ΜΑΘΗΜΑΤΙΚΑ.— **On a convenient category of topological algebras, I.** *by Anastasios Mallios**. Ἀνεκοινώθη ὑπὸ τοῦ Ἀκαδημαϊκοῦ κ. Φίλωνος Βασιλείου.

1. Introduction. By a *topological algebra* we mean in the sequel a complex linear associative algebra equipped with a Hausdorff topology making it a topological vector space with a separately continuous multiplication.

The *spectrum* (or *Gel'fand space*) of such an algebra is, by definition, the set of all non-zero, continuous, multiplicative, linear forms of the algebra, endowed with the topology of pointwise convergence. Hence, it is a subset of the weak topological dual of the respective topological vector space underlying the topological algebra considered.

Now, it is quite obvious that the latter space may be, in general, the empty set. However, there do exist important examples, even of non locally convex topological algebras, for which the preceding topological space is non vacuous: e.g., commutative, complete, locally bounded algebras with identity (cf. [**63**; p. 13, Proposition 4.2]), or colimits in the category of topological vector spaces of commutative Banach algebras with identity (cf. also [**32**; p. 108], and [**31**; p. 214, Proposition 2.1]).

Thus, *it will be part of our hypothesis for a given topological algebra that its spectrum is not the empty set,* so that *the latter space will be,* by its definition, *a Hausdorff completely regular space.*

Now, «compactly generated spaces» (shortly: *k-spaces*) constitute indeed (cf. N. Steenrod [**51**]) a «convenient category» of topological spaces, so that it would be of course interesting, at least, to have the spectrum of a given topological algebra to be a k-space. This, however, is not always the case, whenever we are outside the category of complete normed algebras (: Banach algebras), even for the very particular, but important as well, subcategory of Fréchet locally m-convex algebras [**40**], as this has been pointed out by A. G. Dors [**13**]. On the other hand, Fréchet algebras admitting «functional representation» have always spectra, which are k-spaces [**60**], a fact which is actually true for a much

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more general class of topological algebras than the preceding one (cf., for instance, [**34**; p. 474, Theorem 3.1], and Theorem 3.2 below). The algebras in question still admit functional representation, retaining, however, the separate continuity of the multiplication, a fact which provides at least a bigger frame for the appropriate description of the class of topological algebras for which the pertinent result is valid (cf. Theorem 3.2 in the sequel), towards a more categorical context, for the same class of algebras, based on their property of being «k-algebras», and whose spectra are k-spaces, as this has been treated in Ref. [**14**] (: ibid.; Corollary 3.13).

The class of topological algebras, we are dealt with, is thus wide enough to include almost all the important topological algebras which appear in the applications, while their particular structure permits at the same time to extend and enlighten as well many of the standard results, which have been obtained in the particular cases previously considered in the literature. Besides, whenever the spectra of the same algebras are k-spaces, the «topological algebra spectrum-functor» naturally behaves «contravariantly» relative to the well-known «Michael decomposition» of a given locally m-convex algebra, when restricted to the respective particular class of topological algebras under consideration (cf. Theorem 3.3 below).

In the last two sections of this discussion we are concerned with applications of the technique developed so far within the preceding class of topological algebras, *firstly*, in connection with certain considerations of the current russian literature, in particular (cf. [**43**], [**15**]), and referred to what might be thought of as «homotopy theory within the context of Banach algebras», and also recently discussed, in an extended form, by J. L. Taylor in Ref. [**53**], [**54**], [**55**], however in the same context of the Banach algebras theory. Now, it is proved that the class of topological algebras considered herewith is, so far, the more natural frame for this kind of applications of the general theory, the Banach algebras theory framework being quite technical and in fact unnecessary. The same may also be considered as an extension to more general topological algebras of the well-known program firstly initiated, for Banach algebras, by G. Šilov [**50**], and then subsequently set forward by R. Arens [**6**], including the well-known Arens-Royden Theorem [**5**]. In this concern, it also seems to be possible to extend within the same context analogous considerations by J. C. Wood in Ref. [**61**], as well as similar results in the work of M. Karoubi [**25**], [**26**]. Within the same context (cf. § 6 below), we also give an extended version of some recent results of V. Ya. Lin in Ref. [**28**], referred to the theory of «matrices depending (continuously) on parameters». *Secondly,* we are dealt with «sectional representations» of topological algebras, a subject which we intend to discuss, in a more detailed manner, somewhere else.

In this context, another possible direction of applications, which seems to be naturally embodied within the preceding class of topological algebras, is connected with certain considerations by G. Tomassini in Ref. [**56**], referred to what might be viewed as «topological algebraic geometry», hence its natural connection with «topological algebra sheaves» [**35**], being thus in agreement with the current literature [**19**], as well as with certain «differential geometric considerations in topological algebras theory» very recently applied, in particular, by S. A. Selesnick in Ref. [**47**], however within the same special framework of Banach algebras theory.

We conclude with a short Appendix, where we briefly comment on certain recent results in Ref. [**3**] and [**17**] which are also quite naturally fitted within the same class of topological algebras discussed herein.

The results obtained so far in connection with the above considerations are mostly given without proofs, the present discussion being essentially of a preliminary nature, intending thus to return with more details in some future publications.

2. Spectrally barrelled algebras. By a *Fréchet topological algebra* we mean a topological algebra in the sense of the preceding section, for which the underlying topological vector space is complete and metrizable (: Fréchet, in the «polish sense»). This class of algebras includes, of course, all Banach algebras and even more all the Fréchet locally convex and / or the locally m-convex algebras (: R. Arens - E. A. Michael [**4**], [**40**]). As an immediate consequence of the definition, the topological algebras in question have (jointly) *continuous multiplication* (: continuity in both variables).

On the other hand, the preceding class of algebras is contained in that of *barrelled topological algebras,* which again may be, in particular, locally convex or locally m-convex ones, and which are characterized by the respective property (i. e., barrelledness) of the underlying the given algebra topological vector space.

In this respect, this is actually a common device in characterizing special classes of topological algebras, in terms of particular properties of the respective topological vector spaces, the properties in question being, however, not the genuine algebraic analogon of the respective linear ones, or even more (: «Šilov's point of view») of the appropriate «spectral» (: topological algebra spectrum) ones, as this will presently become clear from the context below.

