MECHANICS.— **The influence of the form of inclusion on the sign of Poisson's ratio of fiber composites**, by *Pericles S. Theocaris*, National Academy of Athens, *G. E. Stavroulakis*, Technical University of Crete, and *P. D. Panagiotopoulos**, Aristotle University of Thessaloniki, Greece.

ABSTRACT

Materials with specific microstructural characteristics and composite structures are able to exhibit negative Poisson's ratio. This result has been proved for continuum materials by analytical methods in previous works of the first author, among others[1]. *Furthermore, it has been shown to be also valid for certain mechanisms, composites with voids, and frameworks, and has been recently verified for micro-structures optimally designed by the homogenization approach. For microstructures composed of beams it has been postulated that non-convex shapes (with re-entrant corners) are responsible for this effect* [2]. *In this paper it is numerically shown that mainly the shape of the re-entrant corner (non-convex, star-shaped) microstructure does influence the apparent (phenomenological) Poisson's ratio. The same is valid for continua with voids, or for composites with irregular shapes of inclusions, even if the individual constituents are quite usual materials. Elements of the numerical homogenization theory are reviewed and used for the numerical investigation.*

1. INTRODUCTION

Composite materials present usually a certain non-homogeneous and isotropic microstructue. Only on the macroscale it is possible to accept these materials as quasi-homogeneous and according to the case and isotropic. By using the method of optimal topology design by numerical homogenization [1,2] a choice of the appropriate quantities for the constituents of the microstructure was achieved on the respective characteristic unit-cell. The origin of this micro-macro approach can be traced back to the modelling of elactic

* Π. ΘΕΟΧΑΡΗΣ, Γ. Ε. ΣΤΑΥΡΟΥΛΑΚΗΣ, Π. Δ. ΠΑΝΑΓΙΩΤΟΠΟΥΛΟΣ.— **Ἡ ἐπιρροὴ τῆς μορφῆς τοῦ ἐγκλείσματος εἰς τὸ πρόσημο τοῦ λόγου Poisson σὲ ἰνοπλισμένα σύνθετα ὑλικά.**

framework in continuous structures [3,4]. By this procedure the overall elastic moduli of the anisotropic structure can be evaluated through the stiffnesses of the members composing the unit-cell walls.

Applying these ideas it is further possible to explain the appearance of negative Poisson's ratios and their effects on the behaviour of the structure [3-7]. It has been recently shown that, by an appropriate selection of the anisotropic properties of the material, or the structure, and especially, by varying its values of Poisson's ratios, a beneficial effect on their strength characteristics and particularly by a reduction of the stress concentration factors due to geometric discontinuities of the structures can be achieved [9-12]. In particular, the advantages of using materials with negative Poisson's ratio have been already appreciated since they permit, among others, a reduction of the stress concentration factors and the production of layered composite panels and beams, which allow for smooth treatment by cold metal forming processes [13,14].

In this paper ways for designing materials with negative Poisson's ratios will be indicated, based on configurations of arrays of inclusions with polygonal shapes with re-entrant corners. These non-convex two-dimensional cellular microstructures, where the inclusions are made of a material of lower moduli than the moduli of the matrix of the structure, are convenient to create composites with negative Poisson's ratios of different amounts, depending on the ratio of the moduli of the constituents of the composite, as well as on the shape of the inclusions. It has been shown in this paper, by applying methods of numerical analysis, that, while the choice of the material properties of the individual constituents of the composite does not influence significantly this effect, the shape of the star-shaped microinclusions is mainly responsible for this phenomenon.

2. TRUE BOUNDS OF POISSON'S RATIOS IN ANISOTROPIC BODIES

The positiveness of the stiffness **C**, and the compliance **S**, tensors in anisotropic materials imposed by thermodynamic principles, based on the fact that the elastic potential should remain always a positive quantity, and the potitive definiteness of these two tensors for any anisotropic material implies that the following four eigenvalues of the minimum polynomial for **S** to be expressed by [15]:

$$\lambda_1 = \frac{1+\nu_{23}}{E_{23}} = \frac{1}{2G_{23}} \qquad \lambda_2 = \frac{1}{2G_{12}} = \frac{1}{2G_{13}}$$

$$\lambda_3, \lambda_4 = \frac{(1-\nu_{23})}{2E_{23}} + \frac{1}{2E_{12}} \pm \left\{ \left(\frac{1-\nu_{23}}{2E_{23}} - \frac{1}{2E_{12}} \right)^2 + \frac{2\nu_{12}^2}{2E_{12}^2} \right\}^{1/2} \tag{1}$$

The above values for the four roots of the minimum polynomial for **S** are simplified expressions for the transversely isotropic body where the 2, 3-principal directions correspond to the transverse plane of symmetry of the material, so that $E_{12} = E_{13}$ and $\nu_{12} = \nu_{13}$. From the above relationships (1) and for positiveness of E_{23} and G_{23} one finds easily for the components of Poisson's ratios ν_{23} and $\nu_{12} = \nu_{13}$ the expressions [16,17]:

$$(\nu_{23}) < 1 \quad \text{and} \quad |\nu_{12} = \nu_{13}| < \left(\frac{(1-\nu_{23})\, E_{12}}{2E_{23}} \right)^{1/2} \tag{2}$$

It should be pointed out and emphasized thet positiveness of the elastic potential is guaranteed only when both above inequalities hold, fact which has been sometimes overlooked in the literature and has led to inaccurate conclusions [17]. Then, for orthotropic solids the following system of relations must hold [17]:

$$|\nu_{12}| < \left(\frac{E_{11}}{E_{22}} \right)^{1/2}, \quad |\nu_{23}| < \left(\frac{E_{22}}{E_{33}} \right)^{1/2}, \quad |\nu_{13}| < \left(\frac{E_{11}}{E_{33}} \right)^{1/2} \tag{3}$$

and

$$2\nu_{12}\nu_{23}\nu_{13} \frac{E_{33}}{E_{11}} < \left(1 - \nu_{12}^2 \frac{E_{22}}{E_{11}} - \nu_{23}^2 \frac{E_{33}}{E_{22}} - \nu_{13}^2 \frac{E_{33}}{E_{11}} \right) \tag{4}$$

For the transversely isotropic body these relations reduce to the simpler ones:

$$|\nu_{12}| = |\nu_{13}| < \left(\frac{E_{11}}{E_{22}} \right)^{1/2}, \quad |\nu_{23}| < 1 \tag{5}$$

and

$$\nu_{12}^2 \nu_{23} < \left[(1-\nu_{23}^2) \frac{E_{11}}{2E_{22}} - \nu_{12} \right] \tag{6}$$

It can be readily derived from these relations that the inequalities (4) or (6) are more restrictive and severe than the respective inequalities (3) or (5) and therefore they are the relationships which should be considered for evaluating limits of variation of Poisson's ratios in composites. Application of these relationships may then protect the researcher from admitting excessive bounds for this important mechanical property (see for example the excessive value for $\nu_{23}=1.97$ given in ref. [**17**] for the transverse Poisson's ratio of some particular composite).

For the isotropic elastic materials the bounds of Poisson's ratio values are reduced to the well known limits varying between -1.0 and $+0.5$. The right-side limit corresponds to incompressible materials with the rubbery materials and especially polymers approaching this limit. The negative values for Poisson's ratio appear in special substances and especially those presenting weak values for the bulk modulus and strong values for their respective shear modulus. The lower limit of the negative unit is an extreme value, which may be achieved only in very special structures of substances. In all other cases the possibility of the appearance of a negative Poisson's ratio, at least in one direction of loading, is not excluded from the theory of general anisotropic elasticity. Instances of this effect will be reviewed in this section [**18,19**].

Thus, single crystals with a polygonal structure at the atomic level are reported to have negative Poisson's ratio along some directions of loading. Such materials are reported to be the cadmium [**20**], the single crystal of pyrite [**21**] and the lattice structured pyrolytic graphite [**22**]. On the other hand, thermomechanically treated low density open-cell thermoplastic polymeric foams are materials, which eventually exhibit negative Poisson's ratio. It is of interest to remark that such materials are usually porous and have a spongy nature, with a lot of voids and a complicated microstructure. From the microstructural picture of the latter materials, which exhibits non totally convex cells, containing also cells with re-entrant corners, a number of micro-structures and mechanisms have been proposed for an explanation and the study of this effect [**13**, **23-25**]. However, these examples are not actually materials which can be found in normally, in applications but, as manufacturing technology and micromechanics attain a higher level of development, the possibility of constructing materials with this microstructures as prototypes grows continuously. On the other hand, it should be remarked that almost all structures in living creatures are practically composed by a combination of such materials.

Cellular microstructures composed of beams have been used with success for the modelling of linear and nonlinear elastic properties of two-dimensional and three-dimensional cellular materials or honeycombs; the results correlated well with experimental measurements [**3**,**4**,**7**]. It should be noted that experience gathered up-to-now indicates that all usual materials and composites with positive values for Poisson's ratio should be formed from units containing exclusively or predominantly convex cells, wheres foamy materials with very high porosity with non-convex cells presenting re-entrant corners, are convenient to create substances with negative Poisson's ratio [**3**, **14**, **23**].

The importance of creating materials with negative Poisson's ratio has been recognized with respect to modern structural analysis applications especially in the aerospace industry. It was recognized that, these materials should normally have a very high shear modulus relatively to their respective bulk modulus. This is appreciated, if the material is used in a sheet or beam form, as it is actually the case in most structural applications, where materials having a high shear modulus than a high bulk modulus are beneficial [**14**]. Moreover, the deformation patterns of elastic structures made of this kind of materials generally differ from the ones made of classical materials (see ref. [**13**] for a detailed description). This latter effect requires a new way of thought for the design of structural elements of structures, but at the same time, opens new possibilities for applications. For example, a sandwich panel or beam with core made of this new material will exhibit a dome-like double curvature on flexure, fact which allows an improved cold metal-forming treatment for the production of shells from initially plane panels, thus reducing the stress concentration factors which, in turn, enchance the crack and fatigue strength of structures.

