III gives the values of that difference which are given by

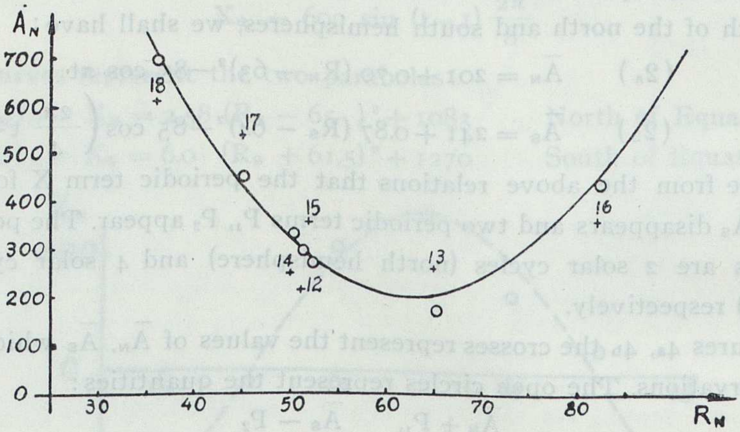


Fig. 4a. The crosses represent the observed values of $\overline{A}_N$. The open circles represent the quantity $\overline{A}_N + P_1$ and the curve the parabola:

$$\overline{A}_N + P_1 = 201 + 0.70\,(R_N - 63)$$

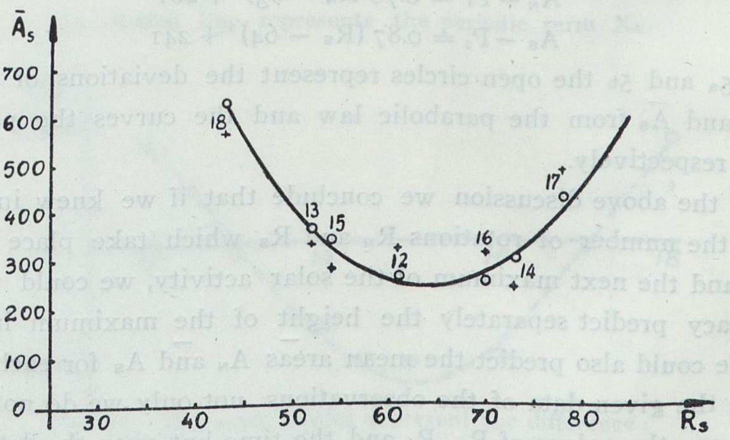


Fig. 4b. The crosses represent the observed values of $\overline{A}_S$. The open circles represent the quantity $\overline{A}_S - P_2$ and the curve the parabola:

$$\overline{A}_S - P_2 = 241 + 0.87\,(R_S - 64)^2$$

the observations (Tables I_a, I_b). It also gives the values resulting from relation (4).

According to relation (4) the difference $R_S - R_N$ for the present 19th cycle must be -19 ± 1. The maximum of the south hemisphere will precede the maximum of the north one by about 19 synodic rotations. But in the next 20th cycle the above difference is expected to be $+24\pm1$ rotations and the maximum of the north hemisphere will precede that one of the south hemisphere. This prediction is made with due reservation for the data of the observations do not refer but to 7 only cycles.

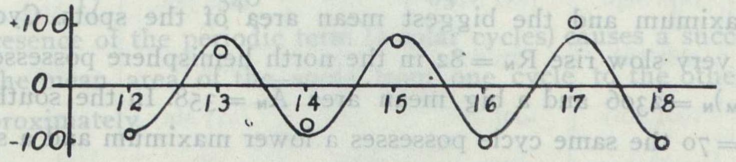


Fig. 5a. The open circles represent the difference:
$\overline{A}_N - [201 + 0.70 (R_N - 63)^2]$
The curve represents the periodic term: $P_1 = 85 \cos \pi t$

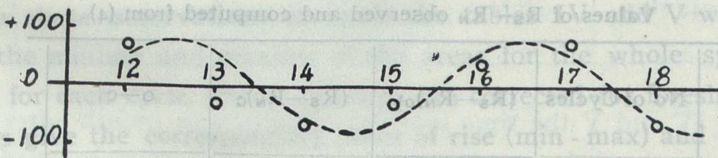


Fig. 5b. The open circles represent the difference:
$\overline{A}_S - [241 + 0.87 (R_S - 64)^2]$
The curve represents the periodic term:

$$P_2 = 85 \cos\left(\frac{2\pi}{4} t + \frac{\pi}{4}\right)$$

From the above investigation we conclude that both the maximum and the mean areas of the spots (umbra + penumbra) per cycle in the two solar hemispheres can be expressed analytically as a function of the number of synodic rotations R_N and R_S of the Sun. The analytic relations in question are composed of two parts; One of them depends upon the square of the number R_N and R_S (parabola), the other constituting a periodic function of the time with principal periods 2, 4 and 8 solar cycles. Of the periodic terms the term X possesses a particular interest. This term has a period of 8 solar cycles, appears only in the maximum area and represents a difference of phase by 180° between the two hemispheres.

The algebraic part of the analytic relations represents parabola in both relations (1), (2) and (3). The vertices of these parabolas have almost the same abscissa $\overline{R} = 64$. This number of rotations plays a particular role for the height of the maxima $(A_M)_N$ and $(A_M)_S$ as well as for the mean areas of the spots per cycle. Really, as bigger the difference $|R-64|$ is taken as an absolute value as higher is the maximum and as greater is the mean areas of the spots per cycle in both hemispheres. In other words, *the cycles which present a very fast or a very slow rise possess in general a high maximum and a big mean area.* Thus, cycles 17 and 18 which present a fast rise in the north hemisphere possess the highest maximum and the biggest mean area of the spots. Cycle 16 which presents a very slow rise $R_N = 82$ in the north hemisphere possesses a high maximum $(A_M)_N = 2306$ and a big mean area $\bar{A}_N = 358$. In the south hemisphere where $R_S = 70$ the same cycle possesses a lower maximum and a smaller mean area, $(A_M)_S = 1196$, $\bar{A}_S = 314$. On the contrary, the cycles whose time of rise is near 64 rotations, as for example cycle 13 of the north hemisphere, possess a low maximum and a small total area of the spots.

TABLE III

Values of R_S—R_N observed and computed from (4)

No of Cycles	$(R_S-R_N)_{ob}$	$(R_S-R_N)_C$	o—c
12	+ 10	+ 11	− 1
13	− 13	− 13	0
14	+ 23	+ 23	0
15	+ 4	+ 3	+ 1
16	− 12	− 12	0
17	+ 33	+ 32	+ 1
18	+ 7	+ 7	0

The above phenomenon becomes complicated by the presence of the periodic terms whose amplitude is perceptible and particularly of the term X in the maximum area and of the terms with periods 2 and 4 solar cycles in the mean area per cycle. Thus in cycle 15, where the periodic term X takes its maximum value $(X = +700)$ in the north hemisphere and its minimum one $(X = -600)$ in the south hemisphere, the height of the maximum is $(A_M)_N = 2174$ and $(A_M)_S = 1085$ respectively although the speeds of rise do not differ very much $(R_N = 52,\ R_S = 54)$. Similarly we have bigger mean area for cycle 17 in the north hemisphere than for cycle 16 and this is due to the presence of the periodic term with period 2 solar cycles. Note that cycle 16 shows a very slow rise while cycle 17 a very fast one but with almost the same absolute value of the difference $|R_N - c|$:

cycles	$\|R_N - c\|$	$\bar{A}_N$	Periodic term (2 solar cycles)
16	20	358	-85
17	17	540	$+85$

The presence of the periodic term (2 solar cycles) causes a successive fluctuation of the mean area of the spots from one cycle to the other by 10-40 per cent approximately.

II. The Areas of the Sunspots on the Total Sun's Visible Hemisphere in Relation to the Time of Rise.

The observations of Greenwich [1 : 7 – 16] give the «Mean Daily Areas of Sunspots and Faculae for each Calendar Month» for the time interval 1874–1954. From these observations we made the tables IV and V which give the epochs of the minima and maxima of the areas for the whole spots (umbra + penumbra) for each cycle. These numbers are corrected for foreshortening. The same tables give the corresponding times of rise (min - max) and descent (max - min) respresented by T and T′ respectively and expressed in months. The maximum areas of the whole spots and umbrae are represented by A_M and U_M respectively and are expressed in millionths ot the Sun's visible hemisphere.

TABLE IV

Maximum Areas of the whole spots corrected for Foreshortening

Number of Cycles	Date of Commen-cement	Date of Maximum	Maximum Areas : A_M	Date of Minimum	T Months	T′ (Months)
12	1878 Sept.	1883 July	2066	1889 Jan.	59	66
13	1889 Feb.	1893 Aug.	2340	1901 Apr.	55	92
14	1901 May	1907 Feb.	2453	1913 May	70	75
15	1913 Juin	1917 Aug.	2978	1923 Feb.	51	66
16	1923 Mar.	1929 Dec.	3084	1933 Dec.	82	48
17	1934 Jan.	1937 July	3363	1944 Apr.	43	81
18	1944 May	1947 Apr.	3950	1954 Jan.	36	81

TABLE V

Maximum Areas of the Umbrae corrected for Foreshortening

Number of Cycles	Date of Commencement	Date of Maximum	Maximum Areas : U_M	Date of Minimum	T_u (Months)	T'_u (Months)
12	1878 Sept.	1882 Nov.	378	1889 Jan.	51	74
13	1889 Feb.	1893 Aug.	376	1901 Apr.	55	92
14	1901 May	1907 Feb.	380	1913 May	70	76
15	1913 Juin	1917 Aug.	471	1923 Feb.	51	66
16	1923 Mar.	1929 Dec.	592	1933 Dec.	82	48
17	1934 Jan.	1937 July	592	1944 Apr.	43	81
18	1944 May	1947 May	607	1954 Jan.	37	80

In tables IV and V the time interval between one month after the minimum and the month of the maximum is taken as time of rise. The time interval between one month after the maximum and the month in which the next minimum took place is taken as time of descent. A_M and U_M represent the biggest observed values of the areas of the whole spots and umbrae corrected for foreshortening, which are given by the respective tables of Greenwich [1 : 10 - 16]. Cycle 18 of table V in which as maximum has been taken the one corresponding to May 1947 ($U_M = 607$) instead of the corresponding to February 1949 ($U_M = 631$) is an exception to the above rule.

As tables IV and V show the maxima and minima of the whole spots and umbrae take place at the same time moments. A difference is observed only in cycle 12 where the maximum area of the whole spots took place in July 1883 while the maximum area of umbrae 8 months earlier (Nov. 1882).

a) *Analytic relations for the whole spots (Umbra + Penumbra).*

Based on table V the maximum monthly area A_M of the whole spots on the Sun's visible hemisphere is given as a function of the time of rise T by the following relation :

$$A_M = 2172 + 2.40\,(T-65)^2 + 300\,\sin\,(t-1)\,\frac{2\pi}{8} \tag{5}$$

Table VI shows that there is a satisfactory agreement between the values of A_M given by the observations and those computed from relation (5). The mean square error of the difference $(A_M)_{ob}-(A_M)_c$ is $\pm 35{,}1$. The individual differences are smaller than 3 per cent of the corresponding values of A_M.

TABLE VI

Number of Cycles	T Months	$(A_M)_{ob}$	$(A_M)_c$	o - c
12	59	2066	2046	+ 20
13	55	2340	2412	— 72
14	70	2453	2444	+ 9
15	51	2978	2942	+ 36
16	82	3084	3078	+ 6
17	43	3363	3334	+ 29
18	36	3950	3978	— 28

Relation (5) is of the same form with the relations (1) which give the maximum areas $(A_M)_N$ and $(A_M)_S$ (north and south of the equator respectively) as a function of the number of R_N and R_S. The amplitude of the periodic term $X=300 \sin (t-1)\frac{2\pi}{8}$ is here only smaller. If the epochs of maxima and minima of the solar activity (north and south of the equator) coincided then relation (5), which gives the maximum area A_M for the total Sun's visible hemisphere, would be the same with relation (1d) which gives the sum $(A_M)_N+(A_M)_S$. Also, the periodic term X in relation (5) would have as amplitude not 300 but 100 approximately. But the above do not happen and the maximum A_M on the total sun's visible hemisphere takes place sometimes near the epoch of the maximum in the north part of the equator and sometimes near the epoch of the maximum in the south part of the equator.

From relations (1d) and (5) we have

$$D(A)=204+6.0\,(R_S-61.5)^2+2{,}53\,(R_N-65)^2- \tag{6}$$
$$-2.40\,(T-65)^2-200 \sin (t-1)\frac{2\pi}{8}$$

where

$$D(A) = (A_M)_N + (A_M)_S - A_M$$

Table VII gives the values of the above difference D(A) which are given by the observations and relation (6). These relations are significant and this is due to the fact that the quantities $(A_M)_N$, $(A_M)_S$ and A_M refer to different time moments.

TABLE VII

cycles	12	13	14	15	16	17	18
D(A)ob	+ 753	+ 463	+ 1397	+ 281	+ 418	+ 1691	+ 2495
D(A)c	737	476	1348	278	519	1678	2461
o − c	+ 16	− 13	+ 49	+ 3	− 101	+ 13	+ 34

If instead of the real maxima $(A_M)_N$, $(A_M)_S$ we take the values of the areas in the months in which the maximum area A_M takes place on the Sun's visible hemisphere, the above differences almost disappear.

