

του εἰς τὰ ἀνώτερα ἰδανικά καὶ καθόλου εἰπεῖν διὰ τὴν εὐγένειαν τῆς ψυχῆς του. Διὰ τοῦτο καὶ ἡ μνήμη του θὰ παραμείνῃ ἐπὶ μακρὸν ἐν τῇ ᾿Ακαδημίᾳ ἔντιμος καὶ προσφιλῆς.

ΑΝΑΚΟΙΝΩΣΕΙΣ ΜΗ ΜΕΛΩΝ

ΑΝΑΛΥΤΙΚΗ ΓΕΩΜΕΤΡΙΑ.— **The principle of geometrical equivalence and some of its consequences to the theory of curves, by**

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*The problem under study.*

Let the analytic relations  $f_1(a_1, b_1)=0, f_2(a_2, b_2)=0, \dots, f_n(a_n, b_n)=0$  be given and suppose that they represent one and the same plane curve C in the distinct coordinate systems  $(a_1, b_1), (a_2, b_2), \dots, (a_n, b_n)$  respectively. If the formulae of transformation among these coordinate systems are known then it is possible to go *analytically* from one of the above relations to the other. Thus these relations may be termed as «analytically convertible or equivalent».

As an example, the equations of a circle with center at the origin and radius a are  $x^2 + y^2 = a^2, x'^2 + y'^2 + 2x'y'\cos\omega = a^2$  and  $r=a$  in the rectangular, the oblique and the polar coordinate systems respectively. All these three equations represent the same circle and one can go from one of the three equations to the other by applying the well-known formulae of transformation among the three coordinate systems.

The corresponding dual problem to the previous one, which has never been examined, may be stated as follows: Let the analytic relation  $f(a, b)=0$  be given. If the variables a and b are substituted by  $a_1$  and  $b_1, a_2$  and  $b_2, \dots, a_n$  and  $b_n$  respectively in the given relation then we get the relations  $f(a_1, b_1)=0, f(a_2, b_2)=0, \dots, f(a_n, b_n)=0$ . Let  $(a_1, b_1), (a_2, b_2), \dots, (a_n, b_n)$  be distinct plane coordinate systems. Then the above n equations represent curves  $K_1, K_2, \dots, K_n$  which in reality are represented by the same basic analytic relation  $f(a, b)=0$ . Now the problem is the possibility to go *geometrically* from one curve to the other. This means that given any point of one of the above curves one can find its corresponding one

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on any other curve by the use of euclidean geometrical constructions as it shown in the next section. Curves for which such a possibility exists may be characterized as «geometrically equivalent or convertible» and the resulting principle of geometrical equivalence leads to some theorems on the total set of plane curves.

*Geometrically equivalent curves.*

Let the polar and the cathetic coordinate systems be given. In the cathetic system<sup>1</sup> the coordinates of a point P (fig. 1) are  $OA=g$  and the polar angle  $\theta$ . The equation  $f(a,b)=0$  represents two different curves  $f(r,\theta)=0$  and  $f(g,\theta)=0$  in the two coordinate systems in use. In order that these curves are geometrically convertible or equivalent it is necessary that to each point Q of the plane with coordinates  $(r=OQ,\theta)$  there corresponds another point P with coordinates  $(g=OA,\theta)$  such that  $OQ=OA,\theta$  being the same to both systems. This is possible here because if P is given one can construct the semicircle OPA passing through the origin and with center on the axis. Now if another circle is constructed with center at the origin and radius OA the point Q can be found at the intersection of the latter circle with the extension of OP. Evidently the polar OQ is equal to the cathetic OA. Conversely one can go geometrically from the point Q to the point P. Therefore the polar curve  $f(r,\theta)=0$  described by Q and the cathetic curve  $f(g,\theta)=0$  by P, both represented by the same analytic relation, represent two distinct curves which are geometrically convertible. As a general conclusion, while the analytical equivalence rests upon the possibility of finding formulae of transformation among the coordinate systems in use, the geometrical equivalence depends upon the possibility of finding