3. A NUMERICAL HOMOGENIZATION METHOD FOR ADAPTING NEGATIVE EFFECTIVE POISSON'S RATIOS

From a series of experimental results on foams with re-entrant corner cells (e.g.[**23**]) and from the relevant results by applying the numerical homogenization theory (e.g. [**2**]), it can be shown that we may construct microstructures with an adjustable mechanical behaviour, which exhibit positive or negative Poisson's ratio. For the study of the overall mechanical properties of these materials we assume that they are periodic, i.e. the same microstructural pattern is repeated for the whole area of a structure. We assume moreover that the overall mechanical behaviour of the material can be described by the classical elasticity relations. In this framework the homogenization problem is posed as follows: *find the elasticity constants of the continuous model which lead to the same mechanical behaviour as the one of the material with the periodic microstructure.*

To this end a detailed analysis of a representative material cell is performed and a best fit method is followed, as it will be shown in the numerical examples later-on in this paper. The possibility to adjust the overall mechanical properties by changing either the geometric, or the material properties of the microstructure constitutes the inverse (optimal) design problem: *find a microstructure for which the material has a given (or optimal in some sense) mechanical behaviour.*

Let us assume a representative unit-cell of the periodic structure, which for simplicity is considered to be two-dimensional (see Fig. 1). Let the unit cell be orthogonal with dimensions equal to L_1 and L_2 along the two coordinate axes and let it occupy the area Ω with boundary Γ. The boundary is composed of the complementary and non-overlapping parts Γ_1. Γ_2. Γ'_1 and Γ'_2 (i.e. $\Gamma_1 . \cup \Gamma_2 \cup \Gamma'_1 \cup \Gamma'_2 = \Gamma, \Gamma_1 \cap \Gamma_2 = \varnothing$ etc). A unit cell of the real structure (case II in Fig. 1) and a unit cell with the same dimensions of the sought homogeneous structure (case I in Fig. 1) are considered. The cells I and II are subjected to the three unit prestresses:

$$
\begin{aligned}
&\text{case (1):} \quad && \sigma_1 = 1, \quad && \sigma_2 = 0, \quad && \sigma_3 = \tau_{12} = \tau_{21} = 0, \\
&\text{case (2):} \quad && \sigma_1 = 0, \quad && \sigma_2 = 1, \quad && \sigma_3 = \tau_{12} = \tau_{21} = 0 \\
&\text{case (3):} \quad && \sigma_1 = 0, \quad && \sigma_2 = 0, \quad && \sigma_3 = \tau_{12} = \tau_{21} = 1
\end{aligned}
\tag{7}
$$

as it is shown in Fig. 1.

The solution of cell I for these load cases can be based on simple engineering mechanics relations, due to the assumption that the dimensions of the periodic cell are small with respect to the dimensions of the structure.

For the cell II a finite-element method is employed for the solution of the above static analysis problems. Moreover, the following periodicity restraints, which result from technical mechanics considerations, are taken into account (as multipoint constraints) in the above described problems:

-For problems 1 and 2 displacements on boundaries Γ_1, Γ'_1 along the horizontal direction 1 are the same,

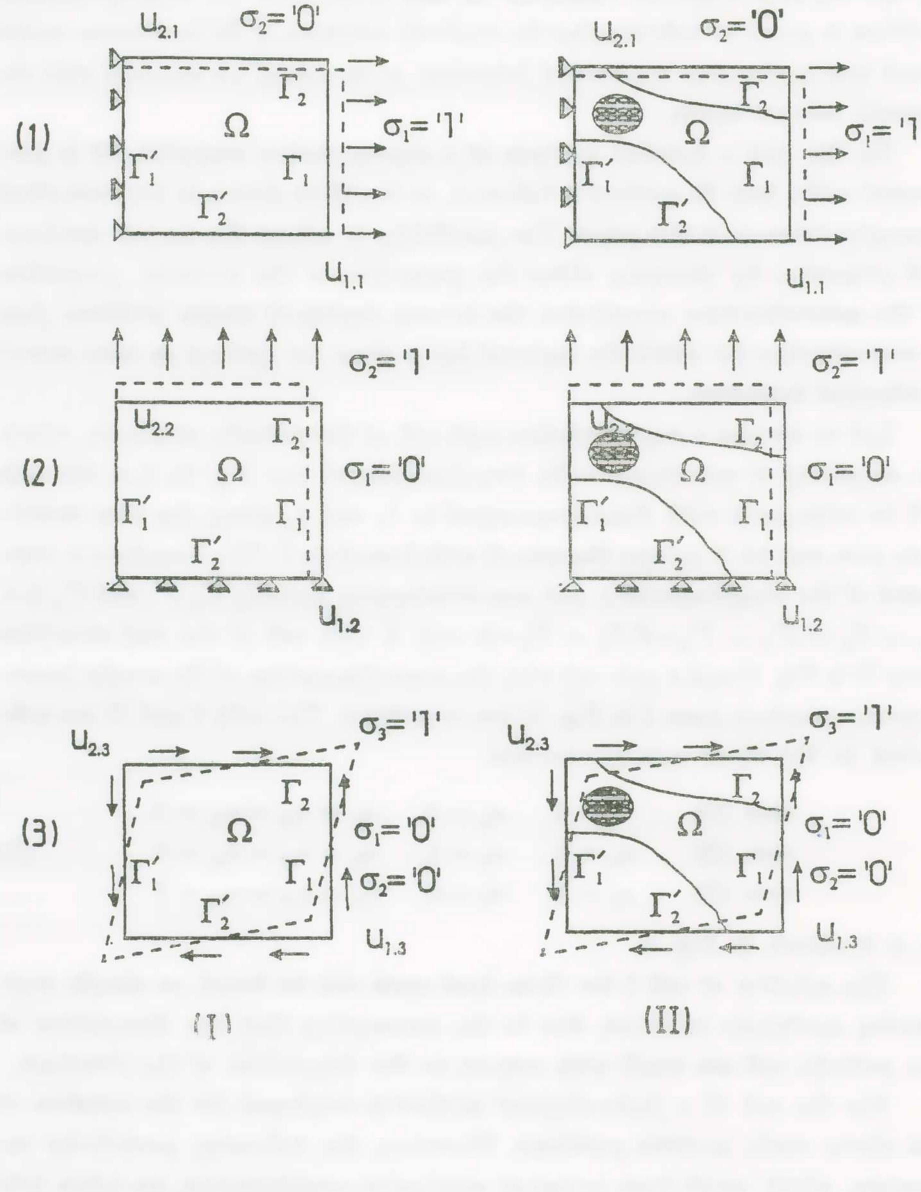

Fig. 1. Elements of the numerical homogenization technique for a unit-cell. Case (I) the homogeneous cell. case (II) the real structure cell. Problems (1) (2) and (3) indicate loading modes of the respective cells.

-For problems 1 and 2 displacements of boundaries Γ_2, Γ'_2 along the vertical direction 2 are the same, and

-For problem 3, boundaries Γ_1. Γ'_1. Γ_2 and Γ'_2 remain straight lines after deformation.

The essence of the energy-based numerical homogenization method is that the parameters of the homogeneous cell I are appropriately chosen, so that it has the same deformation energy with the cell of the real structure (cell II), if both are subjected to the same deformation patterns, which should respect the periodicity assumptions, i.e. they are periodic for the whole structure.

If the parameters, which define the mechanical behaviour of the cell I (e.g. the elasticity constants) are gathered up in the design vector $\boldsymbol{\alpha}$, the numerical homogenization method can be described by the following identification problem:

Find $\boldsymbol{\alpha}$ *as a solution of the optimization problem*:

$$\min_{\alpha \in A_{ad}} \frac{1}{2} \sum_{i=1}^{3} w_i \left\{ \prod_{in}^{I(i)} (e^{(i)},\alpha) - \prod_{in}^{II(i)} (e^{(i)}) \right\}^2 \tag{8}$$

Here A_{ad} is the admissible set for the material parameters of the homogenized cell, i runs over all independent periodic deformation patterns $e(^i)$, which are considered, w_i are appropriate weights, which tranform the multi-objective optimization problem into a classical one, with a cost function as in (8), superscript I or II stands for the quantities of cell I or II respectively and Π_{in} is the internal energy of the considered structure.

The identification problem (8) can be solved, either by classical numerical optimization techiques, or by neural-network based methods, as presented in ref. [**26**]. Here we use a simple procedure, which is based on the optimality criteria method for the solution of a certain class of problems (8).This method avoids the formulation and the solution of large scale optimization problems, and if it can be used, it is considered to be suitable for structural analysis applications [**1**].

Let us assume for simplicity here that all w_i's are equal to one. Moreover we assume that the homogenized unit cell I obeys the classical isotropic elasticity relations, i.e. we have that [**17**].

$$e = \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{Bmatrix} \frac{1}{E} & -\frac{\nu}{E} & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & 0 \\ 0 & 0 & \frac{2(1-\nu)}{E} \end{Bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = K_0\,\sigma \tag{9}$$

The design vector $\boldsymbol{\alpha}$ is chosen as: $\boldsymbol{\alpha} = [\alpha_1, \alpha_2]^T = [1/E, -\nu/E]^T$.

The internal energy is expressed by dΩ for all $i = I, II$, $j = 1,2,3$, where Ω is the area of the considered cell. For simplicity we assume here that $A_{ad} = R^2$.

Under the above assumptions problem (8) reads:

$$\min_{\alpha \in R^2} \left\{ \frac{1}{2}\int_\Omega \left\{ \begin{matrix} (\sigma^{I(1)T}(\alpha)e^{I(1)}(\alpha) - \sigma^{II(1)T}e^{II(1)})^2 + (\sigma^{I(2)T}(\alpha)e^{I(2)}(\alpha) - \sigma^{II(2)T}e^{II(2)})^2 + \\ + (\sigma^{I(3)T}(\alpha)e^{I(3)}(\alpha) - \sigma^{II(3)T}e^{II(3)})^2 \end{matrix} \right\} d\Omega \right\} \tag{10}$$

For the assumed unit stresses (7) and the elasticity relations (9) we get for the unit cell I that:

$$\begin{aligned} &e_1^{I(1)} = \alpha_1\sigma_1^{I(1)} = \alpha_1 && e_2^{I(1)} = \alpha_2\sigma_1^{I(1)} = \alpha_2 \\ &e_1^{I(2)} = \alpha_2\sigma_2^{I(2)} = \alpha_2 && e_2^{I(2)} = \alpha_1\sigma_2^{I(2)} = \alpha_1 \\ &e_3^{I(3)} = 2(\alpha_1 - \alpha_2)\sigma_3^{I(3)} = 2(\alpha_1 - \alpha_2) \end{aligned} \tag{11}$$

with all other components equal to zero.