If instead of the maximum area A_M we consider the mean area $\overline{A}$ per cycle i.e.

$$\overline{A} = \frac{1}{T+T'} \Sigma A$$

TABLE VIII

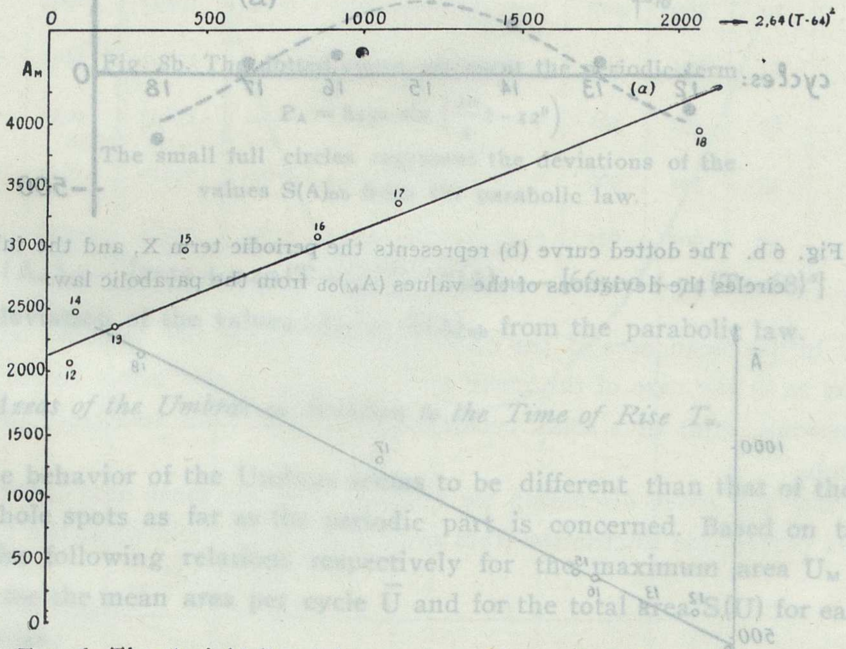
Number of Cycles	T Mont.	$\overline{A}$ obs.	$\overline{A}$ Comp.	$\overline{A}$ o-c	$\overline{A}_{-3,+3}$ obs.	$\overline{A}_{-3,+3}$ Comp.	$\overline{A}_{-3,+3}$ o-c	S(A) obs.	S(A) Comp.	S(A) o-c
12	59	567	533	+ 34	1225	1170	+ 55	70892	70718	− 173
13	55	592	597	− 5	1562	1528	+ 34	86777	87358	− 581
14	70	473	489	− 16	1076	1061	+ 15	68633	68576	+ 57
15	51	676	687	− 11	1654	1750	− 96	78935	79534	+ 401
16	82	658	662	− 4	1419	1489	− 70	85803	80032	+ 5771
17	43	967	943	+ 24	2434	2383	+ 51	119851	121052	− 1201
18	36	1233	1251	− 18	2957	3006	− 49	144283	144052	+ 231
M.sq.E	....	....	....	± 19			± 58			

we shall have

(7) $$\bar{A} = 482 + 0.8\,(T-67)^2$$

Here as in relation (2) the term X does not appear. It is noteworthy that this phenomenon is not only observed for the mean value A of the areas per cycle. Really, if we take the mean value of the areas of the spots near the maximum, i.e. in a time interval between 3 months before the maximum and 3 months after it, that is for the values

$$\bar{A}_{-3,+3} = \frac{1}{7}\sum_{-3}^{+3} A$$



F.g. 6. The straight line (a) represents the parabolic law ; the small circles represent the observed values of A_M.

the same phenomenon is observed. Thus, we have:

(8) $$\bar{A}_{-3,+3} = 1143 + 1.98\,(T-67)^2 - 100\,\cos\pi t$$

Similarly we have:

(9) $$S\,(A) = 66500 + 74\,(T-68)^2 + 8450\,\sin\left(\frac{2\pi}{4}t - 12^\circ\right)$$

where S (A) represents the total sum of the whole spots for each one of the cycles on the Sun's visible hemisphere. The values of A, $\bar{A}_{-3,+3}$ and S (A) given by the observations agree satisfactorily with those computed through the relations (7), (8) and (9) (cl. Table VIII). Exception made of cycle 16 where a sensible disagreement is observed for the values of S (A), in all other cases the differences o - c are smaller than 1 per cent of the corresponding values of $\overline{A}$, $\underset{-3,+3}{\overline{A}}$ and S (A) and the mean sq. errors are respectively

$$\overline{A} : \pm 19, \qquad \underset{-3,+3}{\overline{A}} : \pm 36 \qquad S(A) : \pm 2197$$

Comparing relations (5), (7) and (9) with their corresponding ones (1), (2) and (3) we observe that the amplitude of the periodic terms perceptibly decrea-

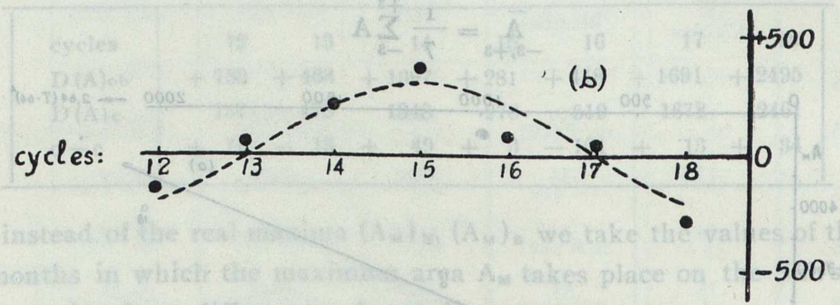


Fig. 6 b. The dotted curve (b) represents the periodic term X, and the full circles the deviations of the values $(A_M)_{ob}$ from the parabolic law.

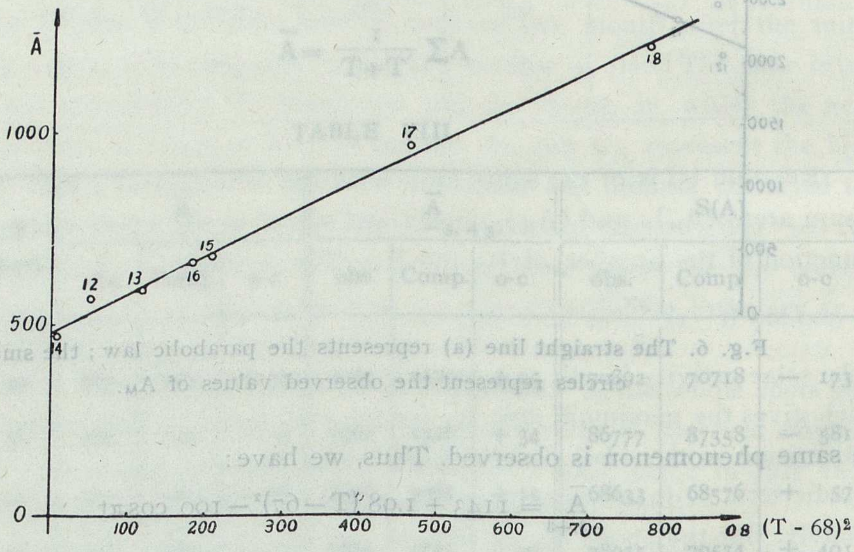


Fig. 7. The straight line represents the parabolic law. The small circles represent the observed values of $\overline{A}$.

ses and the importance of the parabolic law becomes thus more evident. This is shown graphically by figures (6), (7) and (8) where we place on the axis of the abscissas the values of the quantity $a(T-c)^2$, which correspond to each cycle. On the axis of the ordinates we place the corresponding values of A_M, $\overline{A}$ and S (A). The straight lines represent the parabolic law. The small circles represent the valus of A_M, $\overline{A}$, $S(A)$, which are given by the observations. The dotted lines (b) in figures 6_b and 8_b represent the periodic terms

$$300 \sin (t-1)\frac{2\pi}{8}, \qquad 8455 \sin\left(\frac{2\pi}{4}t-12^{\circ}\right)$$

while the small full circles represent the differences:

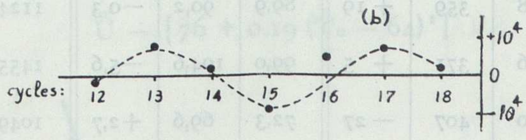


Fig. 8b. The dotted curve represent the periodic term

$$P_A = 8450 \sin\left(\frac{2\pi}{4}t - 12^{0}\right)$$

The small full circles regresent the deviations of the values $S(A)_{ob}$ from the parabolic law.

$$(A_M)_{ob} - [2172 + 2.40(T-65)^2], \quad S(A)_{ob} - [66500 + 74(T-68)^2]$$

i.e. the deviation of the values $(A_M)_{ob}$, $S(A)_{ob}$ from the parabolic law.

b) *The Areas of the Umbrae in Relation to the Time of Rise T_u.*

The behavior of the Umbrae seems to be different than that of the areas of the whole spots as far as the periodic part is concerned. Based on table V we get the following relations respectively for the maximum area U_M of the Umbrae, for the mean area per cycle $\overline{U}$ and for the total area $S(U)$ for each one of the cycles:

$$U_M = 368 + 0.4(T_u - 64)^2 + 70 \sin\left(\frac{2\pi}{8}t - \frac{3\pi}{4}\right) \tag{10}$$

$$\overline{U} = 76 + 0.19(T_u - 64)^2 - 13.2 \cos \pi t \tag{11}$$

$$S(U) = 10490 + 20.1(T_u - 64)^2 - 1450 \cos \pi t + 1000 \sin\left(\frac{2\pi}{4}t - \frac{\pi}{6}\right) \tag{12}$$

As table IX shows the values computed from the above relations for U_M, $\overline{U}$ and $S(u)$ agree very satisfactorily with those given from the observations. The mean square errors are:

$$U_M: \pm 23{,}8, \quad \overline{U}: \pm 3{,}6, \quad S(u): \pm 490$$

The comparison of relations (10), (11) and (12) with the corresponding relations (5), (7) and (9) which refer to the whole Spots presents the following differences with respect to the periodic part:

TABLE IX

Number of Cycles	T_U Months	U_M obs.	U_M Comp.	U_M o-c	$\overline{U}$ obs.	$\overline{U}$ Comp.	$\overline{U}$ o-c	S(u) obs.	S(u) Comp.	S(u) o-c
12	52	378	359	+ 19	89,9	90,2	−0,3	11241	11434	− 193
13	55	376	371	+ 5	99,0	104,6	−5,6	14551	14464	+ 147
14	70	380	407	− 27	72,3	69,6	+2,7	10492	10264	+ 228
15	51	471	499	− 28	118,7	121,3	−2,6	13892	14501	− 609
16	82	592	561	+ 31	120,3	123,3	−3,0	15633	15052	+ 581
17	43	592	569	+ 23	179,1	173,1	+6,0	22210	21640	+ 570
18	37	607	631	− 24	200,5	201,2	−0,7	23460	24193	− 733

1) In the mean area U per cycle of the Umbrae the periodic term X disappears as in the case of the mean area per cycle of the whole Spots. But one more periodic term of 2 solar cycles appears here with relatively perceptible amplitude.

From the above discussion we conclude that the mean area of the Umbrae presents perceptible fluctuation from cycle to cycle and follows at the same time

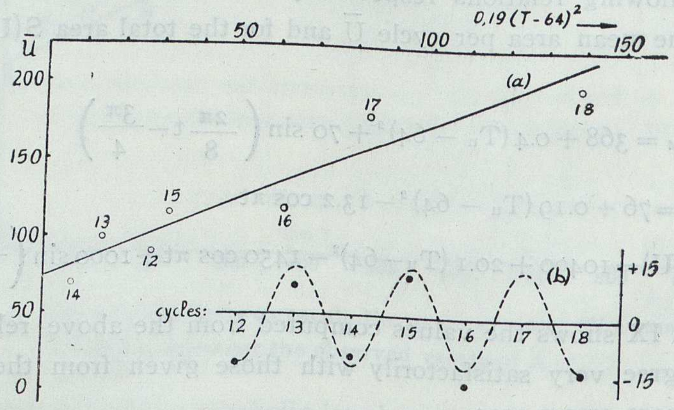


Fig. 9. The straight line (a) represents the parabolic law. The small open circles represent the observed values of $\overline{u}$. On the right below the dotted curve (b) represents the periodic term $-13{,}2 \cos\pi t$; the small full circles represent the deviation of $\overline{u}$ from the parabolic law.

the parabolic law which was followed in all previous cases. This is shown in figure 9 where the values $0.19\,(T_u - 64)^2$ which correspond to each cycle are placed on the axis of the abscissas; the corresponding values of $\overline{U}$ are placed on the axis of the ordinates. The straight line (a) represents the parabolic law. The small circles represent the values of $\overline{U}$ given by the observations. The curves (b) represent the periodic term $K\cos\pi t$ and the small full circles the deviation of the values $\overline{U}$ from the corresponding parabola, i. e. the difference:

$$\overline{U} - [76 + 0.19\,(T_u - 64)^2]$$

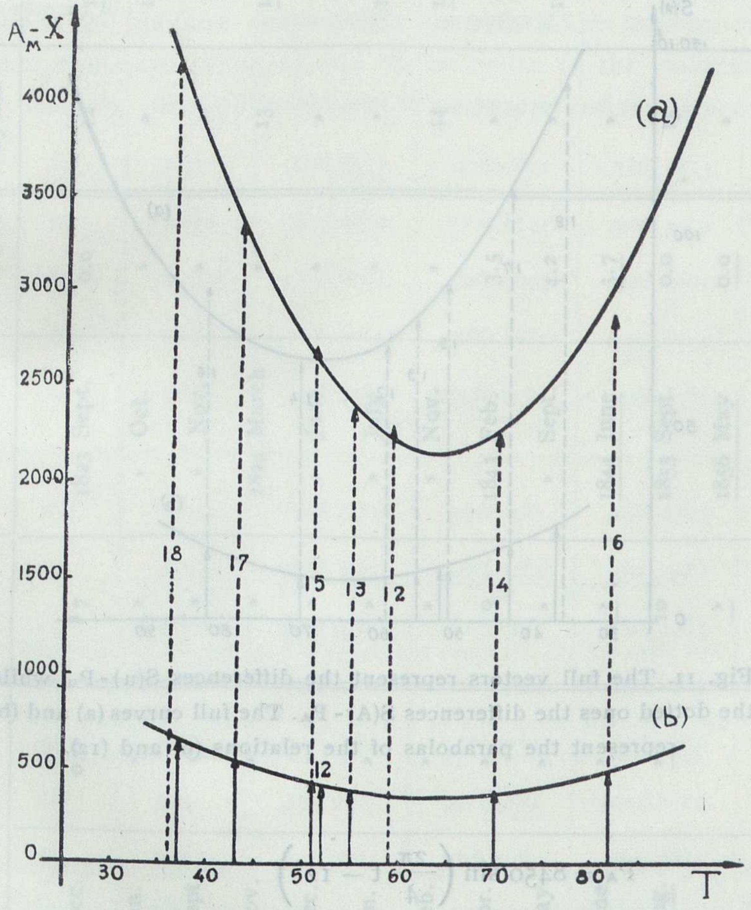


Fig. 10. The full vectors represent the differences $(U_M)_{ob} - X_u$, while the dotted ones the differences A_M-X. The full curves (a) and (b) represent the parabolas of the relations (5) and 10).