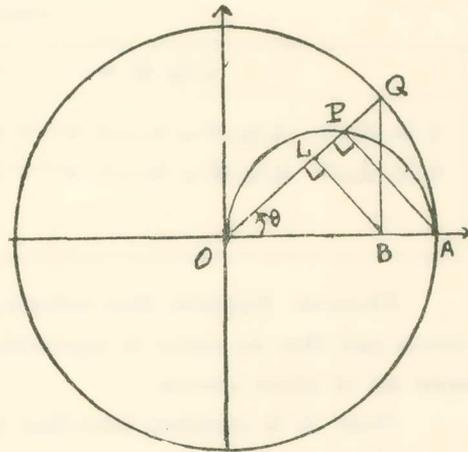


Fig. 1.

<sup>1</sup> C. B. GLAVAS, «Plane coordinate systems in mathematics study», Doctoral Dissertation, New York, Teachers College, Columbia University, 1956, Ch. III.

a geometrical way to go from one coordinate system to the other so that the corresponding coordinates are equal by pairs.

If the point  $L(g=OB, \theta)$  is defined by the cathetic system, the point  $P(r=OP, \theta)$  by the polar one, and the point  $Q(x=OB, \theta)$  by the system  $(x, \theta)$  (fig. 1) then these three systems are geometrically equivalent because  $OB=OP$ . Hence the curves described by each of the above three points are geometrically equivalent curves. Diagram I shows triples  $(A_1, B_1, C_1), (A_2, B_2, C_2), \dots$ , of geometrically equivalent curves. The following theorem is true in relation to these curves.

DIAGRAM I.

	$L(g, \theta)$	$P(r, \theta)$	$Q(x, \theta)$
$f_1(a, b)=0$	$f_1(g, \theta) = A_1=0$	$f_1(r, \theta) = B_1=0$	$f_1(x, \theta) = C_1=0$
$f_2(a, b)=0$	$f_2(g, \theta) = A_2=0$	$f_2(r, \theta) = B_2=0$	$f_2(x, \theta) = C_2=0$
$\underline{\quad}$	$\underline{\quad}$	$\underline{\quad}$	$\underline{\quad}$
$\underline{\quad}$	$\underline{\quad}$	$\underline{\quad}$	$\underline{\quad}$

*Theorem:* Suppose that column  $L(g, \theta)$  contains the total set of plane curves and that no curve is repeated, then the other columns contain the same set of plane curves.

*Proof:* It is assumed here that two curves are different if their equations differ even if the curves belong to the same family. Evidently column  $P(r, \theta)$  of the polar system contains plane curves. For example the curve  $B_1$  geometrically equivalent to  $A_1$  is different from  $A_1$ . For the equation of  $B_1$  is  $f_1(r, \theta)=0$ , while that of  $A_1$  is  $f_1(g, \theta)=0$ . If the formula of transformation  $r=g \cos \theta$  is applied to the latter equation it becomes clear that the two curves  $A_1$  and  $B_1$  are different.

Now since column  $L(g, \theta)$  contains by hypothesis the total set of plane curves and since  $B_1$  is a plane curve then  $B_1$  must be the same with a curve say  $A_k (k \neq 1)$  of the first column  $L$ . The equation  $f_k(g, \theta)=0$  of  $A_k$  can be found if in the equation of  $B_1$  one makes the substitution  $r=g \cos \theta$ . It is impossible that  $B_1$  is the same with another curve  $A_l (l \neq 1)$  of column  $L$ . For  $A_l$  should then be identical to  $A_k$ , which is contrary to the hypothesis of the theorem. Similarly it is shown that one and only one curve of the second column corresponds to each curve of the first one. Hence there exists a one - to - one correspondence between the two columns and conse-

quently column  $P(r, \theta)$  contains the total set of plane curves. Similar conclusion holds for column  $Q(x, \theta)$ .

A corollary of this theorem is that if one may be able to write the total set of plane curves by the use of one coordinate system then one can construct the same curves expressed in another coordinate system geometrically equivalent to the first one.