Relations (11) and (7) written for the cell I are used in (10).

Moreover, the virtual work equality for the cell II reads:

$$\int_\Omega \sigma^{II(j)T} e^{II(j)}\, d\Omega = \int_\Omega S^{II(j)T} u^{II(j)}\, d\Gamma, \qquad j = 1, 2, 3 \tag{12}$$

for all given unit stresses of (7) (i.e. $S^{II(1)} = 1$ on Γ_1, $S^{II(1)} = 0$ on Γ_2, Γ'_2 etc).

Finally, the optimality conditions for (10) are written by means of (11). «*Find* a_1, a_2, *such that:*

$$\int_\Omega \left(\alpha_1 - \sigma^{II(2)T} e^{II(2)}\right) d\Omega \, \frac{\partial \alpha_1}{\partial \alpha_1} + \int_\Omega \left(2(\alpha_1 - \alpha_2) - \sigma^{II(3)T} e^{II(3)} \right) d\Omega \, \frac{\partial \{2(\alpha_1 - \alpha_2)\}}{\partial \alpha_2} = 0 \tag{13}$$

$$\int_\Omega \left(2\,(\alpha_1 - \alpha_2) - \sigma^{II(3)T} e^{II(3)} \right) d\Omega \, \frac{\partial \{2\,(\alpha_1 - \alpha_2)\}}{\partial \alpha_2} = 0 \tag{14}$$

By using (12), the area-integrals are transformed into boundary integrals. Thus, we get the following optimality conditions:

«*Find a_1, a_2 such that:*

$$\left(\alpha_1 \ell_1 \ell_2 - u_1^{II(1)} \ell_2 \right) 1 + \left(\alpha_1 \ell_1 \ell_2 - u_2^{II(2)} \ell_1 \right) 1 + \left(2\,(\alpha_1 - \alpha_2) \ell_1 \ell_2 - \int_\Omega \sigma^{II(3)T} e^{II(3)} \right) 2 = 0 \tag{15}$$

$$\left(2(\alpha_1 - \alpha_2)\, \ell_1 \ell_2 - \int_\Omega \sigma^{II(3)T} e^{II(3)} \right) (-2) = 0 \tag{16}$$

Variable a_1 (the elastic modulus E) results from (14) and (15):

$$\alpha_1 = \frac{u_1^{II(1)} \ell_2 + u_2^{II(2)} \ell_1}{2\, l_1 l_2} \tag{17}$$

Variable a_2 (the Poisson ratio ν) may now be calculated either from (16), or from the elasticity relations (11), which have been assumed to hold true.

Analogous relations can be extracted for the more general case, where the homogeneous model I is assumed to obey the orthotropic elasticity relations, or to general anisotropic elasticity relations [**2**].

4. THE INVERSE PROBLEM OF DEFINING A MATERIAL WITH GIVEN HOMOGENIZED ELASTIC CONSTANTS

The aim of this chapter is to formulate and implement a procedure to define linear elastic materials with prescribed constitutive parameters and presenting a periodic micro-structure, such as fiber composites. Such materials are prone to be defined for their macroscopic behaviour by effective average elastic constants, through an analysis of the micro-structure represented by unit representative cells. Then, the inverse homogenization problem can be formulated analogously to the direct problem of the previous section. We make here the same assumptions and we consider again the cells I and II and the unit stresses (1) $\div$ (3) of Fig. 1.

Now the «homogeneous» cell I is given, i.e. relations (9) are valid and the elastic constants are known and constitute the goal of the optimal design problem. On the other hand, the *real* cell II may now be modified by means of a certain number of design parameters, which are summed up in the design-vector $\boldsymbol{\beta}$. For instance, either elasticity constants of various constituents in a composite structure, or the shape of the inclusions in a reinforced composite, or the type and the shape of the microstructure, may be considered as design variables, by an appropriate choice of the elements of vector $\boldsymbol{\beta}$.

By an analogous reasoning to the one used in the previous section, the optimal design problem reads (cf. (8)):

Find $\boldsymbol{\beta}$ *as a solution of the optimization problem*:

$$\min_{\boldsymbol{\beta} \in B_{ad}} \frac{1}{2} \sum_{i=1}^{3} w_i \left\{ \prod_{in}^{I(i)} (e^{(i)}) - \prod_{in}^{II(i)} (e^{(i)}, \boldsymbol{\beta}) \right\}^2 \tag{18}$$

Here B_{ad} is the admissible set for the design variables $\boldsymbol{\beta}$ and all other quantities are defined after problem (8).

As with problem (8), problem (18) can be solved by means of various methods. A detailed presentation of the solution of this homogenization problem is not undertaken here in this paper, since the method is well established and known. The reader may consult refs. [2], [27] and [28] among others, for analogous recent studies.

Since in the inverse problem it is asked to construct materials with designated properties, it is expected that a number of differently composed bodies may exhibit the same mechanical behaviour. Then, it is chosen for a practical stand-point the goal to construct the simplest material with the given parameters, thus solving an optimization problem, whose cost function must be minimized. If this cost function should be the weight of the structure, then the constraints are expressed by the constitutive parameters to be satisfied and the design variables should define the composition and the topology of the body.

Since the composite materials are periodic structures, they are described by a representative unit-cell, which constitutes the smallest repetitive unit of material, and a calculation of the effective moduli of the substance can be obtained by analyzing only the unit-cell. Considering that the typical composite is a complicated microstructure, an analytic approach for the determination of the properties of the material is rather impossible, and, therefore, a finite-element based numerical method is better suitable, due to its simplicity. Here we are using the homogenization procedure in terms of element mutual energies, which renders the inverse problem better suited for optimization. Then, the optimization problem is formulated as a multiple load minimum weight problem and solved by a modified version of the optimality criterion method proposed in ref. [29].

Since with fiber reinforced materials we are concerned with the general constitutive laws in two dimensional linear elasticity, we consider a case of a particular type of microstructure consisting of a star-shaped inclusion with re-entrant corners, as it is indicated in Fig. 2, and it is related with materials

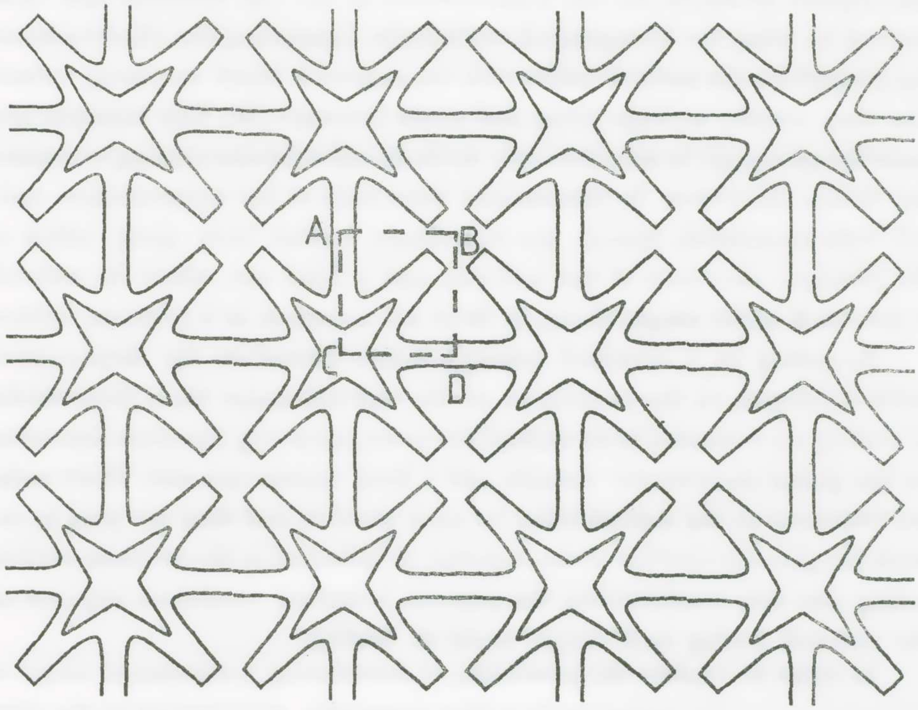

Fig. 2. A periodic fiber-reinforced composite with star-shaped encapsulated inclusions.

of a specific microstructure, which can be modelled by means a truss-like cell. The principal analogue of this example comes from a foamed porous material. Indeed, the truss structure may be a continuum with holes, with the provision for an analytic solution of the problem that none of the holes does intersect the cell boundaries. However, this constraint may be relaxed for the case of a solution based on numerical analysis, provided that the appropriate boundary conditions of the examined cases were conveniently defined. Then, the homogenization relationships can be solved by a finite element approach, for which the individual bars in the truss-like cell are considered as continuum elements, with two modes disposing only of a certain longitudinal stiffness and zero-shear stiffness. In this way the same software, which is used in finding the homogenized coefficients for the truss-like structure, yields also the continuum-like material. Figure 3 presents a periodic composite material with star-shaped inclusions convenient for developing negative Poisson's ratio.

The described asymptotic homogenization procedure provides rigorous convergence estimates for the displacements of the real structure and those derived by using the homogenized coefficients. Concerning the effective material properties, the method tallies with the approach based on energy principles that, employ average stress and strain theorems [**30**]. This techique presents the adventage to use effectively methods and solutions existing for trusses and similar structures. In this method three tests of the representative unit-cell were considered, namely the two simple tension tests along either of the principal directions of the unit-cell, and a third one, where the unit-cell is deformed under simple shear, as these are described in a previous section.

According to a standard homogenization procedure the displacement fields developed on the boundaries of the unit-cell under these three modes of loading are expanded in an asymptotic series, involving functions depending on the global macroscopic variable and a local microscopic one. These series are truncated to the desired order for each problem and they are used to express the global properties of the material, as indicated in the previous section, taking also into consideration the periodic boundary conditions imposed on the unit-cell during each simple mode of loading.