In fig. 10 the curves (a) and (b) represent the parabolas [cl. relations (5) and (10)]:

$$A_M - X = 2150 + 2.64\,(T - 64)^2, \quad U_M - X_u = 366 + 0.4\,(T_u - 64)^2$$

On the other hand the directed segments represent the quantities:

$$(A_M)_{ob} - X, \qquad (U_M)_{ob} - X_u$$

where:

$$X = 300 \sin (t-1)\frac{2\pi}{8}, \qquad X_u = 70 \sin\left(\frac{2\pi}{8}t - \frac{3\pi}{8}\right)$$

In fig. 10 and 11 the curves (a) and (b) represent the parabolas of the relations (9) and (12) while the directed segments the quantities:

$$S'(A) = S(A)_{ob} - P_A, \qquad S(u)_{ob} - A_u,$$

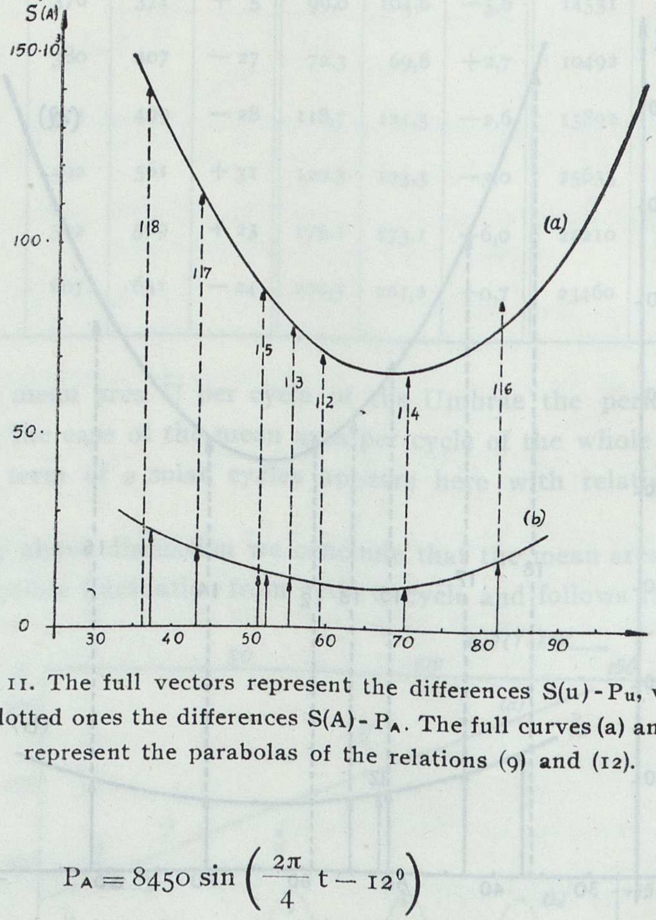


Fig. 11. The full vectors represent the differences S(u) - P_u, while the dotted ones the differences S(A) - P_A. The full curves (a) and (b) represent the parabolas of the relations (9) and (12).

where:

$$P_A = 8450 \sin\left(\frac{2\pi}{4}t - 12^0\right)$$

$$P_u = -1450 \cos \pi t + 1000 \sin\left(\frac{2\pi}{4}t - \frac{\pi}{6}\right)$$

III. Wolf Numbers.

We shall now seek to see if the previously formed analytic relations hold in case we consider the respective Wolf numbers R instead of the areas of the Spots. We shall confine ourselves here only to the time interval 1823 - 1954

TABLE X

Number of Cycles	Date	Value of Min.	Number of Cycles	Date	Value of Min.	Number of Cycles	Date	Value of Min.
7	1821 Dec.	0.0	7	1823 Sept.	0.0	12	1878 Aug.	0.0
»	1822 Jan.	»	»	» Oct.	»	»	1879 Jan.	0.0
»	» Sept.	»	»	» Nov.	»	»	» March	0.0
»	» Nov.	»	»	1824 March	»	13	1889 Jan.	0.8
»	» Dec.	»	»	» June	»	»	» Nov.	0.2
»	1823 Jan.	»	»	» July	»	»	1890 Feb.	0.6
»	» Feb.	»	»	» Nov.	»	14	1901 Apr.	0.0
»	» Apr.	»	9	1843 Feb.	3.5	»	» Dec.	0.0
»	» May	»	»	» Sept.	4.2	»	1902 Feb.	0.0
»	» June	»	»	1844 June	3.7	»	» Apr.	0.0
»	» Aug.	»	10	1855 Sept.	0.0	15	1913 May	0.0
			»	1856 May	0.0	»	» June	0.0
						16	1923 Aug.	0.5
						»	1924 Jan.	0.5

(cycles 7 - 18) because the Wolf numbers are not well known for cycles 1 - 6 (1749 - 1823). The data for cycles 1 - 6 will be taken only as supplementary ones.

As in the case of the areas, the most difficult thing here is the accurate determination of the time of rise which we shall represent by τ. This difficulty should be attributed to the appearance of many minimum values of R of al-

TABLE XI

Number of Cycles	Date of Commencement	Date of Maximum	Date of Minimum	R_M	τ in months	τ′ in months
1	1754 Feb.	1761 May	1766 June	107.2	88	61
2	1766 July	1769 Oct.	1775 July	158.2	41	69
3	1775 Aug.	1778 May	1784 July	238.9	34	74
4	1784 Aug.	1787 Dec.	1798 May	174.0	41	125
5	1798 June	1804 Oct.	1810 Nov.	62.3	77	73
6	1810 Dec.	1817 March	1823 July	96.2	76	76
7	1823 Nov.	1830 Apr.	1833 June	107.1	78	38
8	1833 July	1836 Dec.	1844 June	206.2	42	90
9	1844 July	1847 Oct.	1856 May	180.4	40	103
10	1856 June	1860 July	1867 Jan.	116.7	50	78
11	1867 Feb.	1870 May	1878 Aug.	176.0	39	99
12	1878 Sept.	1884 Jan.	1889 Jan.	91.5	65	60
13	1889 Feb.	1893 Aug.	1901 Apr.	129.2	55	92
14	1901 May	1907 Feb.	1913 May	108.2	70	75
15	1913 June	1917 Aug.	1923 Feb.	154.5	51	66
16	1923 March	1929 Dec.	1933 Aug.	108.0	82	44
17	1933 Sept.	1937 July	1944 Apr.	145 1	47	81
18	1944 May	1947 May	1954 Jan.	201.3	37	80

most the same magnitude in almost all cycles during the period of the minimum of the solar activity (cl. Table X).

The following table XI shows the epochs of the minima and maxima for each one cycle. It also shows the corresponding values of the maximnm R_M and the times τ and τ'. These values were taken from the tables of M. Waldmeir [2] (Tabelle 31) and of Greenwich [1]. The times τ and τ' were calculated as in the case of the areas.

A comparison of the tables IV and XI from cycle 12 to 18 shows that the taken epochs of the maxima and minima in both tables are about the same. The only exceptions are cycles 12 and 16. In cycles 12 the maximum area A_M takes place in July 1883 while the maximum observed Wolf number R_M in

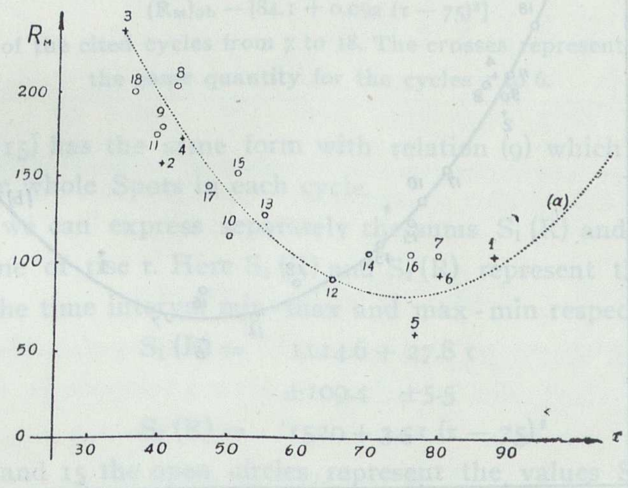


Fig. 12a. The open circles represent respectively for each one of the cited cycles the observed values of R_M. The crosses represent the values of the same quantity, for the cycles 1 to 18. The curve (a) represents the parabola $R_M = 84.1 + 0.092\,(\tau - 75)^2$.

Jan. 1884. In cycle 16 the date of Sept. 1923 is taken as the beginning (Wolf number) and the date of Aug. 1933 as the end instead of the dates March 1923 and Dec. 1933 (table IV) respectively. A disagreement therefore exists for the values T, T′ and τ, τ' between the tables IV and XI for the cycles 12, 16 and 17.

Based on table XI the maximum Wolf number R_M and the mean number $\overline{R}$ per cycle are expressed analytically as a function of the time of rise τ by the following relations:

$$(13) \qquad R_M = 84{,}1 + 0.092\,(\tau - 75)^2 + 25{,}2\,\sin\left(\frac{2\pi}{8}\,t + \frac{5\pi}{8}\right)$$

(14) $\bar{R} = 31.7 + 0.044\,(\tau - 68)^2$

$t = 0, 1, \ldots 11$

cycles: $7, 8, \ldots 18$

Relations (13) and (14) have the same form with relations (5) and (7) which give the maximum area A_M and the mean area $\bar{A}$ per cycle of the Spots. The only difference between them is that the vertex of the parabola in (13) is perceptibly displaced to the right while the periodic term X presents a difference of phase by $22^0.5$.

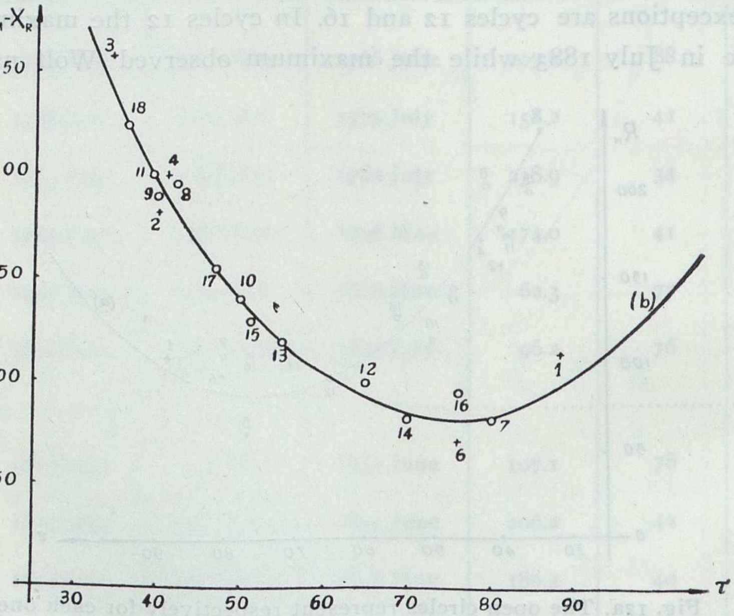


Fig. 12b. The open circles represent respectively for each one of the cited cycles the values of $(R_M)_{ob} - X_R$, and the crosses represent the values of the same quantity of the cycles 1 to 18. The curve (b) represent the parabola

$$(R_M)_{ob} - X_R = 84.1 + 0.092\,(\tau - 75)^2.$$

In fig. 12a and 12b the open circles represent respectively for each one of the cited cycles the values of

$$(R_M)_{ob}, \quad (R_M)_{ob} - X_R$$

and the crosses represent the values of the same quantities of the cycles 1 to 6. The curves (a) and (b) represent the parabola:

$$R_M = 84{,}1 + 0.092\,(\tau - 75)^2$$

We also observe here that the periodic term X_R (cl. fig. 13) appears only in the maximum R_M and not in the mean value $\bar{R}$ per cycle.

As in the case of the areas we can also express here the sum S (R) of Wolf numbers in each one cycle from 7 to 18 as a function of the time τ. Thus:

$$S(R) = 4436 + 4.65\,(\tau - 68)^2 - 500 \sin\left(\frac{2\pi}{4}\,t + 70^0\right) \tag{15}$$

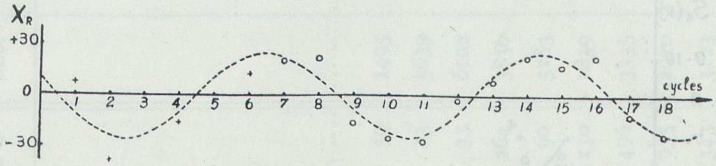


Fig. 13. The doted curve represents the periodic term $X_R = 25{,}2 \sin\left(\frac{2\pi}{8}\,t + \frac{5\pi}{8}\right)$.
The open circles represent the deviation of $(R_M)_{ob}$ from the parabolic law, i.e.
$(R_M)_{ob} - [84.1 + 0.092\,(\tau - 75)^2]$
for each one of the cited cycles from 7 to 18. The crosses represent the values of the same quantity for the cycles 1 to 6.

Relation (15) has the same form with relation (9) which gives the total area S (A) of the whole Spots in each cycle.

Moreover we can express separately the sums $S_1(R)$ and $S_2(R)$ as a function of the time of rise τ. Here $S_1(R)$ and $S_2(R)$ represent the sum of Wolf numbers R in the time interval min-max and max-min respectively. Thus:

$$S_1(R) = \underset{\pm 109.4}{1114.6} + \underset{\pm 5.5}{27.8}\,\tau \tag{16}$$

$$S_2(R) = 1520 + 3.51\,(\tau - 75)^2 \tag{17}$$

In fig. 14 and 15 the open circles represent the values $S_1(R)$ and $S_2(R)$

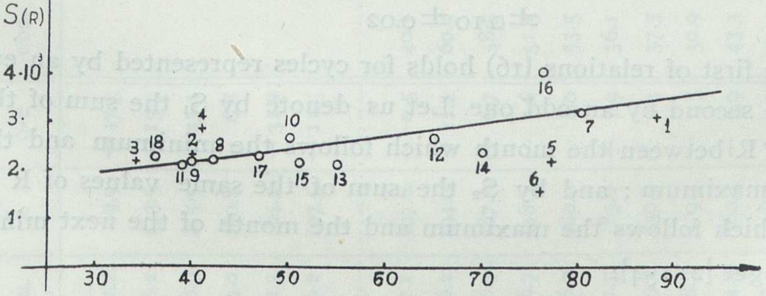


Fig. 14. The open circles represent the observed values of $S_1(R)$ for the cycles 7 to 18. The crosses represent the corresponding values for cycles 1 to 6.

given by the observations for cycles 7 - 18. The crosses represent the corresponding values for cycles 1 - 6.