*Implications of geometrical equivalence.*

1. The usual method for the simplification of the equation of a curve and study of its properties is to transform its equation by the use of another coordinate system. Now if the equation is  $f(g, \theta) = 0$ , one can write this as  $f(r, \theta) = 0$  if the systems  $(g, \theta)$  and  $(r, \theta)$  are geometrically equivalent as the case is here. For example the equation  $g = a\theta$  can be written as  $r = a\theta$ . These curves are geometrically equivalent. But the latter is known (spiral) and one can construct the first one thanks to the principle of geometrical equivalence.

2. Given the geometrically equivalent curves  $K_1, K_2, \dots, K_n$  then it is known that one can go geometrically from one to the other. But all these curves correspond to a common analytical relation  $f(a, b) = 0$ , where  $a$  and  $b$  refer each time to one of the coordinate systems  $(a_1, b_1), (a_2, b_2), \dots, (a_n, b_n)$ . Suppose that the tangent line to a point of one of the given curves, say  $f(a_i, b_i) = 0$ , has been found and that its equation is  $\varphi(a_i, b_i) = 0$ . For another curve  $f(a_m, b_m) = 0$ , whose equation is analytically the same to the previous  $f(a_i, b_i) = 0$ , the tangent line is evidently  $\varphi(a_m, b_m)$ . Note that the two tangent lines as well as their initial curves have common equations but they represent different lines which correspond to the systems  $(a_i, b_i)$  and  $(a_m, b_m)$ . Therefore if one finds the equation of the tangent line to a point of one of  $K_1, K_2, \dots, K_n$  then one can immediately write the equation of the tangent to any other curve. Moreover if such a tangent to one curve is constructed then one can construct the tangent to any other curve.

3 Let the equations  $f(g, \theta, a) = 0$  and  $f(r, \theta, a) = 0$ , where  $a$  is a parameter, of two geometrically equivalent families of curves be given. To find the envelope of the first family is enough to eliminate  $a$  between  $f(g, \theta, a) = 0$  and  $f_a(g, \theta, a) = 0$ . Let  $\varphi(g, \theta) = 0$  be the equation of that envelope. It is clear that the equation of the envelope of the second family is  $\varphi(r, \theta) = 0$ . To avoid repetition one can state analogous conclusions to those of the previous paragraph.

4. Geometrically equivalent curves represented by the same equation have maxima and minima corresponding to common values of the variables.

5. From the above discussion one may conclude that geometrically equivalent curves have those properties in common which are the by-product of the same analytical operations on their common equation.

*Partition theorem of plane curves.*

The geometrical equivalence between two plane coordinate systems is a binary relation of special character. It has been defined at the beginning that  $P(g, \theta) \sim Q(r, \theta)$  if  $g=r$  and given  $P$  one can go to  $Q$  (fig. 1) by means of geometrical constructions. Now this definition leads to an equivalence relation. Really,  $P \sim P$ , because obviously one can go from  $P$  to the same point  $P$ . Therefore the reflexive property exists.

Also if  $P \sim Q$ , then  $Q \sim P$ , for if  $g=r$  and one can go from  $P$  to  $Q$  then  $r=g$  and one can go from  $Q$  to  $P$ . This establishes the symmetric property. And finally if  $L(g, \theta) \sim P(r, \theta)$  and  $P(r, \theta) \sim Q(x, \theta)$  then it is easily seen that  $g=x$  and one can go from  $L$  to  $Q$ . And this shows that transitivity holds.

Now geometrical equivalence between the plane curves  $f(r, \theta)=0$  and  $f(g, \theta)=0$  has been defined in terms of geometrical equivalence of the points  $P(r, \theta)$  and  $Q(g, \theta)$  describing the two curves respectively. Since the latter equivalence constitutes an equivalence relation then this forces a partition of the total set of plane curves into mutually exclusive subsets of equivalent curves according to a well-known theorem of the equivalence relation<sup>1</sup>. With this in mind one can prove the following significant theorem.