In order to explore the possibility of introducing a convenient shape of cross-sections of the inclusions in a fiber composite, contributing to the creation of a negative value for the transverse Poisson's ratio of the composite, we examine the case of the truss-like structure under the form of a convex star, created by a number of beams and rods, whose principal analogy derives from open foam and porous materials. It is indeed anticipated that, in order that a porous material presents negative Poisson's ratio, its porosity should be rather high and the material should be classed in the open-foam materials. We start our investigation with the convex shaped-beam cell of Fig. 2.

The microstructure of the material produced by this cell is schematically shown in Fig. 3. By using the numerical homogenization concepts of section four we model the unit-cell of Fig. 2 by means of two-dimensional beam finite elements, we consider fixed-end boundary conditions (support) at point 1 and a unit load in the horizontal direction applied at point 7. For the above-described cell, with geometric dimensions as in Fig. 2, we assume that the beams have a cross-section equal to unity, a moment of inertia equal to 1000, and they are made of an elastic material with elastic modulus equal to E=1000. For various values of the shear modulus G and for a shear factor equal to 0.3 (resp. to 0.9) the (phenomenological) elastic modulus E, and Poisson's ratio, ν, as they are calculated by the numerical homogenization theory, are plotted in Figs. 4a and b respectively, for low values of the shear modulus G of the structure varying between G=100 and G=1000. For higher values of the shear modulus G, varying between G=1000 and G=10^4, the variation of E and ν is plotted in Figs. 5a, b respectively.

From the above results a negative Poisson's ratio effect is clearly demonstrated. One should nevertheless underline here that the above parametric investigation is extrapolated outside the range of mechanically admissible values for the material constants, in order to give a better visualization of the sought dependence between Poisson's ratio and structural constants for a given cell geometry. In fact, a value of G=333.30 corresponds to a beam material with ν=0.5, which leads to a Poisson's ratio for the microstructure equal to -0.2815 for a material with shear factor equal to 0.3, whereas, for a material with shear factor equal to 0.9, the respective value for Poisson's ratio is equal to -0.1538 for the low range of variation of G ($100 \leqq G \leqq 1000$). However, a value G=1000 corresponds to a beam material with ν= -0.5, which leads to a Poisson's ratio for the microstructure equal to -0,1524 for a low shear factor 0.3 and equal to ν=-0.0120 for a high shear factor. These results indicate that the shape and the geometry of the microstructure and not the material constants of its elements are mainly responsible for a negative Poisson's ratio.

The influence of the shape of the inclusions will be studied subsequently. For this purpose we consider five different shapes and orientations of inclusions, whose forms and orientation are indicated in Fig. 6. Indeed, from the shape of a square cell type of Fig. 3 with four re-entrant sides, whose angles

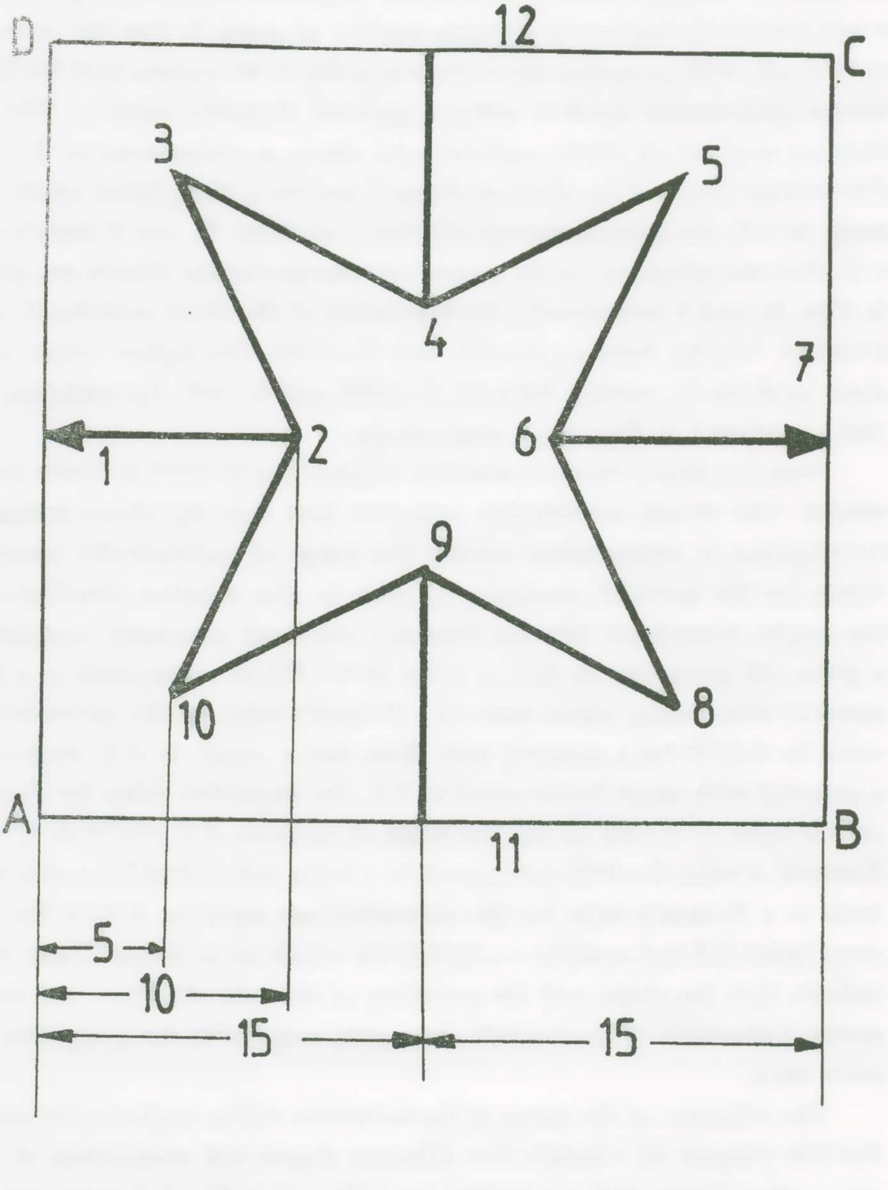

Fig. 3. A star-shaped two-dimensional beam-like cell with re-entrant corners simulating the unit cell of Fig. 2. Finite element discretization and mode numbering.

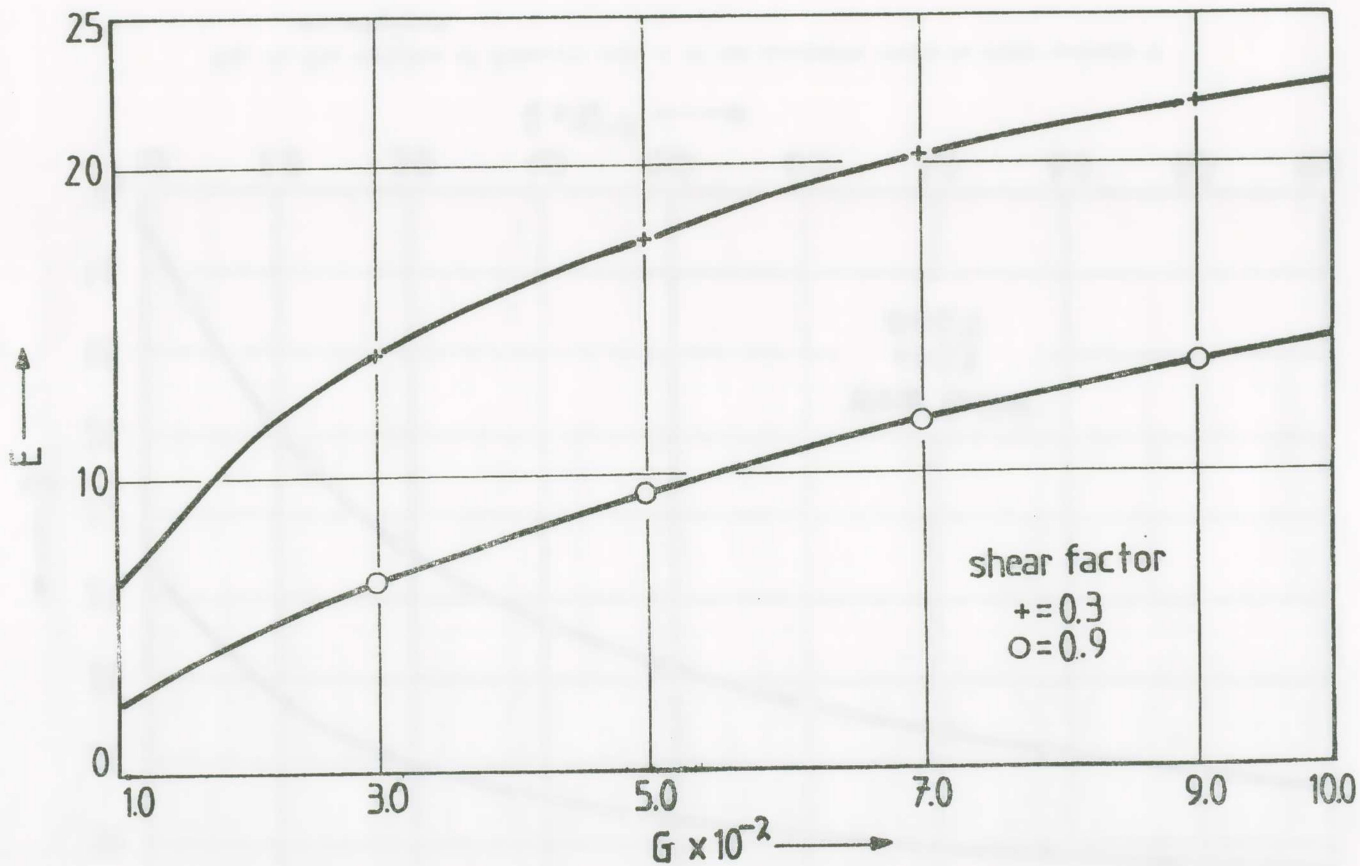

Fig. 4a. The variation of the elastic modulus E of the composite versus its shear modulus G for 100〈G〈1000