As one can see from the table XII the agreement between the observed values of R_M, $\bar{R}$, $S(R)$, $S_1(R)$ and $S_2(R)$ is satisfactory for the cycles 7 to 18. The mean square errors are:

$$R_M \pm 6{,}6, \quad \bar{R} \pm 6{,}4, \quad S(R) \pm 238, \quad S_1(R) \pm 374, \quad S_2(R) \pm 308$$

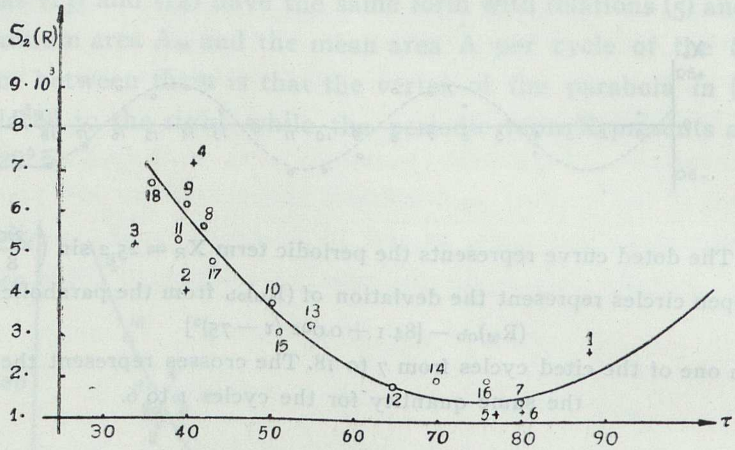


Fig. 15. The open circles represent the observed values of $S_2(R)$ for cycles 7 to 18 and the crosses corresponding values for cycles 1 to 6. The curve represents the parabola $S_2(R) = 1520 + 3.51\,(\tau - 75)^2$.

SMOOTHED VALUES OF R

It is known that if we take instead of the values of R given by the observations the smoothed ones, we get the following relations [2: 152 - 154]:

$$\text{(16)} \quad \begin{aligned} \log R_M^* &= \underset{\pm 0.09}{2.69} - \underset{\pm 0.02}{0.17}\,T \;, & N &= 2, 4, \ldots, 18 \\ \log R_M^* &= \underset{\pm 0.10}{2.48} - \underset{\pm 0.02}{0.10}\,T \;, & N &= 1, 3, 5, \quad 17 \end{aligned}$$

The first of relations (16) holds for cycles represented by an even number, while the second by an odd one. Let us denote by S_1 the sum of the smoothed values of R between the month which follows the minimum and the month of the next maximum; and by S_2 the sum of the same values of R between the month which follows the maximum and the month of the next minimum. Then we shall get [2: 154]:

$$\text{(17)} \quad S_1 = \underset{\pm 3{,}2}{0.4}\, R_M^* + \underset{\pm 340}{2538}$$

$$\text{(18)} \quad S_2 = -\underset{\pm 608}{572} + \underset{\pm 5.9}{40.6}\, R_M^*$$

Let us now see if relations of a similar form with (13) and (15) found above for observed values of R hold in the case of the smoothed values. To

TABLE XII

Number of Cycles	R_M			$\overline{R}$			S(R)			$S_1(R)$			$S_2(R)$		
	obs.	c	o-c	obs.	c	o-c	obs.	c	o-c	obs.	c	o-c	obs.	c	o-c
1	107.2	89.9	+ 17.3												
2	158.2	173.5	− 15.3												
3	238.9	215.5	+ 23.4												
4	174.0	180.8	− 6.8												
5	62.3	94.2	− 31.9												
6	96.2	107.5	− 11.5												
7	107.1	107.6	− 0.5	40.1	36.1	+ 4.0	4738	4421	+ 317	3244	3283	− 39	1495	1552	− 57
8	206.2	194.0	+ 12.2	60.5	61.4	− 0.9	7987	7510	+ 477	2295	2282	+ 13	5639	5342	+ 297
9	180.4	187.1	− 6.7	58.5	66.2	− 7.7	8359	8553	− 194	2260	2227	+ 33	6102	5820	+ 282
10	116.7	118.3	− 1.6	51.5	46.0	+ 5.5	6598	6092	+ 586	2769	2505	+ 264	3830	3714	+ 116
11	176.0	180.0	− 4.0	53.5	68.7	− 15.2	7392	7876	− 484	2109	2199	− 90	5283	5342	− 59
12	91.5	83.6	+ 7.9	36.1	32.1	+ 4.0	4510	4376	+ 134	2692	2922	− 230	1839	1871	− 32
13	129.2	130.6	− 1.4	37.3	39.1	− 1.8	5489	5693	− 204	2153	2649	− 496	3335	2924	+ 411
14	108.2	109.7	− 1.5	30.9	31.9	− 1.0	4483	4507	− 24	2425	3060	− 635	2010	1608	+ 402
15	154.5	160.4	− 5.9	43.3	44.4	− 1.1	5322	5309	+ 13	2199	2532	− 343	3123	3542	− 419
16	108.0	98.3	+ 9.7	41.3	40.3	+ 1.0	4955	5178	− 223	4080	3394	+ 686	2008	1692	+ 316
17	145.1	146.5	− 1.6	57.4	52.1	+ 6.3	7121	6958	+ 163	2349	2415	− 66	4771	4272	+ 499
18	201.3	193.6	+ 7.7	87.1	74.0	+ 13.1	9059	8905	+ 154	2331	2143	+ 188	6729	6588	+ 139

this we shall use the data of M. Waldmeir [2] Tabelle 32. For reasons of clarity we now shall represent the smoothed values by R^*, R^*_M. Table XIII gives for cycles 7 to 18 the epochs of the minimum and maximum; it gives also the corresponding values of R^*_M, τ^*. A comparison of this table with XI shows a significant difference in some cycles between R_M, τ and R^*_M, τ^* (cf. the two last columns of table XIII). The most sensible difference in relation to the times of

TABLE XIII

Number of Cycles	Date of Minim.	Date of Maxim.	R^*_M	τ^* Years	τ^* Months	$R_M - R^*_M$	τ-τ^* Months
7	1823,3	1829,9	71,7	6,6	79	+ 35,4	+ 1
8	1833,9	1837,2	146,9	3,3	40	+ 59,3	+ 2
9	1843,5	1848,1	131,9	4,6	54	+ 48,8	− 14
10	1856,0	1860,1	97,9	4,1	50	+ 18,8	0
11	1867,2	1870,6	140,5	3,4	41	+ 35,5	− 2
12	1878,9	1883,9	74,6	5,0	60	+ 16,9	+ 5
13	1889,6	1894,1	87,9	4,5	54	+ 41,3	+ 1
14	1901,7	1907,0	64,2	5,3	64	+ 44,0	+ 6
15	1913,6	1917,6	105,4	4,0	48	+ 49,1	+ 3
16	1923,6	1928,4	78,1	4,8	57	+ 29,9	+ 19
17	1933,7	1937,5	119,2	3,6	43	+ 25,9	+ 4
18	1944,2	1947,5	151,8	3,3	39	+ 49,5	− 2

rise τ and τ^* are observed in cycles 9 and 16. As for cycle 16 the difference is due to the fact that the maximum value of R_M took place in December 1929 ($R_M = 108.0$) while the maximum value of R^*_M in April 1928, that is in the year in which we have the annual maximum. The difference $\tau - \tau^*$ of cycle 9 is due to a respective difference of the data of minimum and maximum.

Based on table XIII we get the following relations between (R^*_M, τ^*) and $S(R^*)$, τ^* where $S(R^*)$ represents the total sum of the relative numbers R^* for each one cycle:

$$R^*_M = 64.2 + 0.105\,(\tau^* - 68)^2 \tag{19}$$

$$S\,(R^*) = 4260 + 5.12\,(\tau^* - 68)^2 - 500 \sin\left(\frac{2\pi}{4}\,t + \frac{\pi}{4}\right) \tag{20}$$

$$t = 0, 1, \ldots \quad 11$$

$$\text{cycles:} \quad 7, 8, \ldots \quad 18$$

Table XIV shows that relation (19) gives more satisfactory results than re-

TABLE XIV

Number of Cycles	R^*_M From (16)	Dif. o-c	R^*_M From (19)	Dif. o-c	$S(R^*)_{ob}$	S (R*) From (20)	Dif. o-c
1	70.8	+ 15.7	70.9	+ 15.6			
2	140.0	— 24.2	140.7	— 24.9			
3	154.9	+ 3.6	171.7	— 13.2			
4	129.5	+ 11.7	140.7	+ 0.5			
5	61.7	— 12.5	81.9	— 32.7			
6	50.6	— 1.9	64.6	— 15.9			
7	66.1	+ 5.6	76.9	— 5.2	4755	4527	+ 228
8	134.6	+ 12.3	146.5	+ 0.4	7797	7901	— 204
9	104.7	+ 26.9	84.8	+ 46.8	8295	5617	+ 2678
10	98.4	— 0.5	98.2	— 0.3	6557	6262	+ 295
11	138.1	+ 2.4	140.7	— 0.2	7498	7639	— 141
12	69.2	+ 5.4	70.9	+ 3.7	4603	4235	+ 368
13	107.2	— 19.3	84.8	+ 3.1	5538	5616	+ 78
14	61.5	+ 2.7	65.8	— 1.6	4459	4659	— 200
15	120.2	— 14.8	106.2	— 0.8	5330	5955	— 625
16	74.8	+ 3.3	76.9	+ 1.2	4948	4527	+ 421
17	131.8	— 12.6	124.7	— 10.5	7202	7801	+ 599
18	134.6	+ 17.2	152.5	— 0.7	9010	8919	+ 91

lations (16). Exception made of cycle 9 where both relations present a perceptible disagreement, in all the other cycles from 7 to 18 the differences $(R^*_M)_{ob} - (R^*_M)_c$ are smaller when we consider relation (19) than the two relations (16).

In cycle 1 to 6 where the Wolf numbers are not well known relations (16) give more satisfactory results. These cycles, especially 2 and 5, present a perceptible deviation from the parabolic law but one cannot rely on them because of the incomplete knowledge of Wolf numbers.

One can make similar remarks for the total sum $S(R^*)$ of Wolf numbers in each cycle. Relation (20) gives more satisfactory results than the two relations (17) and (18) which separately give the sums $S_1(R^*)$ (min−max) and $S_2(R^*)$

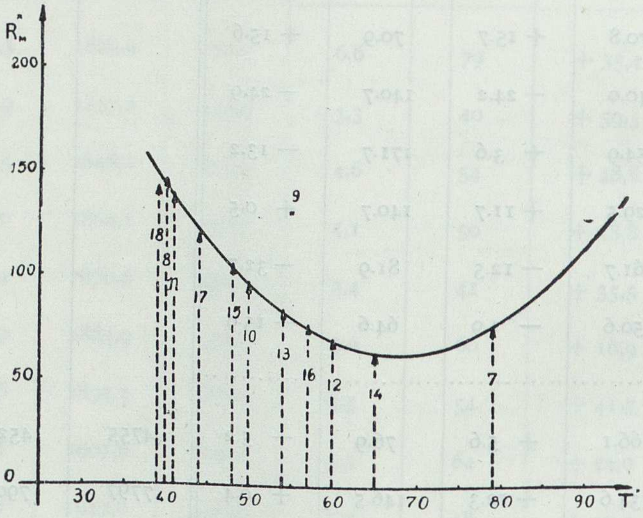


Fig. 16. The dotted lines represent for each of the cited cycles the observed values of R^*_M. The full curve represents the parabola $R^*_M = 64.2 + 0.105\,(\tau^* - 68)^2$.

(max−min). The square errors of the differences o−c, excluding the cycle 9, are:

R^*_M	From relations (16),	cycles 7 - 18,	$\pm$ 10.72
	» » (19),	» » ,	$\pm$ 2.80
	» » (16),	» 1 - 18,	$\mp$ 11.9
	» » (19),	» » ,	$\pm$ 12,2
$S(R^*)$	From relations (17) and (18),	cycles 7 − 18	$\pm$ 629
	» » (20)	, » »	$\pm$ 333

It is my opinion that the observed disagreement in cycle 9 is due probably to the fact that the time of rise τ^* is too big. Really, the minimum observed values of R for this cycle took place in Febr. 1843 (R = 3.5), Sept. 1843 (R = 4.2) and June 1844 (R = 3.7) and the annual minimum in 1843. Similarly the observed maximum R_M takes place in Oct. 1847 (R_M = 180.4) while the annual maximum in 1848 when we have the maximum of R^* (Febr. 1848 $R^*_M = 131.6$). In table XI we took as the epoch of the minimum June 1844 ($R = 3.7$) and as the epoch of maximum Oct. 1847 ($R_M = 180.4$). This is legal and in accordance with the agreed principles. We cannot however do the same thing for the smoothed values of R^*. Moreover it is noteworthy that if we take here the time of rise τ^* near the time of rise τ, i.e. $\tau^* = 42$ months, the difference $(R^*_M)_{ob} - (R^*_M)_c$ disappears. It is this latter result which raises our suspicion lest the taken time of rise $\tau^* = 54$ months.

In fig. 16 and 17 the dotted lines represent for each of the cited cycles

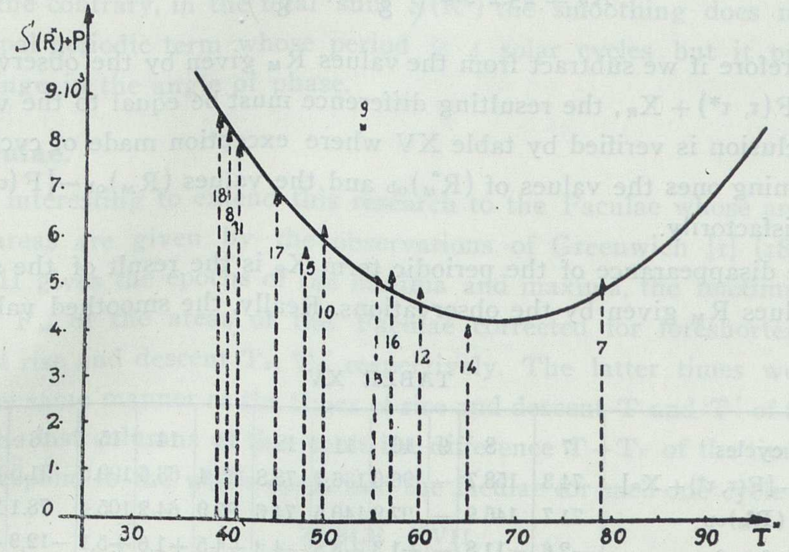


Fig. 17. The dotted lines represent for each of the cited cycles the values of $S(R^*)_{ob} + P$, $\left[P = 500 \sin\left(\frac{2\pi}{4} t + \frac{\pi}{4}\right)\right]$. The full curve represent the parabola:
$$S(R^*) = 4260 + 5.12\,(\tau^* - 68)^2.$$

the values of $(R_M)_{ob}$ and $S(R^*) + 500 \sin\left(\frac{2\pi}{4} t + \frac{\pi}{4}\right)$ while the curves represent the parabolas:

$$R^*_M = 64.2 + 0.105\,(\tau^* - 68)^2, \quad S(R^*) = 4260 + 5.12\,(\tau^* - 68)^2$$

One observes that the disagreement in cycle 9 disappears in both cases if the time of rise τ^* is taken near the value of τ.