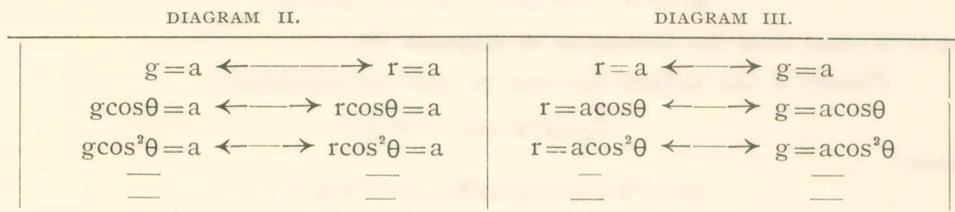
*Theorem:* The total set of plane curves may theoretically be partitioned into subsets of equivalent curves. Each such subset constitutes a group which is isomorphic to the additive integers.

*Proof:* For the proof of this theorem the two equivalent systems  $(g, \theta)$  and  $(r, \theta)$  are used. The curve  $g=a$  is taken as initial curve. This curve represents a circle passing through the origin with center on the axis. But one can start from any other curve.

That the set of plane curves can be partitioned into subsets of equivalent curves is known in advance because the systems in use are equivalent and the relation between  $P(g, \theta)$  and  $Q(r, \theta)$  (fig. 1) is an equivalence relation. It remains to show the way of effectuating that partition. Starting

<sup>1</sup> E. R. LORCH, *Theory of Functions*, New York, Columbia University, 1951, p. 1.

from the curve  $g=a$  (Diagram II) its geometrically equivalent one is found if  $g$  is substituted by  $r$  and it is  $r=a$ . The latter is identical to  $g \cos \theta = a$  due to the transformation  $r = g \cos \theta$ . Now the equivalent curve to the curve  $g \cos \theta = a$  is  $r \cos \theta = a$  which is identical to the curve  $g \cos^2 \theta = a$  and so on and so forth.



If the curve  $r=a$  is taken as initial curve then diagram III can similarly be produced.

Now since the pair of initial curves  $g=a$  and  $r=a$  is a pair of geometrically equivalent curves it is clear that diagrams II and III lead to diagram IV where for reasons of simplicity the equations are expressed in the  $(g, \theta)$  system. All curves of the latter diagram are equivalent curves. And if any other curve is taken as initial curve exception made of the curves of diagram IV one can form similar subsets of equivalent curves.

It is shown now that such a subset (diagram IV) constitutes a group under an operation  $o$  defined as follows:

$$(1) \quad (g \cos^k \theta = a) \circ (g \cos^l \theta = a) = (g \cos^{k+l} \theta = a)$$

Here  $k$  and  $l$  are supposed to be integers.

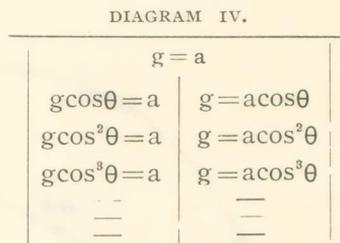
In order that the subset in question constitutes a group it must:

1. The combination under the defined operation of any two elements of the subset produces an element of the same subset. This is true here because in (1)  $k+l$  is an integer too.

2. The defined operation be associative. Really:

$$\begin{aligned} & (g \cos^k \theta = a) \circ \left[ (g \cos^l \theta = a) \circ (g \cos^m \theta = a) \right] = \\ & = \left[ (g \cos^k \theta = a) \circ (g \cos^l \theta = a) \right] \circ (g \cos^m \theta = a) \end{aligned}$$

This relation is true since  $k+(l+m) = (k+l)+m$  holds for the integers.



3. Exist an identity element. Such an element is the initial curve  $(g=a)=(g\cos^0\theta=a)$ :

$$(g\cos^k\theta=a) \circ (g=a) = (g\cos^k\theta=a).$$

And 4. Exist an inverse element to every given element of the subset. Really:

$$(g\cos^k\theta=a) \circ (g\cos^{-k}\theta=a) = (g=a),$$

as it is clear from the formation of diagram IV.

Finally if one defines the one-to-one correspondence

$$(g\cos^k\theta=a) \longrightarrow k,$$

then:

$$(g\cos^k\theta=a) \circ (g\cos^l\theta=a) \longrightarrow k + l.$$

This means that the group in question is isomorphic to the additive integers.