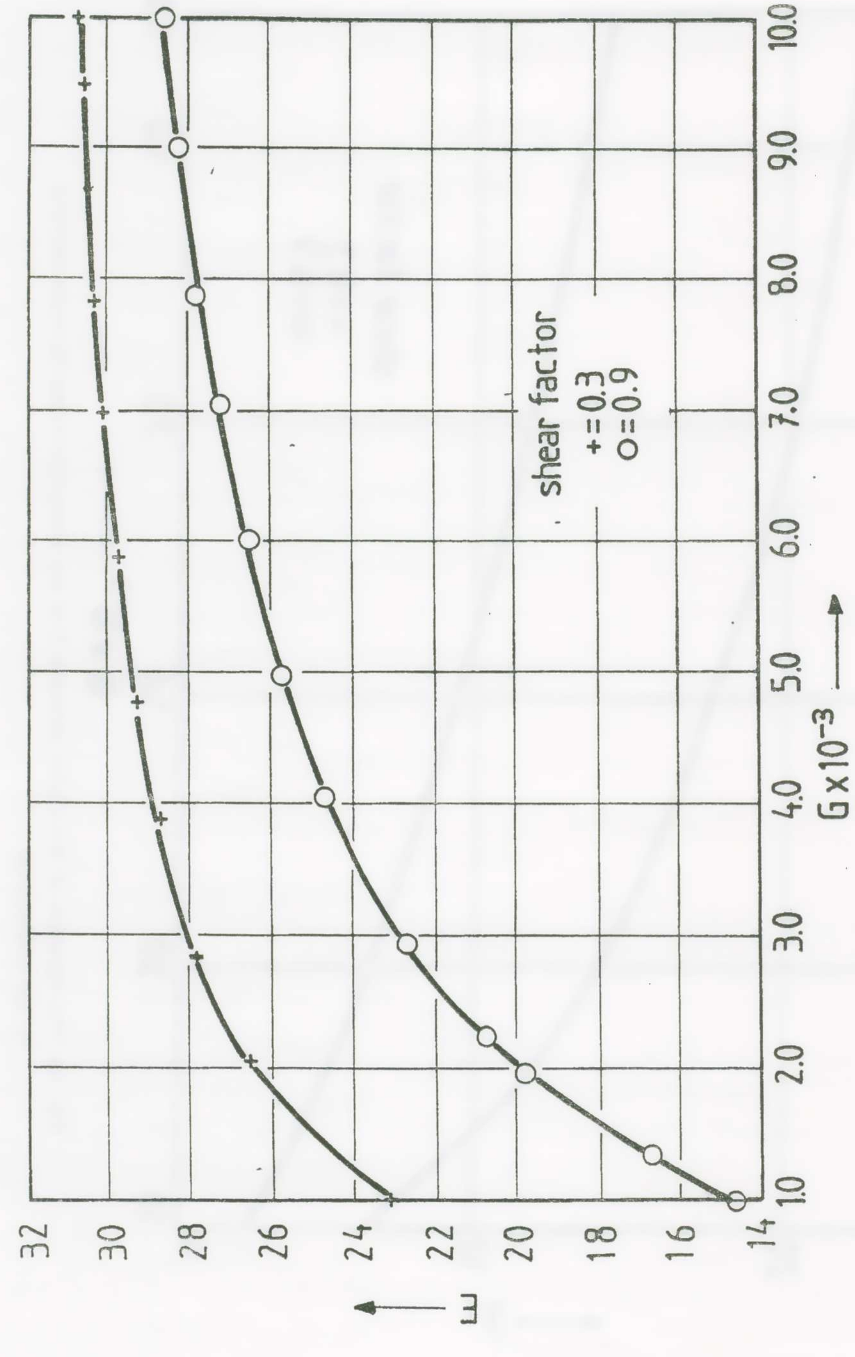

Fig. 4b. The variation of Poisson's ratio ν of the composite versus its shear modulus G for $10^3 < G < 10^4$

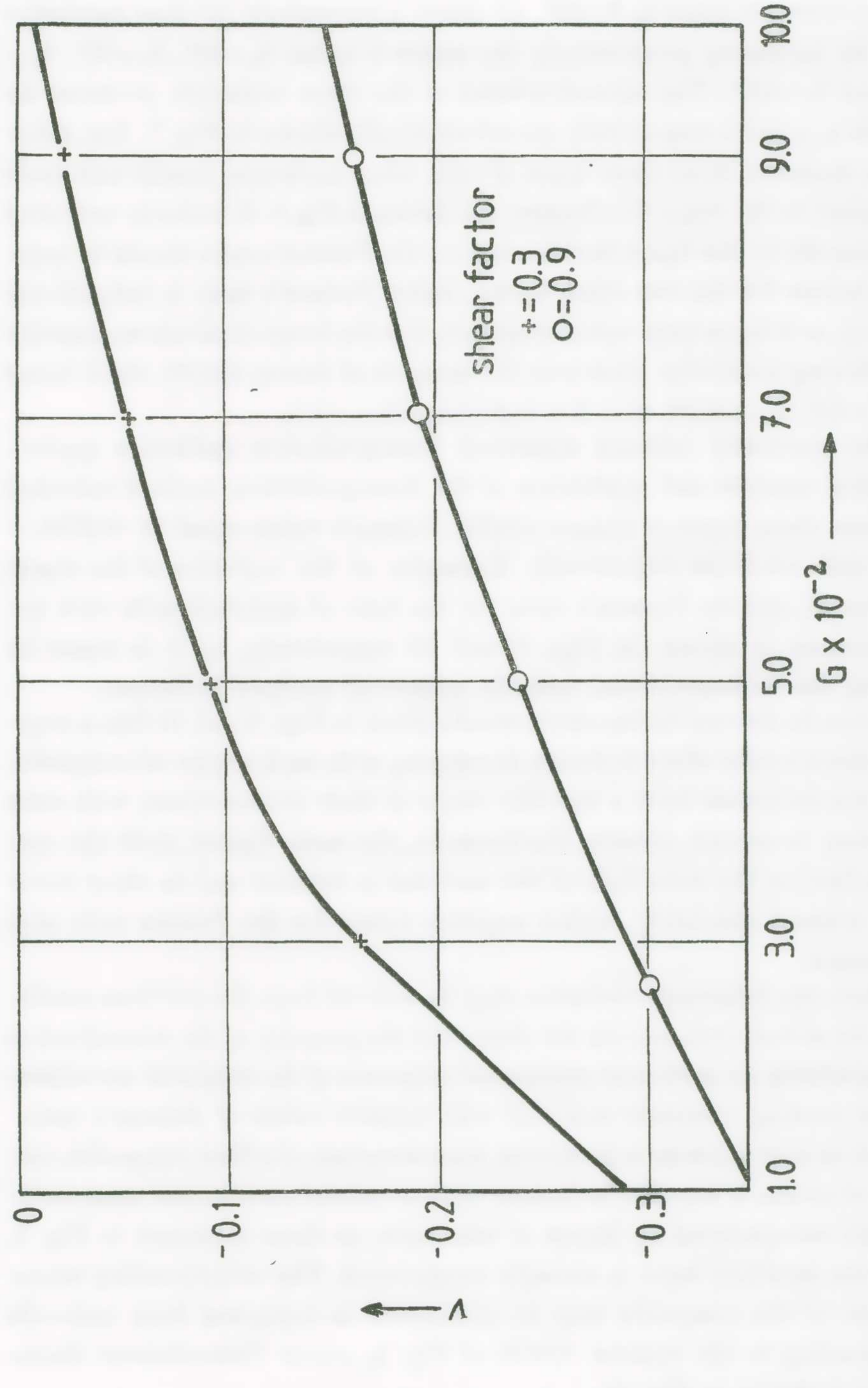

Fig. 5a. The variation of Poisson's ratio ν of the composite versus its shear modulus G for $100<G<1000$.

at its corners are equal to $\theta = 36^{o}$, we create progressively the four successive forms by increasing progressively the angles θ to be: $\theta_b = 61^{o}$, $\theta_c = 90^{o}$, $\theta_d = 134^{o}$ and $\theta_c = 180^{o}$. The microstructures of the three materials produced by the cells a, c and e respectively are schematically shown in Fig. 7. The deformation modes of these three types of cells, when horizontal tensile unit-loads are applied to the respective frames, are shown in Fig. 8. It is clearly indicated schematically in this figure that for case (a) the Poisson's ratio should be negative, whereas for the two other modes, either Poisson's ratio is insignificant (case (c)), or it takes large values (case (e)). For the beam-elements we consider the following constants: cross-area 50, moment of interia 416.66, shear factor 0.9, $E = 10^6$, $G = 333.30$, that is a material with $\nu = 0.5$.

The previously outlined numerical homogenization method is applied. Numerical analysis and application of the homogenization method indicated that these three types of frames exhibit Poisson's ratios equal to -0,2715.+02928 and +0.40134 respectively. Examples of the variation of the elastic modulus, E, and the Poisson's ratio for the type of materials with such microstructures is shown in Figs. 9 and 10 respectively, as it is found by applying the homogenization and the numerical analysis technique.

It can be derived for the above results given in Figs. 9 and 10 that a negative Poisson's ratio effect is clearly developing with such a type of composite, where the inclusions have a star-like shape of their cross-sections, with sides containing re-entrant corners. Furthermore, the same figures yield the conclusion that, as the ratio G/E of the material is reduced and its shear factor is also reduced absolutely, higher negative values for the Poisson ratio may be attained.

Then, the following conclusion may be derived from the previous results. It can be stated that: *mainly the shape and the geometry of the microstructure and secondarily the particular mechanical properties of the composite are responsible for creating composite materials with negative values of Poisson's ratio.*

Let us now examine a particular microstructure of a fiber composite, consisting of arrays of star-like inclusions with re-entrant corners and these inclusions are encapsulated by layers of interfaces, as those indicated in Fig. 2, where the interface layer is strongly exaggerated. The corresponding microstructure of the composite may be considered as composed from unit-cells corresponding to the squares ABCD of Fig. 2, whose finite-element discretization is shown in Fig. 11.