Comparing relations (19) and (20) with the corresponding ones (13) and (15) we see that they have the same form with the difference that in relation (19) the term X does not appear. This means that *because of the smoothing of the* *observed values of R the term of the long period X in the maximum R^*_M disappears.* Really from relations (13) and (16) we have.

$$R_M - R^*_M = F(\tau, \tau^*) + 25{,}2 \sin\left(\frac{2\pi}{8} t + \frac{5\pi}{8}\right), \tag{21}$$

where:

$$F(\tau, \tau^*) = 19{,}9 + 0.092\,(\tau - 75)^2 - 0.105\,(\tau^* - 68)^2$$

Relation (21) shows that the difference of the two maxima R_M and R^*_M is a function of the times of rise τ and τ^* and of the periodic term

$$X_R = 25.2 \sin\left(\frac{2\pi}{8} t + \frac{5\pi}{8}\right).$$

Therefore if we subtract from the values R_M given by the observations the quantity $F(\tau, \tau^*) + X_R$, the resulting difference must be equal to the values R^*_M. This conclusion is verified by table XV where exception made of cycle 9 in all the remaining ones the values of $(R^*_M)_{ob}$ and the values $(R_M)_{ob} - [F(\tau, \tau^*) + X_R]$ agree satisfactorily.

The disappearance of the periodic term X_R is the result of the smoothing of the values R_M given by the observations. Really, the smoothed values of R^*_M

TABLE XV

cycles	7	8	9	10	11	12	13	14	15	16	17	18
$(R_M)_{ob} - [F(\tau, \tau^*) + X_R]$	74,3	158.7	—	96.6	136,7	78.8	83.4	63.6	100.3	91.0	118.3	153.0
$(R^*_M)_{ob}$	71,7	146.9	—	97,9	140.5	74.6	87,9	64.2	105.4	78.1	119.2	151.8
o — c	—2,6	—11.8	—	+1,3	+3,8	—4.2	+4,5	+1,6	+5,1	—12,9	+0,9	—1.2

depend not only upon the observed maximum values R_M but upon their neighboring ones. As we have already seen in the case of the areas if we consider instead of the maximum area A_M the mean values $[\bar{A}_M]_{-3,+3}$ which correspond to the time interval between 3 months before the maximum and 3 months after it, the term X disappears and the mean values in question are represented by relation (8). Thus if we take the mean value of R in the neighborhood of the maximum, i.e.

$$[\bar{R}_M]_{-3,+3} = \frac{1}{7}\sum_{-3}^{+3} R,$$

the periodic term X_R almost disappears and we get:

$$[\bar{R}_M]_{-3,+3} = 62 + 0.071\,(\tau - 75)^2 + 8 \sin\left(\frac{2\pi}{8} t + \frac{5\pi}{8}\right)$$

The observed and calculated values of $[\overline{R}_M]_{-3,+3}$ are given by the following table XVI.

TABLE XVI

Cycles:	7	8	9	10	11	12	13	14	15	16	17	18
$[\overline{R}_M]_{ob\,-3,+3}$	70	151	135	98	147	73	89	64	115	61	123	161
$[\overline{R}_M]_{c\,-3,+3}$	71	142	146	99	146	66	93	71	110	65	115	163
o − c	−1	+9	−11	−1	+1	+7	−4	−7	+5	−4	+8	−2

On the contrary, in the total sum S(R*) the smoothing does not act on the principal periodic term whose period is 4 solar cycles but it provokes a slight change in the angle of phase.

IV. Faculae.

It is interesting to extend this research to the Faculae whose annual and monthly areas are given by the observations of Greenwich [1] (1874-1954). Table XVII gives the epochs of the minima and maxima, the maximum observed value F_M of the areas of the Faculae corrected for foreshortening and the time of rise and descent T_F, T_F' respectively. The latter times were calculated in the same manner as the times of rise and descent T and T′ of the whole spots. In the last columns of this table the difference $T - T_F$ of the times of rise which correspond to the whole spots and the faculae for each one cycle are cited.

TABLE XVII

Number of Cycles	Date of Commencement	Date of Maximum	Date of Minimum	Max. Areas	T_F (Months)	T'_F	$T-T_F$
12	1878 Sept.	1884 Jan.	1889 Feb.	3122	65	61	− 6
13	1889 March	1892 Oct.	1901 April	4080	44	102	+ 11
14	1901 May	1905 Aug.	1913 Aug.	3255	52	96	+ 18
15	1913 Sept.	1917 Aug.	1923 March	3523	48	67	+ 3
16	1923 Apr.	1929 Dec.	1933 Dec.	3631	81	48	+ 1
17	1934 Jan.	1937 Aug.	1944 May	4407	44	82	− 1
18	1944 June	1947 Aug.	1954 June	3594	39	82	− 3

The last column of Table XVII shows that the times of rise T_F and T of the faculae and of the spots do not coincide. The most notable differences are observed in cycles 13 and 14 where the epochs of the observed maxima and minima areas F_M and A_M of the faculae and of the whole spots differ by 11 and 18 months respectively. As in the case of the spots, cycle 16 presents here the following distinct maxima:

1926 Feb. $F_M = 3922$
1928 July $F_M = 3155$
1929 Dec. $F_M = 3631$

We took here Dec. 1929 as the epoch of maximum where a maximum is also observed for the areas of the whole spots and of Umbrae.

Based on table XVII we get the following relations giving the maximum areas F_M and the total areas S(F) of the faculae as a function of the time of rise T_F.

(22) $$F_M = 3075 + 1.8\,(T_F - 65)^2$$

(23) $$S(F) = 151140 + 95\,(T_F - 63)^2 + P_F$$

where

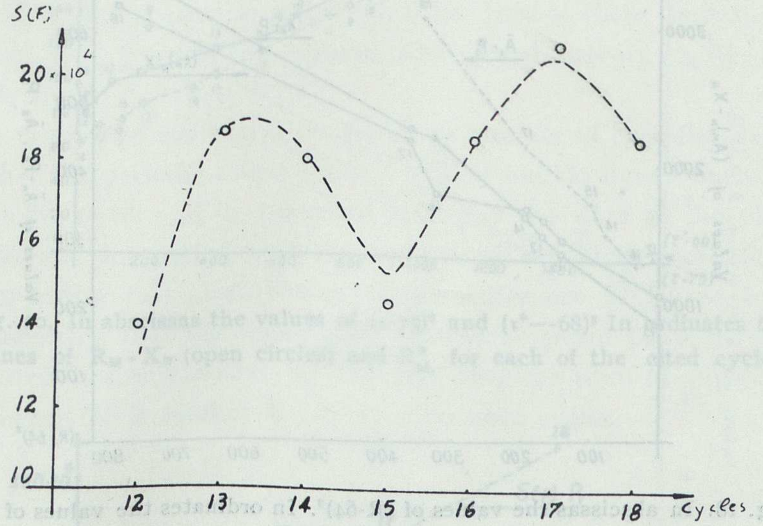
$$P_F = -22900 \sin\left(\frac{2\pi}{3}\,t + 65\right)$$

Table XVIII shows that the agreement between the values of $(F_M)_{ob}$, $S(F)_{ob}$ and the ones given by the above relations is satisfactory. The only notable difference is observed in cycles 17 and 18 for the values of the maximum

TABLE XVIII

Number of Cycles	F_M obs.	F_M Comp.	o-c	S(F) obs.	S(F) Comp.	o-c
12	3122	3075	+ 57	140328	131753	+ 8575
13	4080	3869	+ 211	187440	188407	— 967
14	3255	3379	— 124	180246	182370	— 2124
15	3523	3595	— 72	145278	152748	— 7470
16	3631	3536	+ 95	185278	184892	+ 386
17	4407	3869	+ 538	208040	205170	+ 2870
18	3594	4292	— 698	184908	186093	— 1185

areas F_M. This difference is due to the fact that the maximum area F_M in cycle 17 is bigger than in cycle 18 although the difference $|T_F - 65|$ is bigger in cycle 18. We observe therefore a deviation of the maximum area F_M from the parabolic law in those two cycles.



The open circles represent the observed values of S(F) and the curve the computed ones from (23). In abscissas the successives cycles from 12 to 18.

From the above analysis we conclude that the maximum, the mean and the total areas of the whole spots, umbrae and Faculae as well as the maximum Wolf numbers R_M, R^*_M and their sum per cycles $S(R)$ and $S(R^*_M)$ can be expressed analytically as a function of the «time of rise» expressed either in synodic rotations of the Sun or in months.

The algebraic part of these relations represents a parabola; the remaining one is a sum of periodic terms. It is remarkable that the vertices of these parabolas in all cases have almost the same abscissas lying between 62 - 68 units of the axis of the abscissas in the case of areas, and 68 - 75 in the case of Wolf numbers.

This shows that there exists a particular value of the time of rise, lying between 62 to 68 months in the case of the areas and 68 to 75 months in the case of Wolf numbers. The above value is such that the cycles whose times of rise are near it possess the lowest maximum, the smallest mean value, and the smallest total area or total sum in the case of Wolf numbers. In other words as bigger are the differences $(R-T_o)^2$, $(T-T_o)^2$ as bigger are the values of $(A_M)_N$, $(A_M)_S$, A_M, U_M, F_M, R_M, R^*_M, $\overline{A}$, $\overline{U}$, $\overline{R}$, $S(A)$, $S(U)$, $S(R)$, $S(R^*)$ and $S(F)$.

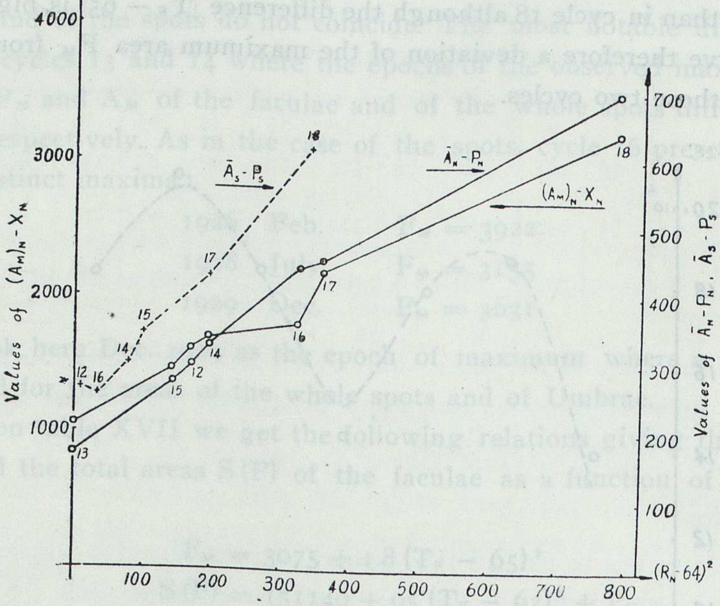


Fig. 18. In abscissas the values of $(R\text{-}64)^2$. In ordinates the values of the quantities: $(A_M)_N - X_N$, $\overline{A}_N - P_n$, and $\overline{A}_S - P_S$ for each of the cited cycles.

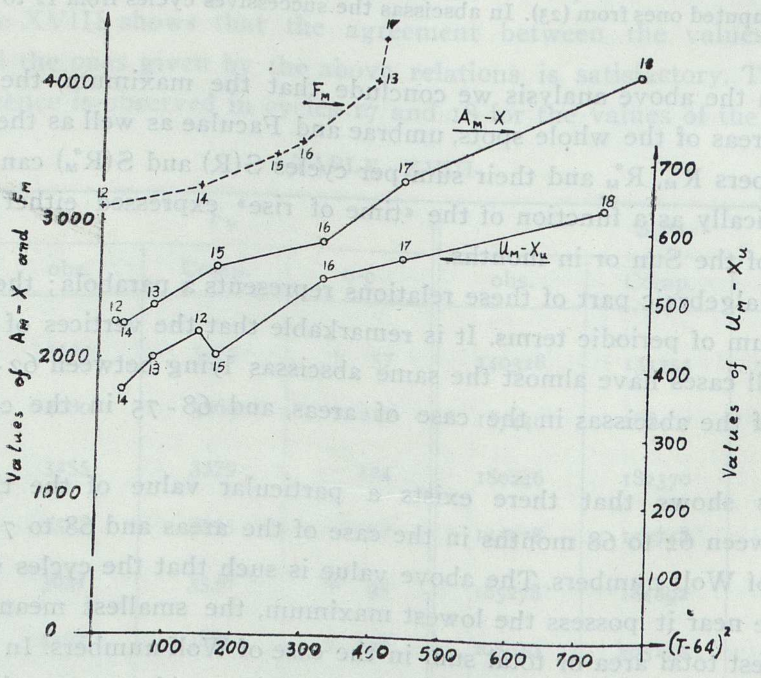


Fig. 19. In abcissas the values of $(T\text{-}64)^2$. In ordinates the values of the quantities $A_M - X_M$, $U_M - X_u$ and F_M, for each of the cited cycles.

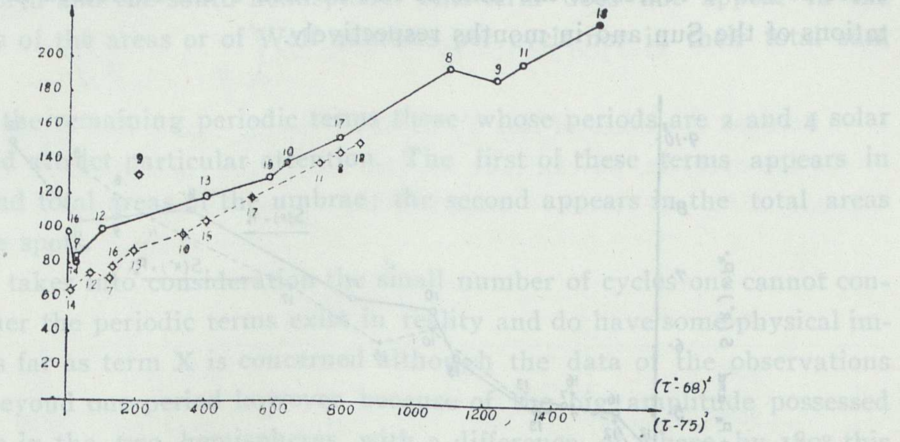


Fig. 20. In abscissas the values of $(\tau\text{-}75)^2$ and $(\tau^*-68)^2$ In ordinates the values of $R_M - X_R$ (open circles) and R^*_{M}, for each of the cited cycles.