*Geometrical interpretation of partition theorem.*

The theorem of partition should find its interpretation on the euclidean plane. First the curve  $g=a$  is constructed (fig. 2), which is the circle

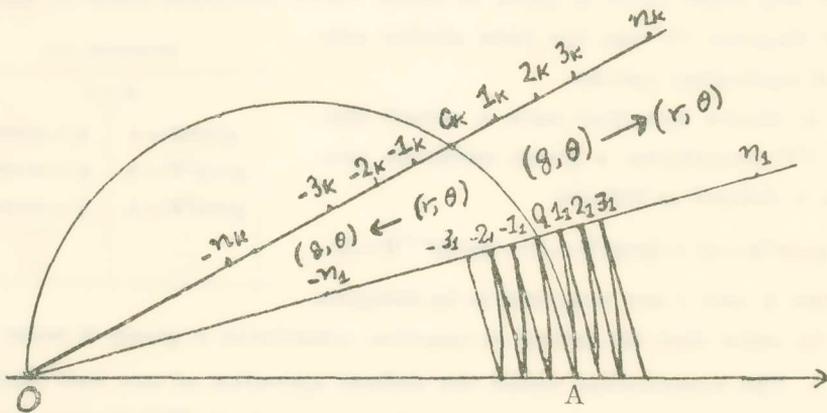


Fig. 2.

$O0_k O_1 A$ . Take the point  $0_1$  of this circle. The geometrically equivalent curve of the latter is  $r=a$  or  $g\cos\theta=a$ . To find the corresponding point to  $0_1$  on the latter curve is enough to draw a perpendicular line on  $O0_1$ , at  $0_1$ , passing through  $A$ , and construct a circle with center at  $O$  and radius  $OA$ . This circle meets the extension of  $O0_1$  at  $1_1$  which corresponds to  $0_1$ . Continuing the same process «perpendicular - circle» one can find the

point  $2_1$ , on  $g\cos^2\theta=a$ , corresponding to  $1_1$ , and so on and so forth. In this process one moves from the system  $(g, \theta)$  to the  $(r, \theta)$  one.

If one follows the inverse process, i.e. from  $1_1$  to  $0_1$ , then  $1_1$  lies on  $r=a$  whose geometrically equivalent curve is  $g=a$ . Therefore to go from  $1_1$  to  $0_1$  requires to follow the process «circle - perpendicular». But  $g=a$  is the same with  $r=a\cos\theta$  whose geometrically equivalent curve is  $g=a\cos\theta$ . The corresponding point of the latter curve to the point  $0_1$  can be found by drawing a circle and then a perpendicular line. The corresponding point is denoted by  $-1_1$  and so on and so forth.

If another point  $0_k$  is taken on  $g=a$  then it is possible by following the above method to find the corresponding points of the equivalent curves of the subset in question.

From the above discussion it becomes clear that on the right side of the initial curve  $g=a$  lie the points of the curves of the first column of diagram IV while on the left side those of the right column.

To complete the geometrical interpretation of the partition theorem the geometrical meaning of the symbol  $\circ$  must be defined in the relation  $(g\cos^k\theta=a)\circ(g\cos^l\theta=a)=(g\cos^{k+l}\theta=a)=(g\cos^{l+k}\theta=a)$ .

By definition the above combination means that if the curve  $(g\cos^k\theta=a)$  is considered as «operating factor» then one should start from the points  $l_1, l_2, \dots$  of the second curve  $(g\cos^l\theta=a)$ . From these points one can find the corresponding ones of the curve  $(g\cos^{l+k}\theta=a)$  through  $k$  successive constructions to the right (of the type perpendicular - circle) or to the left (of the type circle - perpendicular) according as  $k$  is a positive or negative integer respectively. The same definition holds, leading exactly to the same result, if the curve  $(g\cos^l\theta=a)$  is considered as the operating factor.

After the above definition it is easy to interpret geometrically the subsets - groups in question. The combination particularly of two inverse curves  $(g\cos^k\theta=a)$  and  $(g\cos^{-k}\theta=a)$ , which produces the identity element  $(g=a)$ , is characteristically remarkable. Thus, if for example  $k > 0$ , and if the curve  $(g\cos^k\theta=a)$  is taken as operating factor then from the points  $-k_1, -k_2, \dots$  of the inverse curve  $(g\cos^{-k}\theta=a)$  one should make  $k$  successive constructions to the right (of the type perpendicular-circle) when one finds the points  $0_1, 0_2, 0_3, \dots$  of the initial curve which is the identity element.