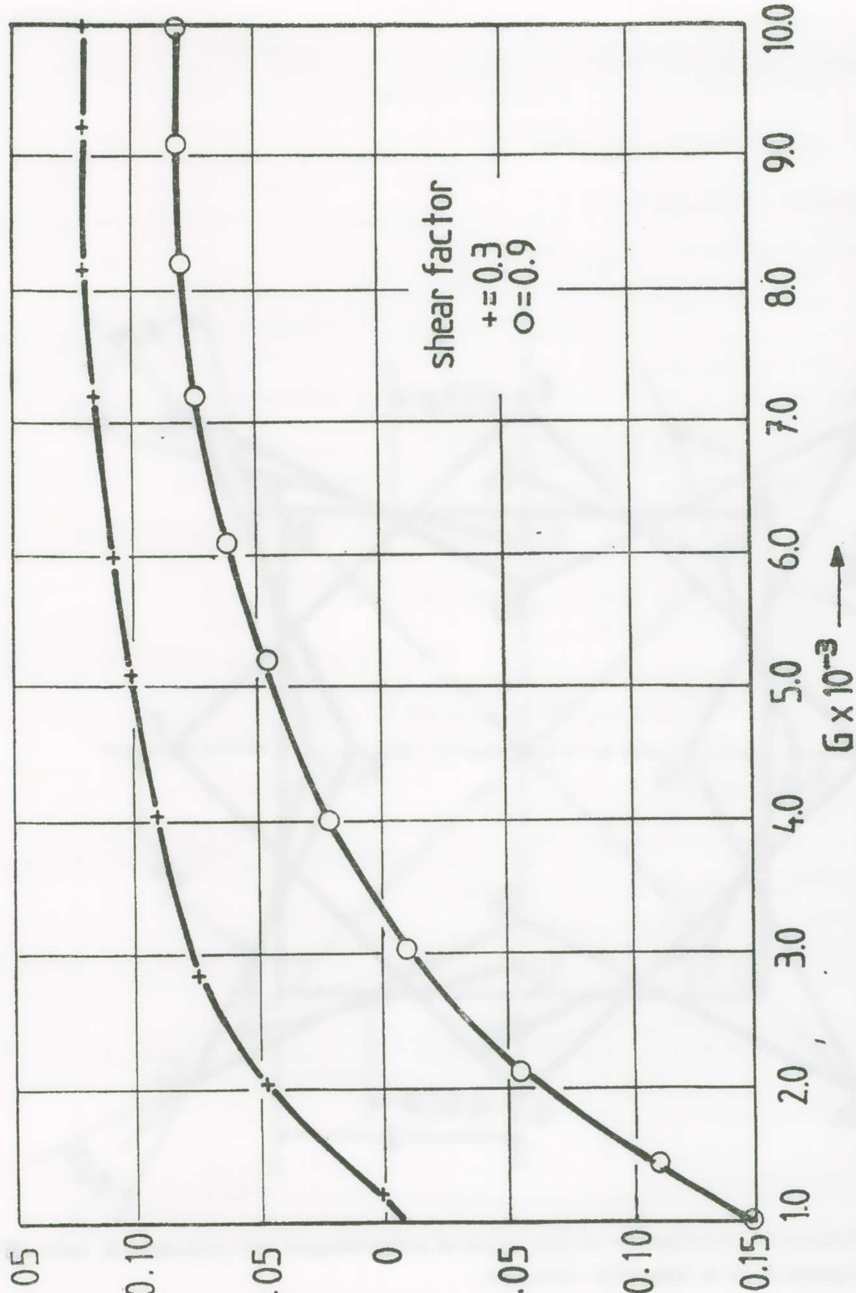

Fig. 5b. The variation of Poisson's ratio ν of the composite versus its shear modulus G and for $10^3 < G < 10^4$.

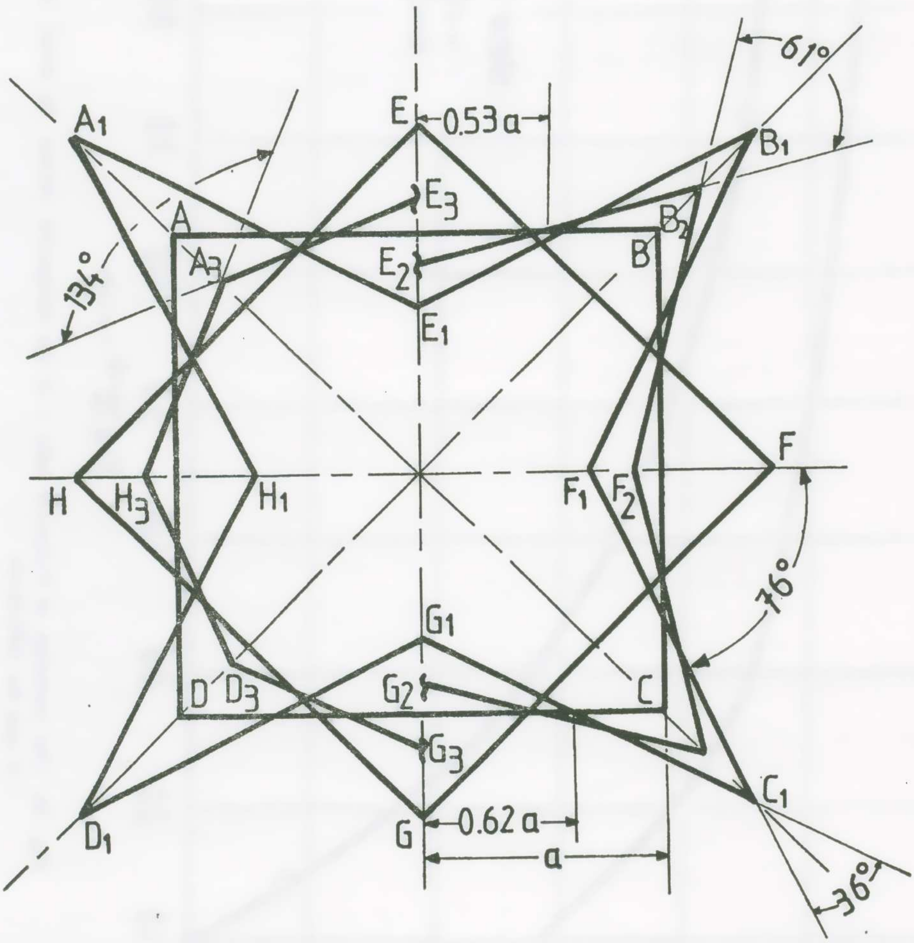

Fig. 6. Parametric investigation of five types of a star-shaped two dimensional unit cell expressed as a truss-like structure.

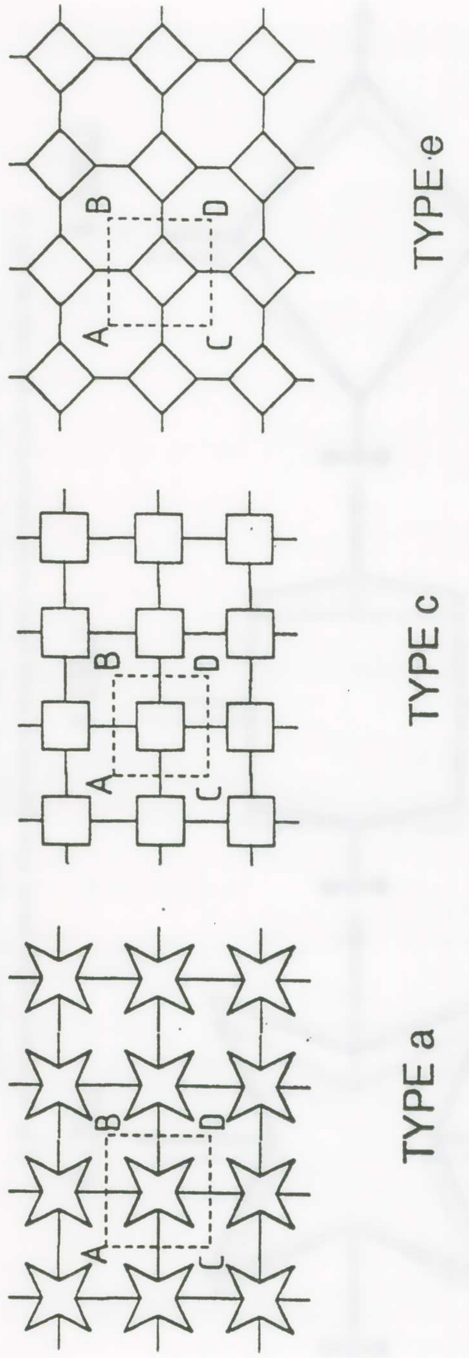

Fig. 7. Three types of microstructures produced by the unit-cells (a), (c) and (e) of Fig. 6.

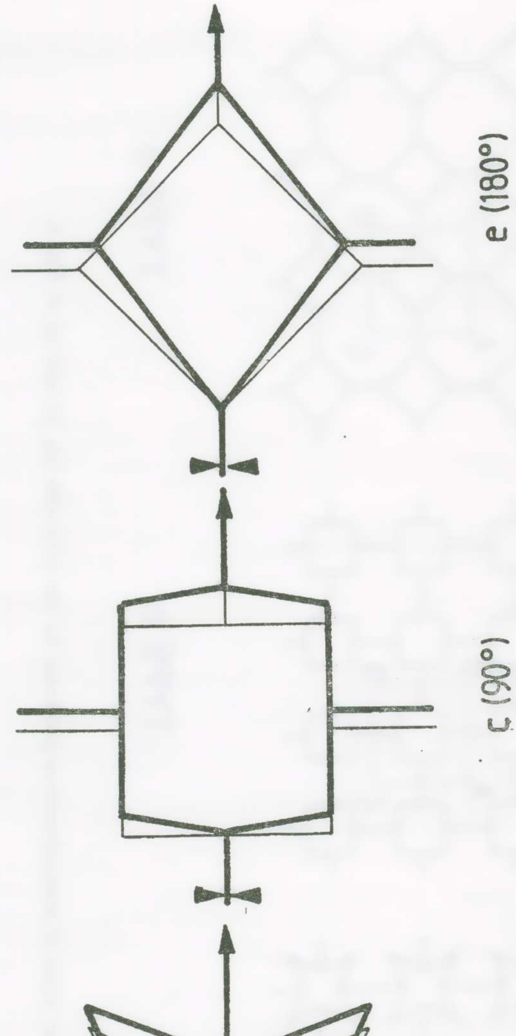

Fig. 8. Initial (thin lines) and deformed (solid lines) configurations for the cells of Fig. 7. The effect of negative, near zero, and positive Poisson's ratio are shown.