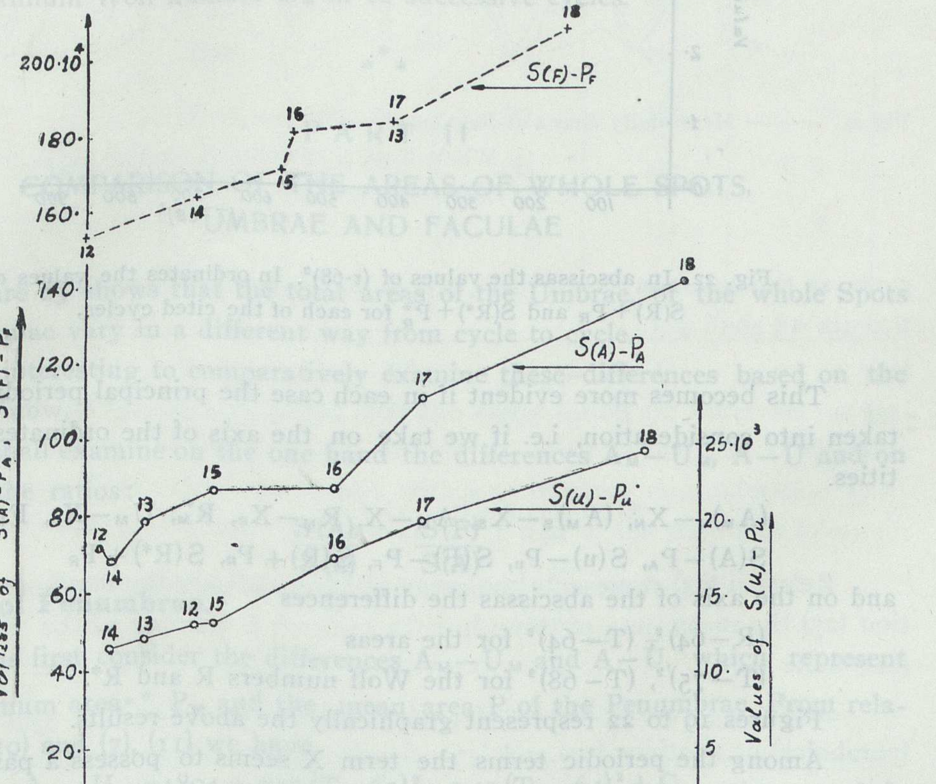


Fig. 21. In abscissas the values of $(T\text{-}64)^2$. In ordinates the values of $S(A) - P_A$, $S(u) - P_u$ and $S(F) - P_F$ for each of the cited cycles.

Here R and T represent the time of rise in each cycle expressed in synodic rotations of the Sun and in months respectively.

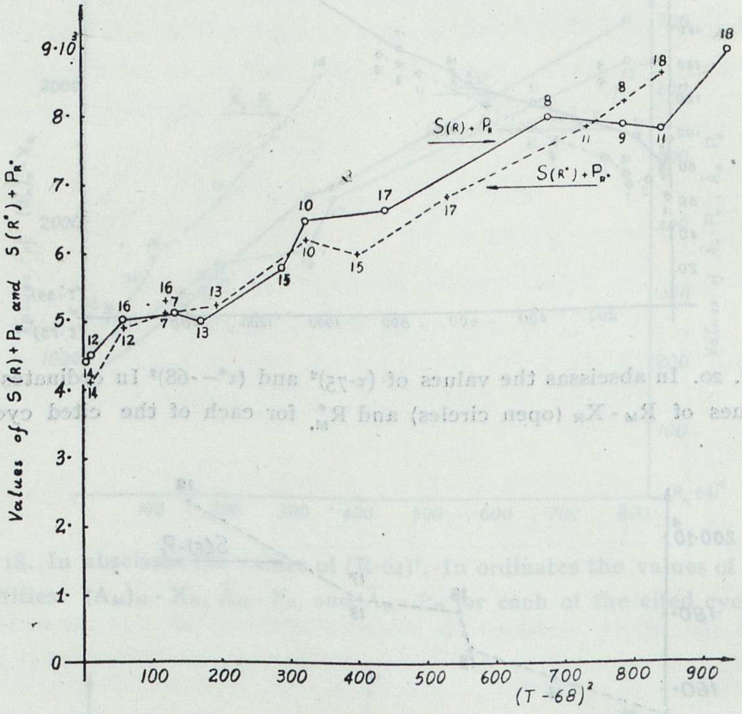


Fig. 22. In abscissas the values of $(\tau\text{-}68)^2$. In ordinates the values of $S(R)+P_R$ and $S(R^*)+P^*_R$ for each of the cited cycles.

This becomes more evident if in each case the principal periodic terms are taken into consideration, i.e. if we take on the axis of the ordinates the quantities.

$$(A_M)_N - X_N,\ (A_M)_S - X_S,\ A_M - X,\ R_M - X_R,\ R^*_M,\ U_M - X_u,\ F_M,$$
$$S(A) - P_A,\ S(u) - P_u,\ S(F) - P_F,\ S(R) + P_R,\ S(R^*) + P_R$$

and on the axis of the abscissas the differences

$(R-64)^2$, $(T-64)^2$ for the areas

$(T-75)^2$, $(T-68)^2$ for the Wolf numbers R and R*.

Figures 19 to 22 respresent graphically the above results.

Among the periodic terms the term X seems to possess a particular importance. The period of this term is 8 solar cycles and appears only in the maximum of the whole spots and Umbrae but not in the maximum of the faculae. The term in question has a significant amplitude when we consider the two Sun's hemisperes separately. Then it presents a difference of phase by 180° between the north and the south hemisphere. This term does not appear in the mean values of the areas or of Wolf numbers per cycle nor in their total sum per cycle.

From the remaining periodic terms those whose periods are 2 and 4 solar cycles should attract particular attention. The first of these terms appears in the mean and total areas of the umbrae; the second appears in the total areas of the whole spots.

If one takes into consideration the small number of cycles one cannot conclude whether the periodic terms exist in reality and do have some physical importance. As far as term X is concerned although the data of the observations do not go beyond one period however because of the big amplitude possessed by this term in the two hemispheres with a difference of phase by 180° this raises our suspicion that the above term possesses certain physical importance. The latter view is strengthened by the fact that the term in question appears in the maximum Wolf number R_M in 12 successive cycles.

* * *

PART II
COMPARISON OF THE AREAS OF WHOLE SPOTS, UMBRAE AND FACULAE

Picture 23 shows that the total areas of the Umbrae, of the whole Spots and of Faculae vary in a different way from cycle to cycle.

It is interesting to comparatively examine these differences based on the data up to now.

We shall examine on the one hand the differences $A_M - U_M$, $\overline{A} - \overline{U}$ and on the other the ratios:

$$\frac{S(A)}{S(u)}, \quad \frac{S(F)}{S(A)}$$

a) **Areas of Penumbrae.**

Let us first consider the differences $A_M - U_M$ and $\overline{A} - \overline{U}$, which represent the «maximum area»* P_M and the mean area $\overline{P}$ of the Penumbrae. From relations (5), (10) and (7), (11) we have

$$A_M - U_M = 1804 + 2{,}40\,(T-65)^2 - 0{,}40\,(T_u - 64)^2 + G$$
$$\overline{A} - \overline{U} = 406 + 0{,}80\,(T-67)^2 - 0{,}19\,(T_u - 64)^2 + 13{,}2 \cos \pi t$$

* The difference $A_M - U_M$ represents the area of the Penumbra at the epoch of the maximum of the solar activity.

where:

$$G = 300 \sin (t-1)\frac{2\pi}{8} - 70 \sin\left(\frac{2\pi}{8}t - \frac{3\pi}{8}\right)$$

Because the times of rise T, T_u of the whole spots and Umbrae coincide, except the cycle 12, we can take approximately the time of rise T of the whole

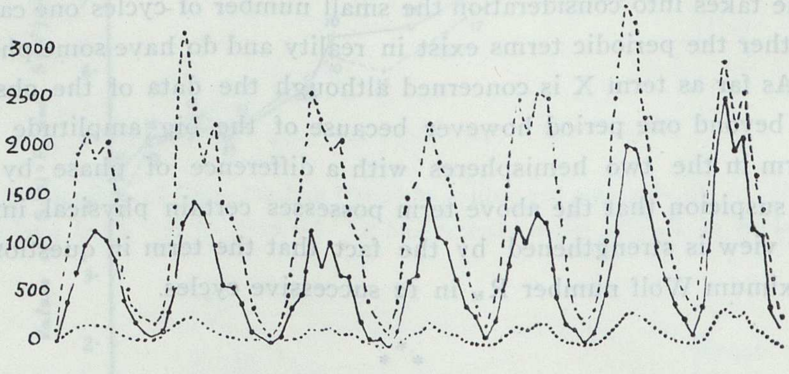


Fig. 23. -------- Mean daily Areas of Faculae for each year.
——— » » » » Whole Spot (Penumbra + Umbra) for each year.
.... » » » » Umbra for each year.

spots as the common time of rise. Then modifying slightly the numerical coefficients we shall get:

(24) $$P_M = 1785 + 2.2\,(T-62)^2 + G$$

(25) $$\bar{P} = 397 + 0.62\,(T-68)^2 + 13.2 \cos \pi t$$

where

$$G = 260 \sin (t-1)\frac{2\pi}{8}$$

Relation (24) represents the areas of Penumbrae at the maximum and relation (25) the mean area of those in each cycle as a function of the time of rise. (See table XIX).

In fig. (24) and (25) the curves (a) and (b) represent the algebraic part (parabolas) of relations (24) and (25); the directed segments represent the values of the quantities

$$P_M - G = (A_M - U_M) - G$$

$$\bar{P} - 13{,}2 \cos \pi t = (\bar{A} - \bar{U}) - 13{,}2 \cos \pi t$$

for each of the cited cycles.

b) **The Ratios** $\frac{S(A)}{S(U)}$, $\frac{S(F)}{S(A)}$

Let us now examine the ratios:

$$q_A = \frac{S(A)}{S(U)} \quad , \quad Q_F = \frac{S(F)}{S(A)}$$

where S(A), S(u) and S(F) represent the total areas of whole spots, Umbrae and faculae respectively in each cycle.

TABLE XIX

Number of Cycles	P_M obs.	P_M Comp.	P_M o-c	p obs.	p Comp.	p o-c
12	1688	1680	+ 8	470	460	+ 10
13	1980	1963	+ 17	497	489	+ 8
14	2071	2048	+ 23	403	413	— 10
15	2481	2417	+ 64	557	563	— 6
16	2498	2682	— 184	538	532	+ 6
17	2771	2755	+ 16	777	771	+ 6
18	3343	3325	+ 18	1032	1045	— 13

Relations (9), (12) and (23) show that the above ratios are functions on the one hand of the times of rise T, T_u, T_f, and on the other of the time t. We can however express these ratios analytically with a satisfactory approximation as a simple periodic function of only the time of rise T and of the time t as follows:

$$(26) \qquad q_A = 6.041 - 0.750 \sin (T-1)\frac{2\pi}{36} - 0.183 \cos\left(\frac{2\pi}{7}\, t + 12°,9\right)$$

$$(27) \qquad Q_F = 1.981 + 0.623 \cos \frac{2\pi}{72}\, T - 0.120 \cos \pi t$$

Table XX shows that the values of the ratios taken from the observations and those computed from the above relations (26) and (27) agree satisfactorily The mean square errors are respectively:

$$q_A : \pm 0.055 \quad , \quad Q_F : \pm 0.105$$

If we consider the ratio $q_P = \frac{\text{Penumbra}}{\text{Umbra}}$ we have

$$q_P = \frac{S(P)}{S(u)} = \frac{S(A) - S(u)}{S(u)} = q_A - 1$$

Therefore it follows from (26)

$$q_P = 5.041 - 0.750 \sin(T-1)\frac{2\pi}{36} - 0.183 \cos\left(\frac{2\pi}{7}t + 12^\circ.9\right) \tag{28}$$

In fig. 25 the directed segments represent the values of q_P taken from the observations for each of the cited cycles while the dotted lines the values of this ratio, which are computed from relation (28).

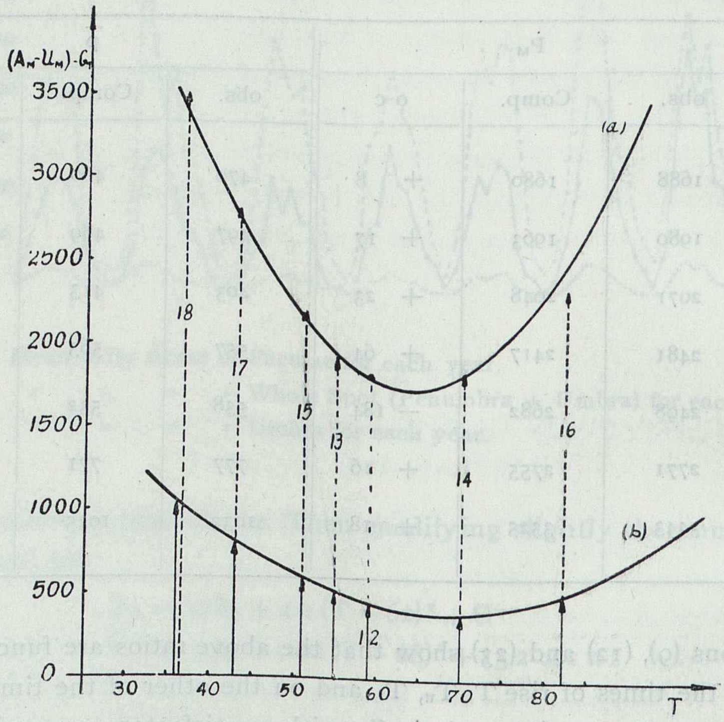


Fig. 24. The dotted directed segments represent the values of the quantities P_M - G. The full directer segments represent the values of $\overline{P}$-13,2 cosπt for each one cited cycles. The full curves (a) and (b) represent the parabolas of the relation (24) and (25).

Thus, the principal part of the variation of the ratio $\frac{\text{Penumbra}}{\text{Umbra}}$ from cycle to cycle is a periodic function of the time of rise T with period 36 months = 3 years; the remaining part is a periodic function of the time with period 7 solar cycles.