Thus, the algebraic analogon of the defining property the preceding class of algebras is that of being the topological algebra *m-barrelled* [**33**], [**34**]. This means, by definition, that every *m-barrel* (i. e., idempotent barrel) in the topological algebra considered is a neighborhood of zero (ibid.). In this concern, we notice that in a locally m-convex algebra (in the sense of Arens - Michael ; ibid.) there always exists, by definition, a local basis consisting of m-barrels (ibid.).

Now, there are even classical examples showing that the «inclusion relation» between the preceding various classes of topological algebras is in each case a genuine one. Besides, it has been proved (cf. [**33**], [**34**]) that, by applying the class of m-barrelled topological algebras, one can extend to this class of algebras, and better explain as well, several basic facts concluded, more or less so far, for the class of Fréchet algebras only, the metrizability of the respective topological vector space being thus involved by excess, the crucial fact in this respect being, of course, the *barrelledness of the space* in question, and indeed its algebraic counterpart, i. e., the *m-barrelledness of the topological algebra* under consideration.

In this respect, a fundamental consequence of the «defining structure property» of an m-barrelled (topological) algebra is that three basic classes of subsets of the spectrum of the algebra coincide. That is, one has the following result (cf., for instance, [**34**; p. 470, Corollary 2. 1]):

Theorem 2. 1. *Let E be an m-barrelled algebra, whose spectrum is $M(E)$. Then, the following classes of subsets of $M(E)$ are the same: 1) the equiconti-*

nuous sets, 2) the (weakly) bounded sets, and 3) the (weakly) relatively compact sets.

(Sketsch of the proof): The crucial step in the proof of the preceding statement is the fact that the (weakly) bounded subsets of the spectrum of an m-barrelled algebra are actually equicontinuous (and this is based on the defining structure property of the algebra; ibid.), so that one concludes the coincidence of the families 1) and 2) above (and this is of a special importance, as we shall see below), and hence the desired result now follows by a direct appealling to the Alaoglu - Bourbaki Theorem. ∎

In connection with the preceding result, we notice that an essential part of the analysis, we are doing below, is the implications which in each case the coincidence of the three preceding classes of sets has for a given topological algebra, this actually leading to the characterization of the particular classes of topological algebras, we are going to consider.

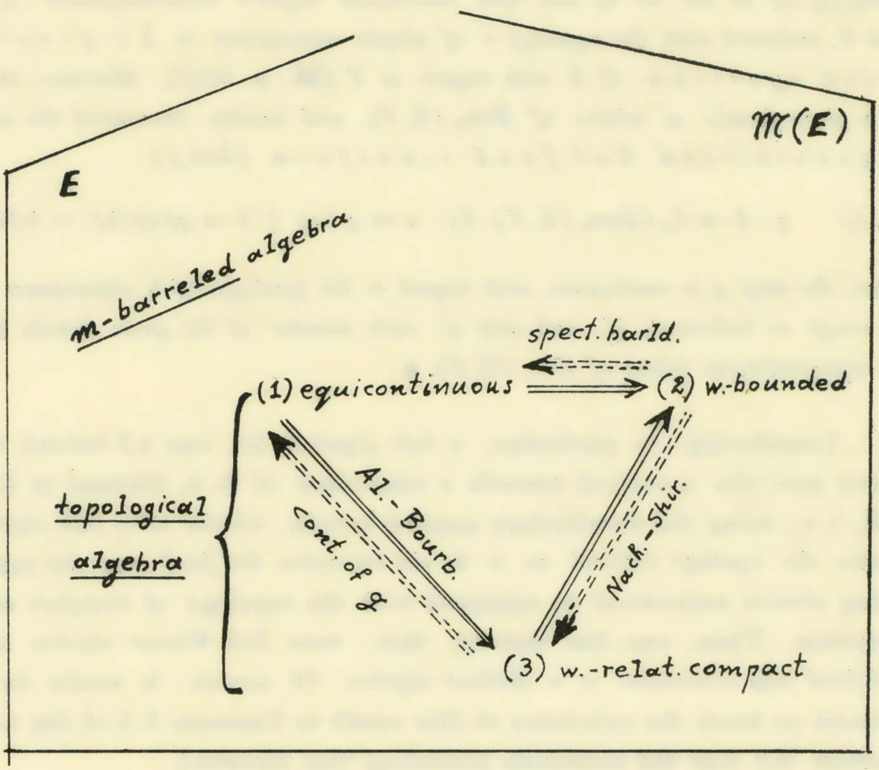
This is schematically explained by the following diagram (sch. 1), which subsumes some basic facts of the analysis which follows:

Now, we remark that *the coincidence of the preceding three classes of sets does not characterize the m - barrelled algebras*: By applying the Alaoglu - Bourbaki Theorem, it is easily seen (cf. sch. 1) that *the coincidence of the three preceding families of sets is equivalent with that of the families 1) and 2)*, in the statement of the above Theorem 2. 1, so that the desired assertion as above would equivalently be proved by showing that *topological algebras for which the families 1) and 2) coincide are not, in general, m-barrelled*: A counterexample has already been given, in a somewhat different context, in Ref. [**33**; p. 306]; besides, the same conclusion follows by some recent considerations of A. K. Chilana in Ref. [**12**], as this will become clear from the discussion below. (In this concern, cf. also Ref. [**38**; p. 154, § 3]).

Thus, we are naturally led to set the following:

Definition 2. 1. We say that a given topological algebra is *spectrally barrelled,* if the equicontinuous and the weakly bounded subsets of its spectrum coincide.

Concerning the preceding definition, the terminology applied intends to remind the analogous situation one has in the case of the weak topological dual of a given barrelled topological vector space, this being now expressed in terms of the spectrum of the topological algebra under consideration.



Sch 1.

Now, before embarking on the various consequences, which the requirement, set forth by Definition 2. 1, has for the structure of a given topological algebra, it would be, at least, more instructive to look at the same program by admitting a weaker condition, than the preceding one, for the spectrum of a given topological algebra.