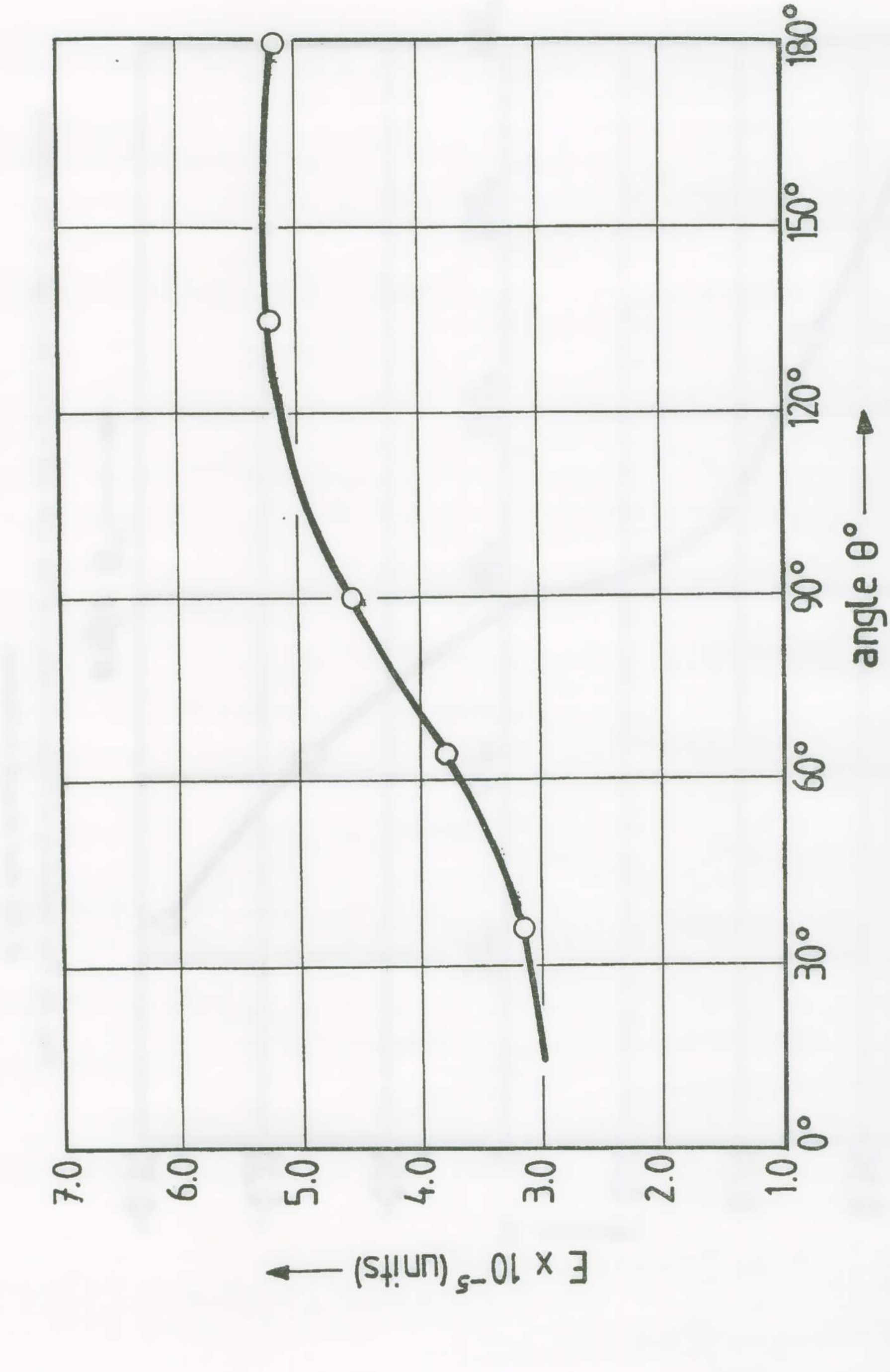

Fig. 9. The variation of the elastic modulus E of the unit cell versus the angle of the corners of the star shaped inclusions.

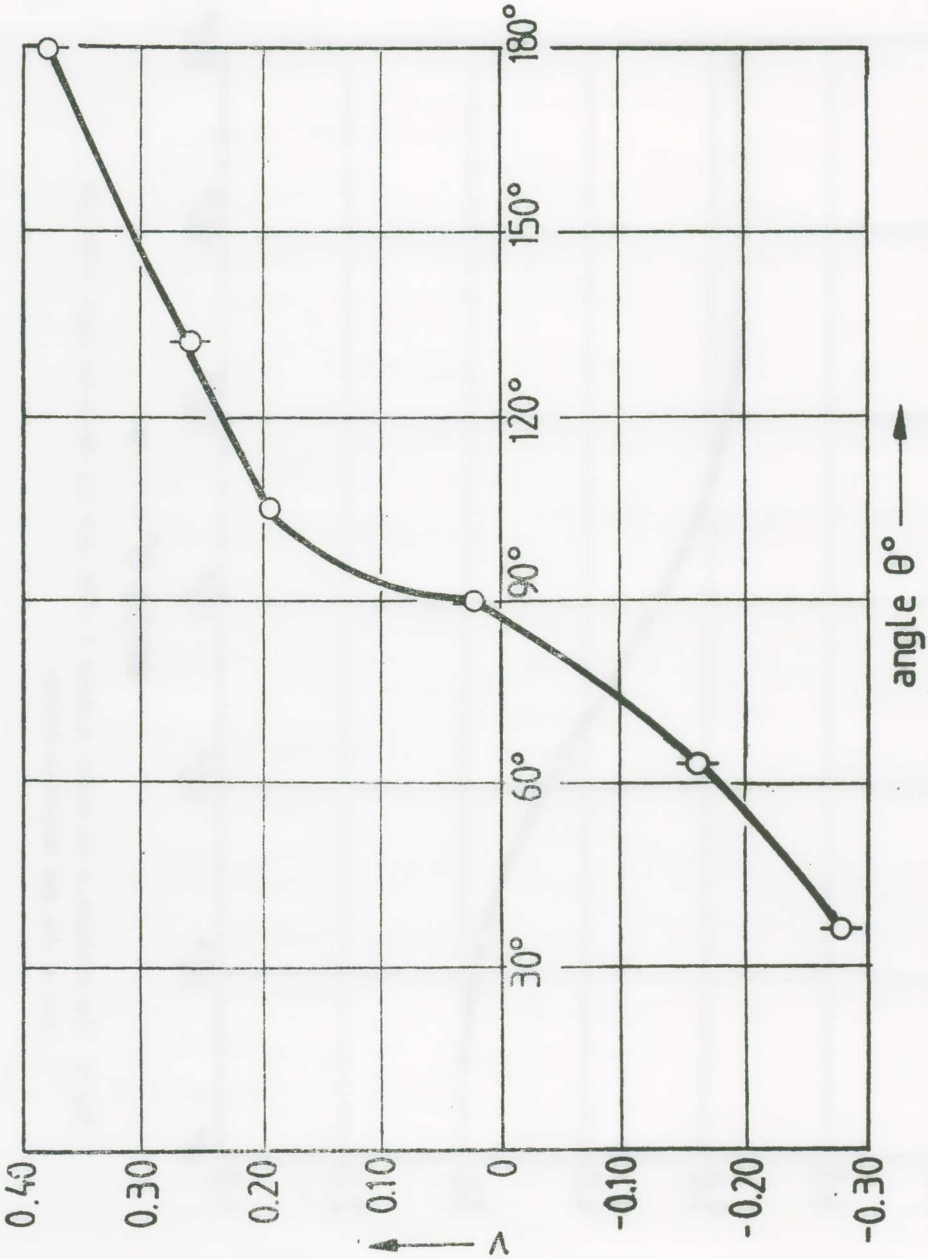

Fig. 10. The variation of Poisson's ratio ν of the unit cell v ersus the angle of the corners of the star shaped inclusions.

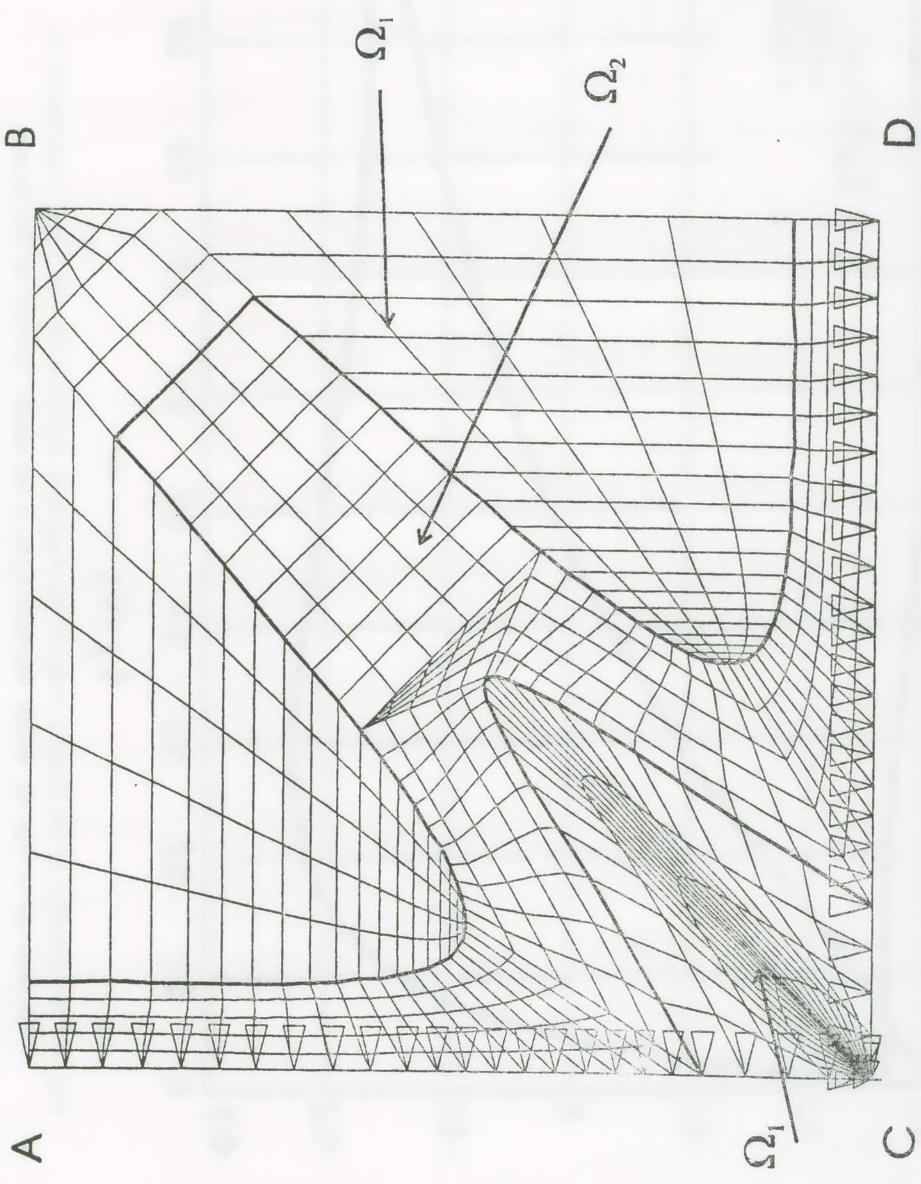

Fig. 11. Finite element model for the analyzed unit cell of Fig. 2.