As for the ratio Q_F of the faculae to the whole spots (Penumbra + Umbra) relation (27) also shows here that the principal part of the change from cycle to

TABLE XX

Number of Cycles	T	q_A obs.	q_A Comp.	q_A o-c	Q_F obs.	Q_F Comp.	Q_F o-c
12	59	6.307	6.361	—.054	2.004	2.125	—.121
13	55	5.964	5.978	—.014	2.139	2.153	—.014
14	70	6.541	6.511	+.030	2.617	2.475	+.142
15	51	5.682	5.753	—.071	1.846	1.940	—.094
16	82	5.489	5.450	+.039	2.159	2.261	—.102
17	43	5.396	5.407	—.011	1.737	1.591	+.146
18	36	6.150	6.045	+.105	1.282	1.238	+.044

cycle is a periodic function of the time of rise T, but with a double period of 72 months = 6 years; the remaining part, which is a periodic function of the time t, possesses a period of 2 solar cycles.

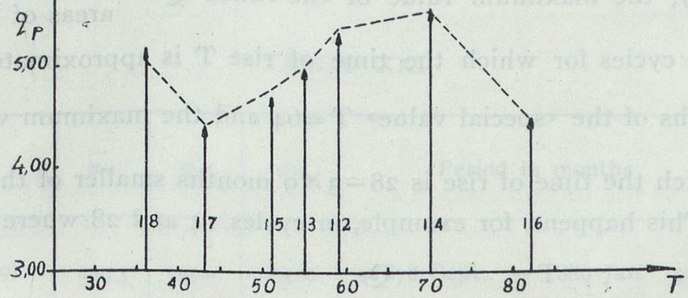


Fig. 25. The directed segments represent the observed values of the ratio q_P; The dotted line represent the values of this ratio computed from relation (28).

From relations (27) and (28) we conclude that, if the periodic terms in relation to the time t are not taken into consideration, then the maximum value of the ratios Q_F and q_P corresponds to those cycles for which the time of rise T is:

Q_F : 72, 144 months

q_P : 28, 64, 100, 136 months

Because up to the present we have not observed cycles for which the time of rise is $T < 30$ months or $T > 110$ months the most interesting cases are those for which we have $T=64$ and $T=72$ months. For the value $T=64$ months, relations (5), (7), (10) and (11) show that we have the minimum value of the mean areas per cycle of the whole Spots and Umbrae ($\overline{A}$, $\overline{u}$) on the one hand and on the other the lower maximum A_M and U_M. Thus we conclude that if the periodic terms in relation to t are not taken into consideration, the maximum value of q_p is observed in those cycles for which we have low maximum and small mean value per cycle of the areas of the whole spots. This happens, for example, in cycles 12 and 14 where:

cycles	T	A_M	U_M	$\overline{A}$	$\overline{u}$	q_p
12	$59=64-5$	2066	378	567	89.5	5.307
14	$70=64+6$	2456	380	473	72.3	5.541

Moreover the minimum value of the ratio q_p takes place in those cycles for which the time of rise is smaller or greater by 18 months of the «special value» $T=64$. The latter happens in cycles 16 and 17, where:

cycles	T	q_p
16	$82=64+18$	4.450
17	$43=64-21$	4.407

Similarly, the maximum value of the ratios $Q = \dfrac{\text{areas of Faculae}}{\text{areas of Spots}}$ takes place in those cycles for which the time of rise T is approximately bigger by $\dfrac{18}{2}=9$ months of the «special value» $T=64$ and the maximum value in those cycles for which the time of rise is $28 = 3\times 9$ months smaller of the «special value» $T=64$. This happens, for example, in cycles 14 and 28 where:

cycles	T	Q_F
14	$70=64+6$	2.617
18	$36=64-28$	1.303

The above results change slightly due to the presence of the periodic terms in relation to the time t. The most important of these periodic terms have periods equal to 7 and 2 solar cycles.

The ratio of the areas of the spots to the corresponding areas of the Umbrae was studied by E. Jensen - J. Nordø and T. S. Ringnes [3] for each year from 1874 to 1952.

For this purpose they compute the quantities

$\overline{q}' = [2 \times \text{mean of daily total areas} / 2 \times \text{mean of daily umbrae areas}]^{1/2}$

$\overline{q} = \dfrac{P}{u}$

where:

$$P = \sqrt{\frac{2 \sum_{1}^{m} A_p}{m}} \quad , \quad u = \sqrt{\frac{2 \sum_{1}^{m} A_u}{m}}$$

A_u and A_p denote the corrected Umbra and total Sunspot areas in terms of one millionth of the solar hemisphere respectively. The number of observations per transit is denoted by m. For the determination of the ratio $\overline{q}$ the above investigators select the material on the basis of definite rules; the ratio $\overline{q}'$ is computed by them on the basis of the observations of Greenwich [1 : 5] without any selection of the principles applied. They find that the curves $\overline{q}$ and $\overline{q}'$ have a very similar run with the solar cycle and especially with the curve of

$$\Pi = \sqrt{2\, \overline{A}_p},$$

where $\overline{A}_p$ is the mean daily sunspot area for the year at which it is published by the observations of Greenwich [1 : 5].

Finally, they conclude «that variation with time in the Penumbra - Umbra ratio of sunspots takes place. These variations must be intimately connected with corresponding changes in the magnetic fields and in the structure of the Spots...».

TABLE XXI

Number of Cycles	$\overline{\pi}_c$	$\overline{q}'_c$	$\overline{q}_c$	Period in months			$\overline{q}_A$
12 (1879 - 89)	29.73	2.500	2.349	1878 Sept.	— 1889 Jan.	59	2.511
13 (1890 - 1901)	31.10	2.386	2.287	1889 Feb.	— 1901 April	55	2.442
14 (1902 - 13)	26 95	2.483	2.360	1901 May	— 1913 May	70	2.557
15 (1914 - 23)	33 86	2.349	2.348	1913 June	— 1923 Feb.	51	2.383
16 (1924 - 33)	35.00	2.308	2.306	1923 March	— 1933 Dec.	82	2.342
17 (1934 - 44)	39.45	2.324	2.305	1934 Jan.	— 1944 April	43	2.323
18 (1934 - 53)	47.97	2 450	—	1944 May	— 1954 Jan.	36	2.480

It might be interesting to see whether the mean values of $\overline{q'}$ and $\overline{q}$ per cycle verify relations of a form similar to (26). Table XXI gives the mean values of $\overline{\pi}$, $\overline{q'}$ and $\overline{q}$ represented by $\overline{\pi}_c$, $\overline{q'_c}$, $\overline{q}_c$ (columns 2, 3, 4) for each cycle. These values were computed from table II of Jensen - Nordø - Ringnes [2]. In the last column of this table one can find the values of the quantity $\overline{q_A} = (q_A)_{ob}^{1/2}$, which are taken from our table XIX (column 3).

It is evident that the values $\overline{q_A}$ and $\overline{q'_c}$ do not coincide, for the first ones were taken from the mean daily areas of the sunspots for each calendar month

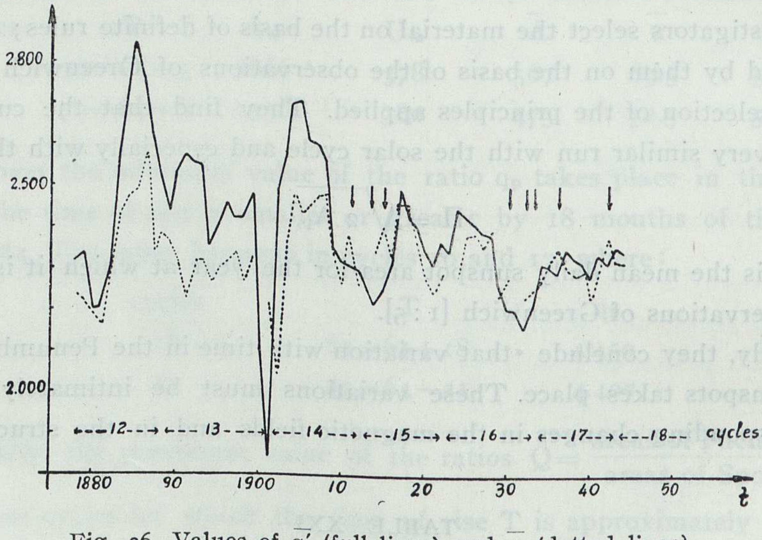


Fig. 26. Values of $\overline{q'}$ (full lines) and $\overline{q}$ (dotted lines).

[1 : 7—16], while the second ones from the mean daily areas of the sunspots for each year [1 : 5—7]. The above table shows that while the values of $\overline{q'_c}$ and $\overline{q_A}$ vary from cycle to cycle in a similar manner the same does not happen for the quantity $\overline{q}_c$. This is due to the selection of the observations based on the rules A, B, C and D set up by Jensen, Nordø and Ringnes for the calculation of the ratio q. The graphical representation (fig. 26) of the annual values of $\overline{q'}$ (full lines) and $\overline{q}$ (dotted lines) shows that the values of $\overline{q}$ are all smaller than the corresponding values of $\overline{q}$. But in the cycles 15 and 17 among the 29 values in total, 8 are greater than the corresponding ones of $\overline{q'}$. And this is the reason for which the difference $\overline{q'_c} - \overline{q}_c$ in cycles 15, 16 and 17 diminishes significantly:

cycles	12	13	14	15	16	17
$\overline{q'_c} - \overline{q}_c$	+0.151	+0.099	+0.123	+0.001	+0.002	+0.019

TABLE XXII

Values of q_A, q'_c, $\bar{q}_c$ and $\bar{\pi}_c$ observed and computed from (29)–(32)

Number of Cycles	q_A obs.	q_A comp.	(o-c). 10^{-3}	q'_c obs.	q'_c comp.	(o-c). 10^{-3}	$\bar{q}_c$ obs.	$\bar{q}_c$ comp.	(o-c). 10^{-3}	$\bar{\pi}_c$ obs.	$\bar{\pi}_c$ comp.	(o-c). 10^{-3}
12	2.511	2.521	− 10	2.500	2.496	+ 4	2.349	2.357	− 8	29.73	28.90	+.83
13	2.442	2.443	− 1	2.386	2.389	− 3	2.287	2.284	+ 3	31.10	30.50	+.60
14	2.557	2.557	0	2.483	2.487	− 4	2.360	2.365	− 5	26.95	28.62	− 1.67
15	2.383	2.397	− 14	2.349	2.342	+ 7	2.348	2.308	+ 40	33.86	32.90	+.96
16	2.342	2.338	+ 4	2.308	2.309	− 1	2.306	2.307	− 1	35.00	35.22	−.92
17	2.323	2.327	− 4	2.324	2.321	+ 3	2.305	2.308	− 3	39.45	40.10	−.65
18	2.480	2.456	+ 24	2.450	2.453	− 3				47.97	49.03	− 1.06

Thus while the values of $\overline{q'_c}$ and $\overline{q_A}$ verify in a very satisfactory way relations of a similar form to (26), i.e.

$$\overline{q_A}=2.459-0.152 \sin (T-1)\frac{2\pi}{36}-0.037 \cos\left(\frac{2\pi}{7}t+13^\circ\right) \tag{29}$$

$$\overline{q'_c}=2.421-0.156 \sin (T-1)\frac{2\pi}{36}-0.038 \cos\left(\frac{2\pi}{7}t-39^\circ\right) \tag{30}$$

the same does not exactly happen for the values of $\overline{q_c}$ taken after a selection of the observations for which we have:

$$\overline{q_c}=2.343-0.095 \sin (T-1)\frac{2\pi}{36}-0.060 \cos\left(\frac{2\pi}{7}t-39^\circ\right) \tag{31}$$

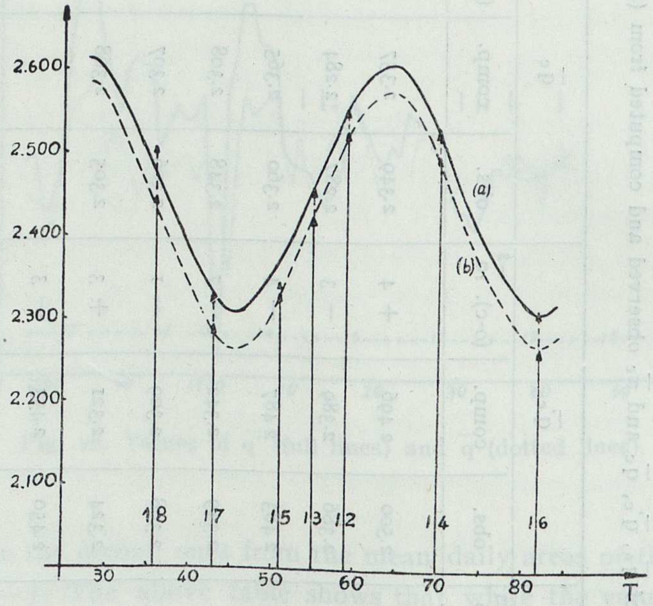


Fig. 27. The directed segments represent the values:

$$(\overline{q_A})_{ob}+0.038 \cos\left(\frac{2\pi}{7}+13^\circ\right), \quad (\overline{q'_c})_{ob}+0.037 \cos\left(\frac{2\pi}{7}-39^\circ\right)$$

The curbes (a) and (b) represent the quantities:

$$2.459-0.152 \sin (T-1)\frac{2\pi}{36}, \quad 2.421-156 \sin (T-1)\frac{2\pi}{36}$$

The selection of the observations diminishes by almost 40 per cent the amplitude of the principal periodic term which depends upon the time of rise T. It (the selection) also increases by an almost equal percentage the amplitude of the term which depends upon the time t.