CONCLUSION. The study of geometry by means only of analytical transformations of the equations of curves had been one sided. After the

establishment of the dual principle of geometrical equivalence the above study becomes complete. Space did not allow to present some applications and especially to make a combination of both analytical and geometrical transformations in a new method for curve tracing and for the solution of differential equations.

#### ΠΕΡΙΛΗΨΙΣ

Ὡς γνωστὸν εἰς τὴν Ἀναλυτικὴν Γεωμετρίαν μία γραμμὴ παρίσταται ἀπὸ περισσοτέρας τῆς μιᾶς ἐξισώσεις, ὅταν ἀναφέρηται εἰς πλείονα τοῦ ἑνὸς συστήματα συντεταγμένων. Ἐὰν ὑπάρχουν δὲ τύποι μετασχηματισμοῦ μεταξύ τῶν συστημάτων τούτων, τότε δύναται τις νὰ μεταβῆ βᾶσει τῶν τύπων τούτων ἀπὸ τῆς μιᾶς ἐξισώσεως εἰς τὴν ἄλλην. Ἐν τοιαύτῃ περιπτώσει αἱ ἐξισώσεις τῶν ἐν λόγῳ γραμμῶν θὰ χαρακτηρίζωνται ὡς ἀναλυτικῶς ἰσοδύναμοι ἢ μετατρέψιμοι.

Ἐνταῦθα ἐξετάζεται τὸ δραστικῶς ἀντίστοιχον τοῦ προηγουμένου θέματος. Μία καὶ ἡ αὐτὴ ἀναλυτικὴ σχέσις, ἀναφερομένη εἰς διάφορα συστήματα συντεταγμένων, παριστᾷ διαφόρους γραμμάς. Αἱ γραμμαὶ αὗται θὰ εἶναι γεωμετρικῶς ἰσοδύναμοι ἢ μετατρέψιμοι, ὅταν ὑπάρχῃ δυνατότης μεταβάσεως γεωμετρικῶς ἀπὸ τοῦ ἑνὸς συστήματος συντεταγμένων εἰς τὸ ἄλλο. Τοῦτο ἀποδεικνύεται ὅτι εἶναι δυνατόν εἰς ὠρισμένας εἰδικὰς περιπτώσεις, ὅπου αἱ ἀντίστοιχοι συντεταγμένοι, αἱ παριστώσαι ὁμοειδῆ μεγέθη, εἶναι ἴσαι κατὰ ζεύγη.

Μερικαὶ συνέπειαι τῆς ἀρχῆς ταύτης τῆς γεωμετρικῆς ἰσοδυναμίας εἶναι, ὅτι δύναται τις νὰ σπουδάσῃ μίαν γραμμὴν ὄχι μόνον διὰ τοῦ μετασχηματισμοῦ τῆς ἐξισώσεώς τῆς, ἀλλὰ διὰ τῆς εὑρέσεως τῆς γεωμετρικῶς ἰσοδύναμου γραμμῆς. Ἡ τελευταία αὕτη δυνατόν νὰ εἶναι εἴτε γνωστὴ εἴτε ἀπλουστέρα εἰς τὴν κατασκευὴν. Ἐν τῇ περιπτώσει ταύτῃ δυνάμεθα νὰ ἐπανέλθωμεν εἰς τὴν ἀρχικὴν γραμμὴν διὰ τῆς γεωμετρικῆς ὁδοῦ.