Thus, *the coincidence of the families (1) and (3) in* (sch. 1) *above characterizes the continuity of the respective Gel'fand map* $g : E \rightarrow C_c(M(E))$ *for a given topological algebra* E, whose spectrum is M(E). This is a strengthen ed (and for locally convex algebras, equivalent) version of a previous result in Ref. [**33**; p. 305, Theorem 3.1]: That is, in a more general context, one has the following, which subsumes and clarifies the particular cases considered hitherto. Namely,

Lemma 2.1. *Let E and F be given topological algebras, and let $Hom_s(E, F)$ be the set of non-zero continuous algebra homomorphisms of E into F, endowed with the topology s of simple convergence in E (: generalized spectrum of E with respect to F [**36**; p. 344]). Moreover, let $\mathfrak{S}$ be a given family of subsets of $Hom_s(E, F)$, and besides considered the map (: generalized Gel'fand transform (ibid.)):*

$$(2.1) \qquad g : E \to C_{\mathfrak{S}}(Hom_s(E, F), F) : x \to g(x) \quad (: h \to g(x)(h) := h(x)).$$

Then, the map g is continuous, with respect to the topology of $\mathfrak{S}$ convergence on its range as indicated, if, and only if, each member of the given family $\mathfrak{S}$ is an equicontinuous subset of $Hom(E, F)$. ∎

Considering, in particular, a full algebra (cf. also § 3 below), the above provides a variant towards a conjecture of E. A. Michael in Ref. [**40**], i.e., using the terminology applied herein, *whether every full algebra carries the topology induced on it by the respective Gel'fand map,* its range being always understood as equipped with the topology of compact convergence. Thus, one has instead, that: *every full Warner algebra with Gel'fand map continuous is a Michael algebra.* Of course, it would be of interest to know the relevance of this result to Theorem 3.2 of the next Section. (Cf. also the comments preceding that theorem).

In this concern, we notice that in Michael's terminology, by a *full algebra* is meant a commutative, complete, semi-simple, locally m-convex algebra for which the respective Gel'fand map is an algebraic onto isomorphism [**40**; p. 35, Definition 8.3]. (Cf. also [**34**; p. 475]).

Within the same context, we also note that *every commutative, semi-simple, topological algebra having the respective Gel'fand map continuous, and which is also a Warner algebra* [**34**; p. 476], *it is actually a Michael algebra.* (This extends Theorem 3.2 in Ref. [**34**; p. 476]; an analogous extension is valid for Corollary 3.2 of the same Ref., p. 477. In this connection, cf. also the respective discussion given in Ref. [**38**; p. 155]).

Thus, several results already concluded for Fréchet algebras, and more generally for m-barrelled topological algebras (cf. [**33**], [**34**]), are also valid for this more general class of algebras, among of which is a form of the classical «Gel'fand - Naimark representation theorem»; however for this, and further details, concerning the class of topological algebras in question, we refer the reader to Ref. [**37**], [**38**].

On the other hand, *the coincidence of the families* (2) *and* (3) *in* (sch. 1) *characterizes the Nachbin - Shirota spaces* (cf. [**37**; p. 104]), *as well as the respective topological algebras* (cf. Definition 2. 2 below):

To be more comprehensive, we recall some of the details of the respective terminology: Thus, given a Hausdorff completely regular topological space X, we shall say that X is a *Nachbin - Shirota space,* whenever by considering X as a subspace of the weak dual space of $C_c(X)$, i. e., $X \underset{\rightarrow}{\subset} (C_c(X))'_s$, via the respective evaluation map, one has that every (weakly) bounded subset of X is also (weakly) relatively compact (cf. [**37**; p. 104]). This is actually the content of the classical homonymous theorem, characterizing the barrelledness of a space of the form $C_c(X)$, with X as above, in this respect cf. also the relevant comments in Ref. [**60**; p. 265, and p. 272].

Thus, it is reasonable to put the following.

Definition 2. 2. Given a topological algebra E, we shall say that it is a *Nachbin - Shirota topological algebra,* whenever its spectrum $M(E)$ is a Nachbin - Shirota topological space.

In case a Nachbin - Shirota topological algebra «admits a functional representation» [**34**], then it is, of course, barrelled (: Nachbin - Shirota Theorem), and a fortiori an m-barrelled one. However, *every m - barrelled topological algebra is Nachbin - Shirota:* This is actually an immediate consequence of Theorem 2. 1 above. More generally, *every spectrally barrelled algebra* (Definition 2. 1) *is a Nachbin - Shirota topological algebra* (cf. (sch. 1)).

In this concern, one has the following theorem which constitutes a strengthened form of a similar result in Ref. [**37**; p. 104, Theorem 2. 1]. That is, we have:

Theorem 2. 2. *Let X be a Hausdorff completely regular space. Then, X* *is a Nachbin-Shirota space if, and only if, it is homeomorphic to the spectrum of a spectrally barrelled algebra.*

Proof: The condition is sufficient, by the preceding comments. Now, the necessity of the stated condition follows by the fact that the space X is homeomorphic to the spectrum of the (locally m-convex topological) algebra $C_c(X)$, which, by hypothesis, is a barrelled one, and hence, a fortiori, spectrally barrelled, and this proves the assertion. ∎

Within the same context, we note that one has an analogous statement to the preceding Theorem 2.2 for barrelled algebras, as well as for m-barrelled ones (ibid.). All these are actually Nachbin-Shirota topological algebras, since they are spectrally barrelled (cf. Theorem 2.1 above), hence the extended form of the previous theorem in relation with similar considerations in Ref. [**37**].

Besides, as a consequence of the preceding, we remark that *a topological algebra of the form* $C_c(X)$, *when X completely regular* (Hausdorff) *space, is spectrally barrelled if, and only if, it is a barrelled one.* Thus, the situation does not contribute towards a characterisation of the completely regular space X via of its being (within a homeomorphism) the spectrum of a spectrally barrelled algebra $C_c(X)$, as is the case with the classical Nachbin-Shirota Theorem (cf. also the relevant comments following Theorem 2.1 in Ref. [**37**; p. 105]).

3. Spectrally barrelled algebras (continued). We are involved in this section in a more detailed discussion of the particular class of topological algebras alluded to at the beginning of this paper, and which are characterized by the coincidence of the three families of sets in (sch. 1) (cf. also Theorem 2.1 above). Thus, one has, at first, the following result which is an immediate consequence of the foregoing. That is, we have.