The quantity $\bar{\pi}_c = \frac{1}{n} \sum \sqrt{2\, A_p}$, which expresses the mean linear dimension of the whole spots from cycle to cycle in $10^{-3} R_\odot$, satisfies the parabolic law:

(32) $$\Pi_c = 28 + 0.025\,(T-65)^2$$

Table XXII shows that the agreement between the values of $\bar{q}_A$, $\bar{q}'_c$, $\bar{q}_c$ and $\bar{\pi}_c$, given by the observations and the ones given by the relations (29) to (32) is very satisfactory. The mean square errors are:

$$\bar{q}_A : \pm 0.011, \qquad \bar{q}'_c : \pm 0.004, \qquad \bar{q}_c : \pm 0.017, \qquad \bar{\pi}_c : \pm 1.00$$

It is evident now that the remarks made for the ratio $q_p = q_A + 1$ are valid for the ratio $\bar{q}_A$ and $\bar{q}'_c$. That is, the maximum value of the ratio $\bar{q}_A$ and $\bar{q}'_c$ takes place in the cycles for which the time of rise is equal to the «special value» T=64 months, i.e. in the cycles with the lowest maximum and the smallest mean value of the areas of the spots per cycles (cycles 12 and 14). And the minimum value takes place in the cycles the time of rise of which is greater or smaller by 18 months than the «special value» (cycles 16 and 17).

cycles	T	$\bar{q}_A$	$\bar{q}'_c$
12	59=64−5	2.511	2.500
14	70=64+6	2.557	2.483
16	82=64+18	2.342	2.308
17	43=64−21	2.323	2.324

In fig. 27 the directed segments represent the values of

$$(\bar{q}_A)_{ob} + 0.038 \cos\left(\frac{2\pi}{7}\, t + 13^\circ\right), \qquad (\bar{q}'_c)_{ob} + 0.037 \cos\left(\frac{2\pi}{7}\, t - 39^\circ\right)$$

which the curves (a) and (b) represent the quantities respectively:

$$2.459 - 0.152 \sin (T-1)\,\frac{2\pi}{36}, \qquad 2.421 - 0.156 \sin (T-1)\,\frac{2\pi}{36}$$

From the observations up to now we conclude that the ratios $\bar{q}_A$ and $\bar{q}'_c$ not only suffer perceptible variations within the 11 year cycle which follows in one way or another the cycle of the spots, but also have perceptible variations from cycle to cycle. The latter depend, for their greater percentage, upon the time of rise T and for their lesser one upon one periodic term whose period is approximately 7 solar cycles. And if the above ratios possess the natural significance attributed to them [4], then the time of rise T must play a particular role not only for the determination of the maximum and the total areas of the spots for each cycle, but also for the structure of the spots and their magnetic fields.

SIGNIFICANT NOTE

The time t in the relations (1), (5) and (13), which give the maximum of the solar activity, can be expressed either in solar rotations or in years. For this purpose if we consider in each cycle the epoch of the maximum of the solar activity we shall have:

$$(1_a)\quad (A_m)_N = 1044 + 2.5 \quad (R_N - 65)^2 + 800 \sin (R_{max} - 540) \frac{2\pi}{1200}$$

$$(1_b)\quad (A_m)_S = 1277 + 5.9 \quad (R_S - 62)^2 + 600 \sin (R_{max} - 540) \frac{2\pi}{1200}$$

$$(5)\quad A_m = 2171 + 2.37 \,(T - 65)^2 + 320 \sin (T_{max} - 1894{,}0) \frac{2\pi}{89}$$

$$(13)\quad R_m = 81.8 + 0.095\,(\tau - 75)^2 + 23 \sin (T_{max} - 1797) \frac{2\pi}{89}$$

The following table gives the necessary elements for the calculations.

Number of Cycles	R_m		A_m		$(A_m)_N$		$(A_m)_S$	
	T_{max}	τ	T_{max}	T	R_{max}	R_N	R_{max}	R_S
7	1830.3	80						
8	1837.0	43						
9	1847.8	40						
10	1860.5	50						
11	1870.4	40						
12	1884.0	65	1883.5	59	389	49	398	61
13	1893.7	55	1893.7	55	546	65	533	53
14	1907.1	70	1907.1	70	696	50	714	74
15	1917.7	51	1917.7	51	854	53	848	54
16	1929.9	76	1929.9	82	1019	82	1003	70
17	1937.5	47	1937.5	43	1128	44	1150	78
18	1947.3	37	1947.3	36	1242	36	1251	43

From the above relations it is shown that the term X possesses a period of 1200 solar rotations or 89 years.

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ΠΕΡΙΛΗΨΙΣ

I. Εἰς τὸ πρῶτον μέρος τῆς παρούσης πραγματείας μελετῶμεν τὰ ἐμβαδὰ τῶν κηλίδων εἰς τὰ δύο ἡμισφαίρια τοῦ Ἡλίου κεχωρισμένως συναρτήσει τοῦ ἀριθμοῦ τῶν ἡλιακῶν περιστροφῶν μεταξὺ τοῦ ἐλαχίστου καὶ τοῦ μεγίστου τῆς ἡλιακῆς δραστηριότητος. Ἔχοντες ὑπ᾽ ὄψει τὰς παρατηρήσεις τοῦ Greenwich εὑρίσκομεν τὰς κάτωθι ἀναλυτικὰς σχέσεις:

$$(A_M)_N = 1083 + 2.58\,(R_N - 65)^2 + 700 \sin (t-1)\,\frac{2\pi}{8}$$

$$(A_M)_N = 1270 + 6.0\,(R_S - 61.5)^2 - 600 \sin (t-1)\,\frac{2\pi}{8}$$

$$\overline{A}_N = 201 + 0.70\,(R_N - 63)^2 - 85 \cos \pi t$$

$$\overline{A}_S = 241 + 0.87\,(R_S - 64)^2 + 85 \cos \left(\frac{2\pi}{4}\,t + \right)$$

$$t = 0,\ 1, \ldots 6$$

$$\text{κύκλοι: } 12,\ 13, \ldots 18$$

ὅπου $(A_M)_N$, $(A_M)_S$ παριστῶσιν ἀντιστοίχως τὸ μέγιστον παρατηρηθὲν ἐμβαδὸν τῶν κηλίδων εἰς τὸ Βόρειον καὶ Νότιον ἡμισφαίριον τοῦ Ἡλίου καὶ $\overline{A}_N$, $\overline{A}_S$ τὰς μέσας τιμὰς αὐτῶν κατὰ κύκλον.

Εἰς τὰς δύο πρώτας τῶν ἀνωτέρω σχέσεων ἀξιοσημείωτος εἶναι ἡ παρουσία ἑνὸς περιοδικοῦ ὅρου μὲ περίοδον 8 ἡλιακῶν κύκλων καὶ μὲ διαφορὰν φάσεως 180° μεταξὺ τῶν δύο ἡμισφαιρίων. Τὸν ὅρον τοῦτον καλοῦμεν ὅρον X.

II. Ἐν συνεχείᾳ μελετῶμεν τὰ ἐμβαδὰ τῶν κηλίδων, τῶν πυρήνων καὶ τῶν πυρσῶν ἐφ᾽ ὁλοκλήρου τοῦ ὁρατοῦ ἡλιακοῦ ἡμισφαιρίου συναρτήσει τοῦ χρόνου ἀνόδου T ἐκπεφρασμένου εἰς μῆνας καὶ εὑρίσκομεν τὰς κάτωθι σχέσεις:

$$A_M = 2172 + 2.40\,(T - 65)^2 + 300 \sin (t - 1\,\frac{2\pi}{8}$$

$$\overline{A} = 482 + 0.8\,(T - 67)^2$$

$$S(A) = 66500 + 74\,(T - 68)^2 + 8450 \sin \left(\frac{2\pi}{4}\,t - \frac{\pi}{15}\right)$$

$$U_M = 368 + 0.4\,(T_u - 64)^2 + 70 \sin \left(\frac{2\pi}{8}\,t - \frac{\pi}{8}\right)$$

$$\overline{U} = 76 + 0.19\,(T_u - 64)^2 - 13.2 \cos \pi t$$

$$S(u) = 10490 + 20.1\,(T_u - 64)^2 - 1450 \cos \pi t + 1000 \sin \frac{2\pi}{4}\,t$$

$$F_M = 3075 + 1.8\,(T_F - 65)^2$$
$$S(F) = 151140 + 95\,(T_F - 63)^2 - 22900 \sin\left(\frac{2\pi}{3}t + 65^\circ\right)$$

ὅπου A_M, $\overline{A}$ καὶ S(A) παριστῶσιν ἀντιστοίχως τὸ μέγιστον ἐμβαδόν, τὸ μέσον τούτου καὶ τὸ ὁλικὸν ἐμβαδὸν τῶν κηλίδων κατὰ κύκλον, U_M, $\overline{U}$ καὶ S(u) τὰς ἀντιστοίχους ποσότητας διὰ τοὺς πυρῆνας τῶν κηλίδων καὶ F_M, S(F) τὸ μέγιστον καὶ τὸ ὁλικὸν ἐμβαδὸν τῶν πυρσῶν κατὰ κύκλον.

III. Ἡ ἀνωτέρω ἔρευνα ἐπεκτείνεται καὶ εἰς τὴν περίπτωσιν ὅπου ἀντὶ τῶν ἐμβαδῶν τῶν κηλίδων θεωροῦμεν τοὺς σχετικοὺς ἀριθμοὺς Wolf καὶ μάλιστα οὐχὶ μόνον εἰς τοὺς 7 θεωρηθέντας κύκλους ἀλλ' ἔτι εἰς 12, ἀπὸ τοῦ 7ου μέχρι τοῦ 18ου. Εἰς τὴν περίπτωσιν ταύτην εὑρίσκομεν τὰς κάτωθι σχέσεις:

$$R_M = 84.1 + 0.092\,(\tau - 75)^2 + 25.2 \sin\left(\frac{2\pi}{8}t + \frac{5\pi}{8}\right)$$
$$\overline{R} = 31.7 + 0.044\,(\tau - 68)^2$$
$$S(R) = 4436 + 4.65\,(\tau - 68)^2 - 500 \sin\left(\frac{2\pi}{4}t + 70^\circ\right)$$
$$S_1(R) = 1114.6 + 27.8\,\tau$$
$$\pm 109.4 \pm 5.5$$
$$S_2(R) = 1520 + 3.51\,(\tau - 75)^2$$
$$t = 0, 1, \ldots 11$$
$$\text{κύκλοι:}\quad 7, 8, \ldots 18$$

ὅπου, R_M, $\overline{R}$ καὶ S(R) παριστῶσιν ἀντιστοίχως τὸν μέγιστον ἀριθμὸν Wolf, τὴν μέσην τιμὴν καὶ τὸ ὁλικὸν ἄθροισμα αὐτῶν κατὰ κύκλον καὶ $S_1(R)$, $S_2(R)$ παριστῶσι τὸ ἀντίστοιχον ἄθροισμα τῶν ἀριθμῶν Wolf κατὰ τὸ χρονικὸν διάστημα ἐλάχιστον - μέγιστον καὶ μέγιστον - ἐλάχιστον τῆς ἡλιακῆς δραστηριότητος.

Αἱ ἀνωτέρω σχέσεις ἰσχύουν καὶ ὅταν ἀντὶ τῶν «παρατηρουμένων» ἀριθμῶν Wolf θεωροῦμεν τὰς λειανθείσας τιμὰς αὐτῶν μὲ τὴν διαφορὰν ὅτι ὁ ὅρος X δὲν ἐμφανίζεται εἰς τὸν μέγιστον ἀριθμόν.

Ἐκ τῶν ἀνωτέρω ἀναλυτικῶν σχέσεων συνάγεται ὅτι:

α) Τὸ μέγιστον, τὸ μέσον καὶ τὸ ὁλικὸν ἐμβαδὸν τῶν κηλίδων, τῶν πυρήνων καὶ τῶν πυρσῶν, καθὼς καὶ ὁ μέγιστος, ὁ μέσος καὶ ὁ ὁλικὸς ἀριθμὸς Wolf κατὰ κύκλον δύνανται νὰ ἐκφρασθῶσιν ἀναλυτικῶς συναρτήσει τοῦ «χρόνου ἀνόδου». Τὸ ἀλγεβρικὸν μέρος τῶν σχέσεων τούτων παριστᾷ π α ρ α β ο λ ὰ ς τῶν ὁποίων αἱ κορυφαὶ ἔχουν τετμημένας κυμαινομένας μεταξὺ στενῶν ὁρίων. Τοῦτο δεικνύει ὅτι ὑπάρχει μία ἰ δ ι ά ζ ο υ σ α τιμὴ τοῦ χρόνου ἀνόδου, $T_0 \simeq 66$ μῆνες, τοιαύτη, ὥστε οἱ κύκλοι τῶν ὁποίων οἱ χρόνοι ἀνόδου εἶναι γειτονικοὶ τῆς τιμῆς ταύτης κέκτηνται τὸ χαμηλότερον μέγιστον, τὴν μικροτέραν μέσην τιμὴν καὶ τὸ μικρότερον ὁλικὸν ἐμβαδὸν κατὰ κύκλον. Οἱ κύκλοι δὲ τῶν ὁποίων οἱ χρόνοι ἀνόδου εἶναι αἰσθητῶς μικρότεροι ἢ μεγαλύτεροι τῆς τιμῆς ταύτης κέκτηνται τὸ ὑψηλότερον μέγιστον, τὴν μεγαλυτέραν μέσην τιμὴν καὶ τὸ μεγαλύτερον ὁλικὸν ἐμβαδὸν κατὰ κύκλον.

β) Μεταξὺ τῶν περιοδικῶν ὅρων οἵτινες ἐμφανίζονται εἰς τὰς ἀνωτέρω ἀναλυτικὰς σχέσεις ἰδιαιτέραν σημασίαν κέκτηται ὁ ὅρος X, τοῦ ὁποίου ἡ περίοδος εἶναι 8 ἡλιακοὶ κύκλοι καὶ ὅστις ἐμφανίζεται μόνον κατὰ τὴν ἐποχὴν τοῦ μεγίστου τῆς ἡλιακῆς δραστηριότητος. Ὁ ὅρος οὗτος, ὅταν ἐξετάζωμεν τὰ ἐμβαδὰ τῶν κηλίδων κεχωρισμένως εἰς τὸ βόρειον καὶ νότιον ἡμισφαίριον τοῦ Ἡλίου κέκτηται σημαντικὸν εὖρος καὶ παρουσιάζει διαφορὰν φάσεως 180° μεταξὺ τῶν δύο ἡμισφαιρίων.

Τέλος, εἰς τὸ δεύτερον μέρος τῆς παρούσης πραγματείας ἐξετάζομεν συγκριτικῶς τὰ ἐμβαδὰ τῶν κηλίδων, τῶν πυρήνων καὶ τῶν πυρσῶν καὶ καταλήγομεν εἰς ἀναλυτικὰς σχέσεις ἐκ τῶν ὁποίων προκύπτει ὅτι οἱ λόγοι τῶν ἐμβαδῶν τῶν ἀνωτέρω σχηματισμῶν κατὰ κύκλον ἐκφράζονται συναρτήσει τοῦ χρόνου ἀνόδου παρουσιάζοντες συνάμα περιοδικὰς μεταβολὰς ἀπὸ κύκλου εἰς κύκλον.