Αἱ γεωμετρικῶς ἰσοδύναμοι καμπύλαι, ἔχουσαι κοινὴν ἐξίσωσιν, ἔχουν καὶ κοινὰς τινὰς ιδιότητας. Οὕτω τὰ μέγιστα ἢ τὰ ἐλάχιστα αὐτῶν ἀντιστοιχοῦν εἰς κοινὰς τιμὰς τῶν μεταβλητῶν, αἱ δὲ ἐξισώσεις τῶν ἐφαπτομένων καὶ περιβαλλουσῶν εἶναι ἀναλυτικῶς αἱ αὐταί. Δεικνύεται ἐπὶ πλέον, ὅτι ἡ γεωμετρικὴ ἰσοδυναμία εἶναι σχέσις ἰσοδυναμίας (Equivalence Relation). Τὸ τοιοῦτον δὲ προκαλεῖ κατανομὴν τοῦ συνόλου τῶν ἐπιπέδων γραμμῶν εἰς ὑποσύνολα καὶ συμφώνως πρὸς ἀποδεικνυόμενον θεώρημα ἕκαστον τῶν ὑποσυνόλων τούτων συνιστᾷ ἀβελιανὴν ὁμάδα συμφώνως πρὸς ὀριζομένην πράξιν. Τέλος τὸ θεώρημα τοῦτο κατανομῆς ἐρμηνεύεται γεωμετρικῶς ἐπὶ τοῦ ἐπιπέδου.

Ἐν συμπεράσματι ἡ δυνατότης μόνον ἀναλυτικῶν μετασχηματισμῶν τῶν ἐξισώσεων ἐπιπέδων γραμμῶν καθίστα μονόπλευρον τὴν σπουδὴν των. Μετὰ τὴν διατύπωσιν τῆς ἀρχῆς τῆς γεωμετρικῆς ἰσοδυναμίας εἶναι δυνατὴ ἡ ἐφαρμογὴ καὶ γεωμετρικῶν μετασχηματισμῶν. Οὕτως ἡ σπουδὴ τῶν ἐπιπέδων γραμμῶν διὰ μετασχηματισμῶν καθίσταται πλήρης. Ὁ διατιθέμενος ἄνωθεν χῶρος δὲν ἐπιτρέπει τὴν παρουσίαν σχετικῶν τινῶν πρὸς τὰ ἀνωτέρω ἐφαρμογῶν. Ἰδιαιτέρως ἐνδιαφέρουσα εἶναι

ἡ περίπτωση συνδυασμοῦ ἀναλυτικῶν καὶ γεωμετρικῶν μετασχηματισμῶν, ποὺ συνιστᾷ κατ' οὐσίαν μίαν νέαν μέθοδον διὰ τὴν κατασκευὴν γραμμῶν καὶ τὴν ἐπίλυσιν διαφορικῶν ἐξισώσεων.

ΝΕΟΕΛΛΗΝΙΚΗ ΦΙΛΟΛΟΓΙΑ.—Ἡ λογοτεχνικὴ ἐπίδρασις τῆς Ἀγίας Γραφῆς ἐπὶ τοῦ ἐθνικοῦ ποιητοῦ Δ. Σολωμοῦ, ὑπὸ Ν. Θ. Μπουγάτσου.  
Ἀνεκοινώθη ὑπὸ κ. Παναγ. Μπρατσιώτου.

Εἶναι γνωστὸν ὅτι ὁ ἐθνικός μας ποιητὴς Διονύσιος Σολωμὸς ἐγνώριζε καλῶς τὴν Ἀγίαν Γραφήν (Παλαιὰν καὶ Καινὴν Διαθήκην), τῆς ὁποίας ἔκαμνε δαψιλῆ χρῆσιν<sup>1</sup>. Ὁ Σολωμὸς ἐκ παραλλήλου πρὸς τὸ βιβλικὸν θέμα πολλῶν ποιημάτων του μᾶς δίδει ὁ ἴδιος εἰς τὰ ἔργα του 24 παραπομπὰς εἰς τὴν Βίβλον, ἡμεῖς ὅμως ἐσημειώσαμεν περὶ τὰ 250 χωρία τῆς Βίβλου, τὰ ὁποῖα χρησιμοποιεῖ οὗτος ἐμφανῶς ἢ ἐμμέσως εἰς τὸ ἔργον του. Ἐπίσης ὁ ἴδιος ὁ Σολωμὸς ρητῶς ὁμολογεῖ ἑαυτὸν γνώστην καὶ θαυμαστὴν τῆς διδασκαλίας καὶ λογοτεχνικῆς ἀξίας τῆς Ἀγίας Γραφῆς εἰς τὸν «Ὑμνον τῆς Ἐλευθερίας», τὸν «Διάλογον» καὶ τὴν «Γυναῖκα τῆς Ζάκυνθος», καὶ ἐμμέσως ὁμολογεῖ ὅτι ἡ ποιητικὴ του τέχνη δὲν εἶναι μακρὰν τῆς βιβλικῆς<sup>2</sup>.