Theorem 3.1. *A given topological algebra is spectrally barrelled if, and only if, it is a Nachbin-Shirota topological algebra, having besides the respective Gel'fand map continuous.* ∎

It is for the same class of topological algebras for which one obtains a strengthened form of a «functional representation» result, which for

Fréchet (locally m-convex) algebras is already essentially due to S. Warner (cf. [**60**; p. 269, Theorem 4]), and for m-barrelled topological algebras has been given in Ref. [**34**; p. 474, Theorem 3.1]. To be more comprehensive, we first give the following.

Definition 3.1. Les E be a topological algebra whose spectrum is $M(E)$. We shall say that E *admits a functional representation,* whenever one has:

$$E = C_c(X), \tag{3.1}$$

within a topological algebraic isomorphism, in such a way that X is homeomorphic to $M(E)$. (In this respect, $C_c(X)$ denotes the set of all complex-valued continuous functions on a given Hausdorff topological space X, equipped with the topology of compact convergence in X).

In this connection, we shall say that a given topological algebra is *full,* if the respective Gel'fand map $g : E \rightarrow C_c(M(E))$ is an algebraic onto isomorphism (we apply the notation of the preceding Definition 3.1; cf. also [**40**; p. 35, Definition 8.3]).

Thus, one obtains the following result, which gives, among other things, sufficient conditions in order the requirements set forth by Definition 3.1 above to be valid, and which also constitutes still another step related to a conjecture of E. A. Michael in Ref. [**40**; p. 36. Cf. also p. 38, Remark after Proposition 8.5]. (In this respect, cf. also [**37**; p. 105, Theorem 2.2], as well as [**38**; p. 153, Theorem 2.1]). That is, we have.

Theorem 3.2. *Let E be a full, spectrally barrelled, Pták locally convex algebra. Then, E admits a functional representation (Definition 3.1). In particular, E has the topology of uniform convergence on the equicontinuous subsets of its spectrum, that is E is a Michael algebra* [**34**], *whose spectrum is, besides, a k-space. Moreover, E is actually an m-barrelled (locally m-convex) algebra.* ∎

Scholium 3.1. The proof of the preceding Theorem 3.2 is essentially based on an analysis of the structure properties for a given spectrally barrelled algebra, exhibited by Theorem 3.1 in the foregoing. (Cf. also the respective proof initially given for m-barrelled algebras in Ref. [**34**; p. 474, Proof of Theorem 3.1]. In this connection, it would be interesting, of course, to have the least sufficient conditions, which would imply, for a given full topological algebra, the situation described by Definition 3.1 above.

In connection with the foregoing, we also notice that Pták (locally convex) algebras have also been applied recently in Ref. [**27**] (in fact, as «fully complete» locally m-convex algebras; ibid., p. 47), by considering *«separable* (topological) *algebras».* In this concern, cf. also Ref. [**30**] and [**39**].

Now, spectrally barrelled algebras which are, in particular, locally m-convex ones, and whose spectra are k-spaces behave, in a most natural way, «contravariantly» relative to the «topological algebra spectrum-functor» and the respective «Michael decomposition» (cf. the following theorem), this being, besides, a characteristic property of the spectra in question to be k-spaces. More precisely, one obtains the following result, which is indeed a compilation of previous ones in Ref. [**37**; p. 100, Theorem 1.1] and [**38**; p. 157, Theorem 4.1]:

Theorem 3.3. *Let E be a complete spectrally barrelled locally m-convex algebra (being a «limit» of Banach algebras E_α, $\alpha \in I$: «Michael decomposition», i.e. $E = \varprojlim E_\alpha$, within a topological algebraic isomorphism [**40**]). Then, the following assertions are equivalent:*

1) $M(E)$ (: the spectrum of E) is a k-space.

*2) The family $\{M(E_\alpha) : \alpha \in I\}$ (cf. [**38**; p. 156]) generates the Gel'fand topology of $M(E)$.*

3) $M(E) = \varinjlim M(E_\alpha)$, within a homeomorphism.

*4) $C_c(M(E))$ is complete, and the space $kM(E)$ (cf. [**37**; p. 99]) is also completely regular.* ∎

Within the same context and as a corollary to previous considerations in Ref. [**38**], we state, in a more explicit form, the following.

Proposition 3.1. *Every (commutative) complete «uniform» [**38**] topological algebra E, with Gel'fand map continuous (in particular, a spectrally barrelled one (Theorem 3.1)), may be identified, within a topological algebraic isomorphism, with its algebra of Gel'fand transforms, being in particular a closed subalgebra of $C_c(M(E))$.*

(Sketsch of the proof): The assertion is an immediate consequence of Theorem 3.1 in Ref. [**38**; p. 155] and the Scholium following its proof (ibid.; p. 155). ∎

On the other hand, *for a commutative and complete spectrally barrelled locally m-convex algebra with an identity element, its spectrum being compact is equivalent with the given algebra to be a Q-algebra or, equivalently, a pseudo-Banach algebra* [**2**]: The last statement is a strengthening of a previous result of Alan-Dales-Mc Clure (ibid.), given recently in terms of the theory of «topological algebras with a bound structure». In this concern, cf. also Ref. [**38**; p. 160, Theorem 5.1].

Now, a given *topological algebra* E is said to be *bounded,* whenever the respective algebra $E^{\wedge} \equiv g(E)$ of Gel'fand transforms of the elements of E (that is, the *Gel'fand transform algebra* of E) consists entirely of bounded (complex-valued, continuous) functions or, what amounts to the same thing, the spectrum $M(E)$ of the given algebra E is a bounded subset of E'_s (: the weak topological dual of E).

In this concern, the following result subsumes and extends as well previous standard conclusions in Ref. [**59**] and [**40**], together with their strengthened form given in Ref. [**34**] (: ibid.; p. 472, Theorem 2.2, and Corollary 2.4). Thus, we have the following.

Theorem 3.4. *Let E be a locally m-convex algebra whose spectrum is M(E). Moreover, consider the following assertions:*

1) E is a Q-algebra.

2) M(E) is equicontinuous.