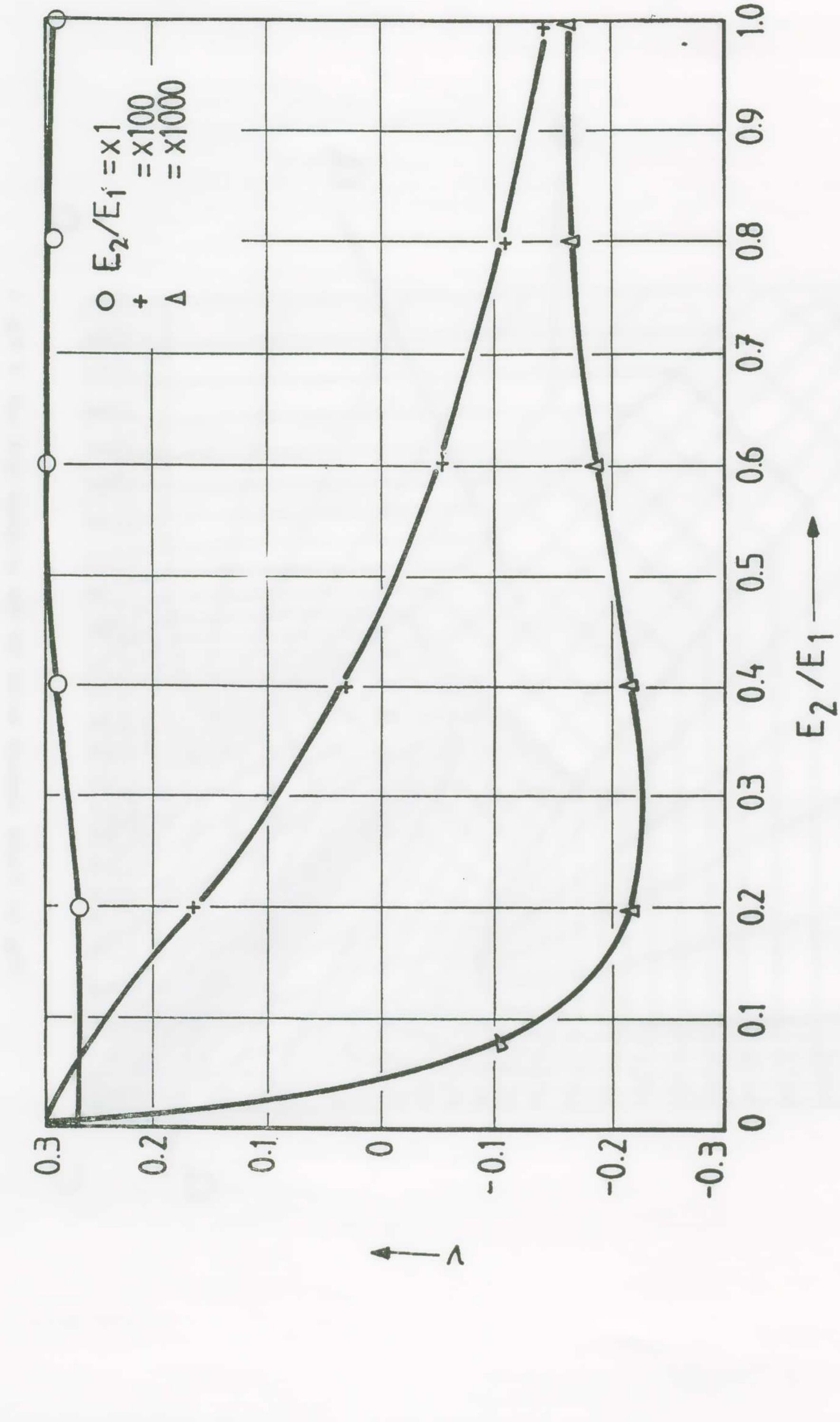

Fig. 12. The variation of the Poisson ratio of the composite indicated by Fig. 2 and represented by the unit-cell of Fig. 11, versus the ratio E_1/E_2 of the elastic modulis of the phases.

For the respective isotropic material, which occupies the region Ω_1 (matrix) of the composite, we consider an elastic modulus $E_1 = 100$ and Poisson's ratio $\nu_1 = 0.3$. For the material of the region Ω_2, that is the reinforcement of the composite, we consider $\nu_2 = 0.30$ and several values for E_2, from a weak material with $E_2 = 10$ to a very strong one with $E_2 = 10^5$. The dependence of Poisson's ratio of the composite from the ratio E_2/E_1 is presented in Fig. 12, where it is clear that for low values of the ratio E_2/E_1 (lower than $E_2/E_1 < 46$) the Poisson ratio of the structure remains positive and above this limit this material parameter becomes negative.

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ΠΕΡΙΛΗΨΗ

Ἡ ἐπιρροὴ τῆς μορφῆς τοῦ ἐγκλείσματος εἰς τὸ πρόσημο τοῦ λόγου Poisson σὲ ἰνο-πλισμένα σύνθετα ὑλικὰ

Ὑλικὰ μὲ ἰδιάζοντα χαρακτηριστικὰ μικροδομῆς καὶ σύνθετες κατασκευὲς μποροῦν νὰ ἐμφανίσουν ἀρνητικὸν λόγον Poisson, ὅπως ἔχει ἀποδειχθεῖ μὲ ἀναλυτικὲς μεθόδους γιὰ συνεχῆ ὑλικά. Τὸ φαινόμενο αὐτὸ ἐμφανίζεται καὶ σὲ μηχανισμούς, σὲ πορώδη ὑλικὰ καὶ σὲ πλαισιακὲς (μικρο)-κατασκευὲς καὶ ἔχει προσφάτως ἐπιβεβαιωθεῖ καὶ σὲ βέλτιστα σχεδιασμένες μικροκατασκευὲς μὲ τὴν βοήθεια τῆς μεθόδου τῆς ὁμογενοποιήσεως.

Μὲ χρήση τῶν μεθόδων τοῦ βελτίστου τοπολογικοῦ σχεδιασμοῦ καὶ μὲ χρήση ἀριθμητικῆς ὁμογενοποιήσεως, μπορεῖ νὰ καθορισθεῖ ἡ ἐπιλογὴ τῶν καταλλήλων ποσοτήτων τῶν συστατικῶν ὑλικῶν καὶ ἡ ὁλικὴ ἐλαστικὴ μηχανικὴ συμπεριφορὰ συνθέτου κατασκευῆς μὲ λεπτομερῆ ἀνάλυση τοῦ ἀντιπροσωπευτικοῦ κελύφους της. Ἡ διαδικασία αὐτὴ μπορεῖ νὰ χρησιμοποιηθεῖ καὶ γιὰ τὴν μελέτη τῆς ἐμφανίσεως ἀρνητικοῦ λόγου Poisson σὲ σύνθετα ὑλικὰ καὶ τὴν ἐπιρροὴ τῶν διαφόρων παραμέτρων σχεδιασμοῦ τοῦ συνθέτου ὑλικοῦ ἐπὶ τοῦ ἐξεταζομένου φαινομένου.

Γιὰ μικροκατασκευὲς ἀποτελούμενες ἀπὸ δοκοὺς ἔχει γίνει ἡ ὑπόθεση ὅτι μὴ κυρτὰ σχήματα (μὲ εἰσερχόμενες γωνίες) γιὰ τὰ ἐγκλωβίσματα ἢ τοὺς πόρους προκαλοῦν τὸ φαινόμενο ποὺ μελετᾶται. Εἰς τὴν παροῦσα ἐργασία ἀποδεικνύεται ἀριθμητικῶς ὅτι κυρίως τὸ σχῆμα τῆς (μὴ κυρτῆς, ἀστεροειδοῦς) μικροδομῆς μὲ εἰσερχόμενες γωνίες ἐπηρεάζει τὸν φαινόμενον λόγον Poisson. Τὸ αὐτὸ ἰσχύει καὶ γιὰ πορώδη ὑλικὰ ἢ γιὰ σύνθετα ὑλικὰ μὲ ἀκανόνιστα σχήματα ἐγκλωβισμάτων, ἀκόμη καὶ ἂν καθένα συστατικό τους ἐμφανίζει τελείως κλασικὴ μηχανικὴ ἀπόκριση. Στοιχεῖα ἀπὸ τὴν θεωρία τῆς ἀριθμητικῆς ὁμογενοποιήσεως, ὅπως χρησιμοποιεῖται εἰς τὴν ἀνακοίνωση παρουσιάζονται ἐν συντομίᾳ καὶ χρησιμοποιοῦνται γιὰ τὴν ἀριθμητικὴ διερεύνηση ποὺ παρατίθεται καὶ ἐρευνᾶται στὴν παροῦσα ἐργασία.