Ἐκ τῆς προσεκτικῆς συγκρίσεως τῶν κειμένων τῆς Ἀγίας Γραφῆς πρὸς τὸ ἔργον τοῦ Σολωμοῦ καταφαίνεται ἡ μεγάλη ἐπίδρασις τῆς Γραφῆς ἐπὶ τοῦ Σολωμοῦ εἰς τὴν φρασιολογίαν καὶ εἰς τὴν ἰδεολογίαν, καθὼς καὶ οἱ κ. Μπαλάνος, Βέης, Τωμαδάκης, Μπρατσιώτης καὶ Φιλιππίδης παρετήρησαν<sup>3</sup>, ἀλλὰ καὶ εἰς τὴν ποιητικὴν του τέχνην. Εἰς τὴν ἀνακοίνωσιν ταύτην παρουσιάζω ἐν γενικαῖς γραμμαῖς τὴν λογοτεχνικὴν ἐπίδρασιν τῆς Ἀγίας Γραφῆς ἐπὶ τοῦ Διονυσίου Σολωμοῦ, (ὡς ἐμφαίνεται ἐκ τῶν ἔργων του), ἐπίδρασιν ἢ ὁποῖα διακρίνεται εἰς τὸ ἐν γένει ὕφος τοῦ Σολωμοῦ, εἰς τὴν χρῆσιν τῆς ἀλληγορίας καὶ ποιητικῶν τινῶν εἰκόνων, ἰδίᾳ ὅμως εἰς τὴν χρῆσιν τῶν σχημάτων τοῦ λόγου τῆς παλλιλογίας καὶ δὴ τοῦ παραλληλισμοῦ.

<sup>1</sup> Κ. ΣΤΡΑΤΟΥΛΗ, Λόγος ἐπικήδειος εἰς τὸν ποιητὴν τῆς Ν. Ἑλλάδος τὸν Κόμητα Δ. Σολωμὸν (Ζάκυνθος, 1857), σ. 7. Δ. Γ. ΚΑΝΑΛΕ, Σολωμὸς καὶ Δώρα Ἰστρία (π. Ἐθν. Βιβλιοθήκη, 4 1868-69, σ. 320). ΙΑΚΩΒΟΥ ΠΟΛΥΛΑ, Ὑποσημειώσεις εἰς τοὺς «Στοχασμοὺς τοῦ Ποιητοῦ» τῶν «Ἐλευθέρων Πολιορκημένων» τοῦ Σολωμοῦ.

<sup>2</sup> Ἴδε ἰδίως «Ὑμνον Ἐλευθερίας» (στρ. 118 κ.ἐξ.), «Διάλογον» («Ἀγιώτατα λόγια...»), «τὴν Γυναῖκα τῆς Ζάκυνθος» (5, 10).

<sup>3</sup> Δ. ΜΠΑΛΑΝΟΣ, Ν. Ἐστία, 15, 1934, σ. 524-42. Ν. ΒΕΗΣ, Ν. Ἐστία, 27, 1940, σ. 342. Ν. ΤΩΜΑΔΑΚΗΣ, Φιλολογικά, Ἀθήναι, 1935, σ. 13. Ὁ Σολωμὸς καὶ οἱ ἀρχαῖοι (ἈΦ., 1943), σ. 84-94. Ὁ Σολωμὸς καὶ ἡ Ἀγ. Γραφή, εἰς π. «Ὁ Αἰώνας μας», 1949, σ. 173-4. Π. ΜΠΡΑΤΣΙΩΤΗΣ, Ἡ Ἀποκάλυψις τοῦ Ἰωάννου (ἈΦ., 1950), σ. 58. Δ. ΦΙΛΙΠΠΙΔΗΣ, Ἐκκλησία, 24, 1947, σ. 105.