3) M(E) is a weakly relatively compact subset of E'.

4) $M(E)^+$ (: $= M(E) \cup \{0\}$, the e x t e n d e d s p e c t r u m of E) is a weakly compact subset of E'.

5) E is a bounded algebra.

*Then, one has the following implications: 1) ⟹ 2) ⟹ 3) ⟹ 4) ⟹ 5). Furthermore, in case the algebra E is, in particular, a commutative, advertibly complete [**59**], spectrally barrelled (locally m-convex) algebra, then all the preceding five assertions are equivalent.*

Proof: The assertion can be deduced by Theorem 2.2 and Corollary 2.4 in Ref. [**34**; p. 472], by taking also into account Theorem 3.1

in Ref. [**33**; p. 305], these results being converted within the present more general context by an immediate application of the reasoning developed in the foregoing (cf. too Ref. [**59**; p. 7, Theorem 6, and Proposition 10]). ∎

In a less technical terminology, one can state the preceding Theorem 3. 4 into the form of the following.

Corollary 3. 1. *Let E be a commutative, complete, spectrally barrelled, locally m-convex algebra, whose spectrum is $M(E)$. Then, all the five assertions of the preceding Theorem 3. 4 are mutually equivalent.* ∎

The preceding Corollary 3. 1 subsumes a statement given by Professor W. Żelazko at the «Bordeaux Colloquium of Functional Analysis (4-8 May, 1975), in terms of barrelled algebras (cf. also [**64**]), the latter result being also the initial motive for the present form of the above Theorem 3. 4; this was, essentially, an immediate application of previous considerations in Ref. [**33**], [**34**], now being used within the more general context of spectrally barrelled algebras, as it has been developed in Ref. [**38**] and the foregoing (cf. also the proof of the said theorem given above).

Within the same context, we actually get the following extension of the analogous result of W. Żelazko in Ref. [**64**], even under much weaker conditions than those in the preceding Corollary 3. 1. That is, we have:

Theorem 3. 5. *(W. Żelazko). Let E be a commutative, advetribly complete [**59**], locally m-convex algebra, having an identity element and the respective Gel'fand map continuous. Then, the following assertions are equivalent:*

1) Every maximal ideal of E is of codimension one.

2) Every maximal ideal of E is closed.

3) The algebra is bounded.

4) The range of (the Gel'fand transform) $\hat{x}$, for every $x \in E$, is a compact subset of $\mathbb{C}$.

5) E is a Q-algebra.

6) The spectrum $M(E)$ of the given algebra E is a compact space.

Proof: This is a consequence of Żelazko's Theorem in Ref. [**64**; p. 5] (the same Theorem is actually valid for advertibly complete algebras; however cf. cond. iv) of the same result), plus [**59**; p. 7, Theorem 6] and [**33**; p. 305, Theorem 3. 1]. (In this respect, it suffices the given algebra to be advertibly complete, taking into account the respective results of *E. Killam:* Pacific J. Math. 12 (1962), 581 - 588), and this finishes the proof. ∎

As a consequence, one obtains a strengthening of Żelazko's statement in the same Ref. [**64**; p. 4] in the sense that *a topological algebra satisfying the conditions of the preceding Theorem 3. 5 has a dense maximal ideal of infinite codimension if, and only if, it is not a Q-algebra.*

The author is much indebted to Professor W. Żelazko for having providing him with a copy of his reprint of Ref. [**64**], which constitutes a more detailed account of his talk at the Bordeaux Colloquium.

Now, *the category of spectrally barrelled topological algebras admits «colimits»:* A special case of this assertion (i. e., for locally convex spectrally barrelled algebras) has already been given in Ref. [**38**; p. 157, Lemma 4. 2]. However, the general case is given by the proposition which follows. That is, we have.

Proposition 3. 2. *Let $(E_\alpha)_{\alpha \in I}$ be a family of spectrally barrelled algebras, together with a family $(f_\alpha)_{\alpha\in I}$ of algebra morphisms, with $f_\alpha : E_\alpha \to E$, $\alpha \in I$, where E is a given algebra, such that the respective vector space E is the linear span of $\bigcup \{f_\alpha(E_\alpha) : \alpha \in I\}$. Then, the algebra E endowed with the finest vector space topology, defined by the family $\{(E_\alpha, f_\alpha) : \alpha \in I\}$ (*-inductive-limit topology on E in the terminology of Ref. [**24**]), is a spectrally barrelled topological algebra.*

Proof: Let $M(E)$ be the spectrum of E, the algebra E being topologized as in the statement above, and let $B \subseteq M(E)$ be a bounded subset. Then, each one of the sets $B \circ f_\alpha = \{h \circ f_\alpha : h \in B\}$, $\alpha \in I$, is a weakly bounded subset of $M(E_\alpha)^+$, with $\alpha \in I$, and hence, by hypothesis, equicontinuous, so that the same is concluded for the set $B \subseteq M(E)$ (cf. also Ref. [**24**; p. 286, the remarks after Definition 2. 1), and this finishes the proof. ∎

On the other hand, *the category of spectrally barrelled algebras admits «topological tensor products»:* Cf. Ref. [**38**; p. 158, Lemma 4. 3, as well as

p. 159, Corollary 4. 1]. Furthermore, *the completion of a given spectrally barrelled topological algebra* (: continuous multiplication) *is an algebra of the same kind* (: ibid.; p. 160, Remark).

Now, we are going to consider spectrally barrelled algebras having a prescribed set of generators. These algebras exhibit several properties of a particular significance, as we shall see below, and will be discussed next in the following section.

4. Spectrally barrelled algebras with a prescribed set of topological generators. Thus, given a topological algebra E, we shall say that a family $(x_\alpha)_{\alpha\in I}$ of elements of E is (a *set of topological generators*, or simply) a *set of generators* of E, whenever the algebra E coincides with the smallest closed subalgebra of it, containing the given family.

Now, let I be a non-empty set of indices, and let the space $\mathbb{C}^I$ (where $\mathbb{C}$ denotes the complexes) be given the respective cartesian product topology. Thus, for every compact subset K of the space $\mathbb{C}^I$, let $C(K)$ denote the Banach algebra of complex-valued continuous functions on K, with respect to the «uniform norm», and let $P(K)$ be *the uniform closure in $C(K)$ of the algebra of polynomials on K* (the polynomials considered being taken with respect to a family of indeterminates $(z_\alpha)_{\alpha\in I}$), so that $P(K)$ is a commutative Banach algebra with an identity element; now, one defines the *polynomially convex hull* of K (we denote it by $\hat{K}$), as the spectrum of the Banach algebra $P(K)$, i. e., one has by definition, the relation:

$$\hat{K} := M(P(K)). \tag{4.1}$$

On the other hand, *one has the following relations:*

$$K \underset{\rightarrow}{\subset} M(P(K)) \underset{\rightarrow}{\subset} \sigma_{P(K)}\left((z_\alpha)_{\alpha\in I}\right) \subseteq \mathbb{C}^I, \tag{4.2}$$

where $\underset{\rightarrow}{\subset}$ denotes bicontinuous injections (: homeomorphisms) defined in the first case by the obvious «evaluation map» and in the second one by the corresponding map to the relation (4.5) in Theorem 4.1 below. Thus, one obtains, in particular, the relation:

$$M(P(K)) = \sigma_{P(K)}\left((z_\alpha)_{\alpha\in I}\right), \tag{4.3}$$

within a homeomorphism, where the second member of the last relation defines the *joint spectrum* of the family $(z_\alpha)_{\alpha \in I}$, when it is considered as consisting of elements of the algebra $P(K)$ (cf. also the following Theorem 4.1).

Now, a subset S of $\mathbb{C}^I$ is said to be *polynomially convex,* whenever for every compact $K \subseteq S$ the polynomially convex hull $\hat{K}$ of K (cf. the rel. (4.1) above) is contained in S or, what amounts to the same thing, *whenever one has the relation:*

$$S = \bigcup_{K \subseteq S,\ \text{compact}} \hat{K}. \tag{4.4}$$

Concerning the terminology applied in the foregoing, the reader is also referred to Ref. [**46**], where one finds the analogous setting for (complex, commutative and unital) Banach algebras, and which also was, to a great extend, a motivation to the preceding. In this concern, we also refer the reader to [**52**]. Now, the following result specializes to the analogous one, given in the latter Ref. within the Banach algebras theory (ibid.; p. 25, Theorem 5.8). That is, we have:

Theorem 4.1. *Let E be a topological algebra, whose spectrum is M(E), and let $A \equiv (x_\alpha)_{\alpha \in I}$ be a set of generators of E. Then, the following map:*

$$\phi : M(E) \to \mathbb{C}^I : f \to \phi(f) := (\hat{x}_\alpha(f))_{\alpha \in I} \tag{4.5}$$

defines a continuous bijection of M(E) onto its range $Im(\phi) \equiv \sigma_E(A)$ (: the joint spectrum of the given family A). In particular, if E is a commutative, complete, spectrally barrelled, locally m-convex algebra with an identity element in such a way that the map ϕ is a homeomorphism onto its range $Im(\phi)$, then the latter set is a polynomially convex subset of $\mathbb{C}^I$.

Proof: The map ϕ is obviously continuous by the definition of the topologies in $M(E)$ and $\mathbb{C}^I$. Besides, it is easily seen that the same map is one-to-one, based on the fact that E is, by hypothesis, «topologically generated» by the given family A, and *this proves the first part of the assertion.* On the other hand, suppose, *in particular,* that the algebra E satisfies the conditions set forth by the second part of the statement above, and let $E = \varprojlim E_\lambda$ be the corresponding *Michael decomposition of E into Banach algebras,* associated to a local basis, say $(U_\lambda)_{\lambda \in \Lambda}$, of E [**40**]. Thus, one obtains a *k-covering family of the spectrum of E,* by considering the spectra $\mathcal{M}(E_\lambda) \equiv \mathcal{M}(E) \cap U_\lambda^\circ$ of the Banach algebras E_λ, $\lambda \in \Lambda$ (cf., for instance, [**38**; p. 156, Lemma 4.1]). Therefore (ibid.), for every compact set $K \subseteq \mathcal{M}(E) \underset{\varphi}{\cong} \sigma_E(A) \subseteq \mathbb{C}^I$, there exists an index $\lambda \in \Lambda$ such that one has the relation:

$$K \subseteq \mathcal{M}(E_\lambda) \cong \sigma_{E_\lambda}(\dot{A}), \tag{4.6}$$

where by $\dot{A}$ is denoted the «canonical image» of the given family A in each one of the algebras E_λ, $\lambda \in \Lambda$. Now, the latter set in the above rel. (4.6) is a compact, polynomially convex subset of $\mathbb{C}^I$ (cf. [**52**; p. 25, Theorem 5.8]). Thus, by considering the polynomially convex hull $\hat{K}$ of K, one obtains:

$$\begin{aligned} \hat{K} := \mathcal{M}(P(K)) \underset{\text{homeo.}}{\xrightarrow{\subset}} \mathcal{M}\big(P(\sigma_{E_\lambda}(\dot{A}))\big) \underset{\text{homeo.}}{\cong} \sigma_{E_\lambda}(\dot{A}) \\ \cong \mathcal{M}(E_\lambda) \xrightarrow{\subset} \mathcal{M}(E) \underset{\varphi}{\cong} \sigma_E(A), \end{aligned} \tag{4.7}$$

where, by definition, $\mathcal{M}(E_\lambda)$ is a compact subset of the spectrum of the given algebra E, *which proves the second part of the assertion* (cf. also rel. (4.4) above), and this finishes the proof of the theorem. ∎

In particular, one has the following Corollary, expressing within the present more general context, the analogous situation one has in the special case of Banach algebras theory (cf., for instance, Ref. [**52**; p. 25, Theorem 5.8]). That is, one obtains.

Corollary 4.1. *Let E be a commutative, complete, spectrally barrelled, locally m-convex algebra with an identity element and compact spectrum $\mathcal{M}(E)$. Moreover, let $(x_\alpha)_{\alpha \in I}$ be a set of generators of E. Then, $\mathcal{M}(E)$ is homeomorphic to a compact polynomially convex subset of $\mathbb{C}^I$.* ∎

As an immediate consequence of the preceding, one obtains the following result, which is of a particular significance for the applications considered in the sequel (cf. Section 5 below). In this respect, cf. also Ref. [**37**; p. 109, Theorem 4.1], the latter result being also a special case of the above Theorem 4.1, and of the following corollary as well, for algebras having compact spectra. Thus, we have:

Corollary 4.2. *Let E be a commutative, complete, n-generated, spectrally barrelled, locally m-convex algebra, having an identity element and compact* *spectrum $M(E)$. Then, the latter space is homeomorphic to a compact, polynomially convex subset of* $\mathbb{C}^n$. ∎

The above corollary specializes, of course, to the well-known analogous result for commutative, unital, finitely-generated Banach algebras (cf., for instance, [**22**; p. 66, Theorem 3. 1. 15]), as well as to a similar one for Fréchet (locally m-convex) algebras, given by R. M. Brooks in Ref. [**10**; p. 149, Theorem 2. 2]).

Scholium 4. 1. In connection with the preceding Theorem 4. 1, we remark that *the map* ϕ, given by the rel. (4. 5), *defines a homeomorphism onto its range if, and only if, there exists a local basis of the given algebra E which determines a k-covering family for the set* $\mathrm{Im}(\phi) = \sigma_E(A)$, *where the algebra E satisfies the conditions in the second half of the said theorem:* The assertion can easily be concluded by applying the argumentation in the proof of Lemma 4. 1 in Ref. [**37**; p. 108], whose the preceding constitutes an extended form within the class of spectrally barrelled algebras.

The preceding Scholium has a special bearing on Theorem 3. 2 in Ref. [**20**; p. 461]. Besides, the respective Corollary 3. 3 of the same Ref. [**20**] is subsummed in Corollary 4. 2 given above. Furthermore, it is easy to see that *Theorems 3. 4 and 3. 6* (ibid.; p. 461 and p. 462) *hold also true within the present more general context.*

We are now ready to deal with the applications alluded to in the Introduction of this paper, by using the technique developed in the foregoing. These are discussed next in the following Sections. (: These will constitute the Sesond Part of this study, published separately).

Due to its length, the present paper will be appeared in two Parts, according to its natural conceptual division (within the same journal), the first one of which is included herewith. The literature cited at the end of the paper is referred to both Parts.

Remarks (added in proof). The relation $M(\mathrm{E}) = \varinjlim M(\mathrm{E}_\alpha)$, valid within a homeomorphism, in Prop. 3 of Theorem 3. 3, holds actually true without the assumption the algebra E to be complete, this being added only in order the «Michael's formula» to be valid, i.e. $\mathrm{E} = \varprojlim \mathrm{E}_\alpha$, within a topological algebraic isomorphism of the topological algebras involved. In this concern, we still notice that, in view of specific applications, one can dispense with the assumption, regarding the definition of a locally m-convex algebra, the latter to be a topological algebra; instead, we assume that the given algebra is a topological vector space for which there exists a local basis consisting of multiplicative (: idempotent) and convex (abbr. to *m-convex*) sets. It is then proved that such an algebra is a topological algebra which has a local basis consisting of m-barrels, i.e. the standard definition is fulfilled. (More details are given in the author's: Topological Algebras. Selected Topics (to appear)). Finally, a step forward can be made, concerning Theorem 3.2 above, by applying recent results of D. Rosa (: Pacific J Math. 60 (1975), 199-208). Thus, Theorem 3.2 is valid, by using standard argumentation, when considering *«Pták algebras»* in the (weakened) sense of D. Rosa (i.e., *B-complete algebras;* ibid.). In this respect, the same theorem can also partly be improved, as it concerns the functional representation asserted, by considering only *infra-Pták* (or *B_r-complete*) *algebras* (ibid.).

ΠΕΡΙΛΗΨΙΣ

Εἰς τὴν παροῦσαν ἐργασίαν, ἡ ὁποία ἀποτελεῖ τὸ πρῶτον μέρος μιᾶς ἐκτενεστέρας μελέτης, δίδονται τὰ κυριώτερα μέχρι τοῦδε ἀποτελέσματα, τὰ ἀναφερόμενα εἰς τὴν γενικὴν θεωρίαν μιᾶς νέας κατηγορίας τοπολογικῶν ἀλγεβρῶν, τῶν *«φασματικῶς κυλινδροειδῶν»* (: spectrally barrelled, πρβλ. [**38**]). Αἱ ἐν λόγῳ τοπολογικαὶ ἄλγεβραι χαρακτηρίζονται ἀπὸ τὸ γεγονὸς ὅτι *κάθε ἀσθενῶς φραγμένον ὑποσύνολον τοῦ φάσματος εἶναι ἰσοσυνεχὲς* (αὐτόθι· σελ. 153, Definition 2.1). Ἀποδεικνύεται ὅτι ἡ ἐν λόγῳ ἰδιότης τοῦ φάσματος εἶναι τὸ κατ' ἐξοχὴν ἐκεῖνο στοιχεῖον, τὸ ὁποῖον κατ' οὐσίαν χρησιμοποιεῖται εἰς τὰς μέχρι τοῦδε ἐφαρμογὰς τῶν εἰδικωτέρων τοπολογικῶν ἀλγεβρῶν, ὅπως εἶναι αἱ ἄλγεβραι Fréchet [**40**] καὶ ἡ γενίκευσίς των, αἱ m-κυλινδροειδεῖς [**37**]. Ἡ διαπίστωσις τοῦ ἐν λόγῳ φαινομένου κατὰ τὴν προηγουμένην μελέτην τῶν τελευταίων ὡς ἄνω ἀλγεβρῶν ὡδήγησεν εἰς τὸν ὁρισμὸν τῆς ὑπὸ συζήτησιν κατηγορίας τῶν τοπολογικῶν ἀλγεβρῶν.

Ἰδιαίτερον ἐνδιαφέρον παρουσιάζει ἐπίσης τὸ γεγονὸς ὅτι ἡ ὁρίζουσα ἰδιότης τὰς ἐν λόγῳ ἀλγέβρας εἶναι καθαρῶς «φασματική» (ἀναφερομένη εἰς τὴν τοπολογίαν τοῦ φάσματος τῆς θεωρουμένης ἀλγέβρας, ἐξ οὗ καὶ ἡ χρησιμοποιουμένη ὁρολογία), γεγονὸς τὸ ὁποῖον δύναται νὰ θεωρηθῇ ὅτι περιλαμβάνεται εἰς τὸ γνωστὸν πρόγραμμα ἐρεύνης, τὸ ὑπὸ τοῦ Ρώσου μαθηματικοῦ G. E. Šilov τὸ πρῶτον εἰσαχθέν [50], δηλαδή, «τὸ κατὰ πόσον τὸ τοπολογικὸν φάσμα μιᾶς (τοπολογικῆς) ἀλγέβρας καθορίζει τὴν ἄλγεβραν» (πρβλ. ἐπίσης [6]). Σχετικῶς, ἀξιοσημείωτος ὑπῆρξεν ἡ διαπίστωσις ὅτι εἰς ὅλας τὰς μέχρι τοῦδε ἐφαρμογὰς τῶν m-κυλινδροειδῶν ἀλγεβρῶν, ὅπως αὗται ἐκτίθενται ἐν [33], [34] καὶ [37] (ἀποτελοῦσαι ἐπέκτασιν προηγουμένων ἐφαρμογῶν τῆς βιβλιογραφίας δι' ἀλγέβρας Fréchet ἤ, γενικώτερον, κυλινδροειδεῖς) εἶναι ἡ ἐν λόγῳ ἰδιότης τοῦ φάσματος ἐκείνη, ἡ ὁποία χρησιμοποιεῖται καὶ ἡ ὁποία οὕτως ἐμφανίζεται ὡς γενικεύουσα τὴν «γεωμετρικὴν ἰδιότητα», ἡ θεωρουμένη ἄλγεβρα νὰ εἶναι κυλινδροειδὴς ἤ, ἀκόμη γενικώτερον, m - κυλινδροειδὴς (: δομικὴ - structural - ἰδιότης τῶν θεωρουμένων ἀλγεβρῶν). *Γεωμετρικὸς* (δηλαδή, δομικὸς) *χαρακτηρισμὸς τῆς ἐν λόγῳ ἰδιότητος θὰ ἦτο ἐνδιαφέρον νὰ δοθῇ* ἐν συνδυασμῷ πρὸς τὸ ἀνωτέρω πρόγραμμα τοῦ Šilov.

Ἡ ὡς ἄνω κατηγορία τῶν τοπολογικῶν ἀλγεβρῶν περιλαμβάνει ἐπίσης, ὅπως ὑπῆρξε προηγουμένως ἡ περίπτωσις διὰ τὰς m - κυλινδροειδεῖς ἀλγέβρας [37], τὰς ἐνδιαφερούσας διὰ τὰς ἐφαρμογὰς *τοπολογικὰς ἀλγέβρας Nachbin - Shirota* (πρβλ. Definition 2. 2 τῆς παρούσης μελέτης), καθὼς καὶ ἐκείνας μὲ *«συνεχῆ ἀπεικόνισιν Gel'fand»* (πρβλ. Lemma 2. 1, ὡς ἀνωτέρω). Μία ἰδιαιτέρα ἐφαρμογὴ τῆς τελευταίας κατηγορίας ἀλγεβρῶν ὑπῆρξε καὶ ἡ ἰσχὺς ἐν προκειμένῳ ἑνὸς προσφάτου ἀποτελέσματος τοῦ Πολωνοῦ μαθηματικοῦ W. Żelazko, ἀναφερομένου εἰς τὸν χαρακτηρισμὸν «μεγίστων ἰδεωδῶν μὲ πεπερασμένην συνδιάστασιν» μιᾶς τοπικῶς m - κυρτῆς ἀλγέβρας [40], ἀρχικῶς διατυπωθέντος εἰς τὰ πλαίσια τῆς θεωρίας τῶν κυλινδροειδῶν ἀλγεβρῶν (πρβλ. [64] καὶ ἀνωτέρω Theorem 3. 5).

Φασματικῶς κυλινδροειδεῖς ἄλγεβραι μὲ «πεπερασμένον πλῆθος (τοπολογικῶν) γεννητόρων» ἔχουν ἰδιαιτέραν σημασίαν διὰ τὰς ἐφαρμογάς, ἐν συνδυασμῷ μὲ τὴν θεωρίαν τῶν ἀναλυτικῶν συναρτήσεων περισσοτέρων μιγαδικῶν μεταβλητῶν (πρβλ. ἀνωτέρω Corollary 4. 2). Τοῦτο εἶχεν ἤδη προηγουμένως ἀποδειχθῆ διὰ τὰς m - κυλινδροειδεῖς ἀλγέβρας [37], ἐπέκτασις ἐν προκειμένῳ ἑνὸς παλαιοτέρου ἀποτελέσματος διὰ «πεπερασμένως παραγομένας» ἀλγέβρας Fréchet [10]. Σχετικῶς, εἶναι τὸ θέμα τοῦ δευτέρου μέρους τῆς παρούσης μελέτης, ἡ λεπτομερὴς ἀνάλυσις τῶν ἐν λόγῳ ἐφαρμογῶν ἀναφέρεται εἰς *ὁμοτοπικὰς ἀναλλοιώτους* τοῦ φάσματος τοπολογικῶν ἀλγεβρῶν, αἱ ὁποῖαι ἐν προκειμένῳ εἶναι *φασματικῶς κυλινδροειδεῖς* ἢ *ὡρισμένα* κατάλληλα *παράγωγα τούτων*, καθὼς καὶ εἰς ἐφαρμογὰς τῆς θεωρίας *«πινάκων ἐξαρτωμένων συνεχῶς ἀπὸ παραμέτρους»*. Μελετῶνται ἐπίσης θέματα παραστάσεως τοπολογικῶν ἀλγεβρῶν μέσῳ ἀλγεβρῶν, τῶν ὁποίων τὰ στοιχεῖα εἶναι συνεχεῖς τομαὶ καταλλήλων χώρων τοπολογικῶν ἀλγεβρῶν (: sectional representations), ὑπὸ τὴν ἔννοιαν τῶν **[35]**, **[36]** (πρβλ. ἐπίσης **[21]**).

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