ΦΥΣΙΚΗ.— **Matter Condensation in Nuclei and Black Holes,** *by C. Syros**. Ἀνεκοινώθη ὑπὸ τοῦ Ἀκαδημαϊκοῦ κ. Κ. Ἀλεξοπούλου.

In a number of recent papers Sawyer [**1**, **4**], Canuto and Chitre [**2**] and Migdal [**3**] have discussed the conditions and the possible forms of the matter condensation. The discussion given by Sawyer and Scalopino [**5**] is based on a field theoretic model for the charged pions interacting with the nucleon field via a σ_3^- coupling, while Migdal derives his conclusions from a critical condition for the π^- condensation in a neutron gas. Since, as Sawyer pointed out, a strong assumption was made by Migdal [**3**] concerning the Fermi energy and the chemical potential of the pion, and since the proper polarization in the propagator depends on the assumptions made, it appears worth examining the condensation problem from a quite different point of view.

The present discussion will be concerned with the condensation of particles having integral or half-integral spin and any mass. As a check of the validity of our conclusions the limiting cases of the Bose-Einstein and the Fermi-Dirac distributions will be derived from the non-equilibrium distributions given here. We shall take the point of view of statistical mechanics and we shall consider the N-particle Liouville equation $(N \leqslant \infty)$ in which the particles interact via spin-dependent but otherwise constant forces.

Two systems of particular interest will be considered here: statistical properties of the atomic nucleus (finite density) and the conditions under which black holes (infinite density) may be formed as a result of the Einstein condensation.

In doing so we assumed that the system is contained in a space region, R_1^{3N}, of finite volume, V^{3N}. The nuclear forces acting on each particle is given by

$$\mathbf{F}^{(n)} = \mathbf{a}^{(n)} + \mathbf{b}^{(n)}(\sigma_n), \tag{1}$$

where $\mathbf{a}^{(n)}$ is the spin-independent force and $\mathbf{b}^{(n)}(\sigma_n)$ is any vector dependent only on the spin.

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* Κ. ΣΥΡΟΥ, Συμπύκνωσις τῆς ὕλης εἰς πυρῆνας καὶ μελανὰς ὀπάς.

On the complement, R_0^{3N}, of the 3N-dimensional co-ordinate space, R^{3N}, the forces are assumed to vanish identically.

If $\{\mathbf{p}^{(n)}, \mathbf{q}^{(n)} \mid n = 1, 2, \ldots N\}$, with $\mathbf{p}^{(n)} \in P^{3N}$ and $\mathbf{q}^{(n)} \in R^{3N}$, are the linear momenta and the co-ordinates respectively, where P^{3N} is the 3N-dimensional momentum space, then the Liouville equation, $\mathscr{L}f = 0$, reads explicitly

$$\{\partial_t + \sum_{n=1}^{N} [\mathbf{p}^{(n)} \cdot \nabla^{(n)} + \mathbf{F}^{(n)} \cdot \nabla'^{(n)}]\} f = 0 \tag{2}$$

everywhere on $P^{3N} \otimes R_i^{3N}$. Similarly we have

$$[\partial_t + \sum_{n=1}^{N} \mathbf{p}^{(n)} \cdot \nabla^{(n)}] f_0 = 0 \tag{3}$$

everywhere on $P^{3N} \otimes R_0^{3N}$ $(R^{3N} = R_i^{3N} \cup R_0^{3N})$.

In both Eqs. (2) and (3) $\nabla^{(n)} \equiv \partial_{\mathbf{q}^{(n)}}$ and $\nabla'^{(n)} \equiv \partial_{\mathbf{p}^{(n)}}$.

It is found that an appropriate form for the distribution function satisfying Eq. (2) is given by

$$f_i = \bar{N} \cdot \exp(-s_j); \text{ on } R_i^{3N}, \ (j = 1, 2, 3) \tag{4}$$

where $\bar{N}$ is a normalisation constant and $s_j = s_j(\mathbf{p}^{(1)}, \ldots \mathbf{p}^{(N)}; \mathbf{q}^{(1)}, \ldots \mathbf{q}^{(N)}; t)$.

In particular we have the forms [**6**]

$$s_1 = \sum_{n=1}^{N} [\alpha \cdot \varepsilon_n \cdot t / \frac{h}{2\pi} - \mu_n \cdot \mathbf{F}^{(n)} \cdot \mathbf{q}^{(n)} + 1/2 \cdot \mu_n \cdot (\mathbf{p}^{(n)} - \nu_n \cdot \mathbf{F}^{(n)})^2] + \vartheta, \tag{5}$$

$$\equiv \sum_{n=1}^{N} (\psi_n^1 + \vartheta_n)$$

$$s_2 = \sum_{n=1}^{N} \mu_n [1/2 (\mathbf{p}^{(n)})^2 - \mathbf{F}^{(n)} \cdot \mathbf{q}^{(n)}] + \vartheta, \tag{6}$$

$$\equiv \sum_{n=1}^{N} (\psi_n^2 + \vartheta_n)$$

$$s_3 = \sum_{n=1}^{N} \varkappa_n \mathbf{p}^{(n)} \cdot \mathbf{F}^{(n)} \wedge \mathbf{q}^{(n)} + \vartheta, \tag{7}$$

$$\equiv \sum_{n=1}^{N} (\psi_n^3 + \vartheta_n),$$

where α, μ_n, ν_n, $\varkappa_n$ are given parameters and $\mathscr{E} = \sum_{n=1}^{N} \varepsilon_n$ is the total energy of the nucleus. ϑ i s a p h a s e of Eq. (4) and will be determined presently. Concerning the distribution function on R_0^{3N} we can either assume that $f \cong 0$, but in a more physical way we find from Eq. (3) the distribution

$$f_0 = \bar{N}_0 \cdot \exp(-r_j)\,;\ \text{on } R_0^N, \tag{8}$$

where

$$r_j = r_j(\mathbf{p}^{(1)}, \ldots \mathbf{p}^{(N)};\ \mathbf{q}^{(1)}, \ldots \mathbf{q}^{(N)};\ t)\,;\ \mathbf{q}^{(n)} \in R_0^N. \tag{9}$$

We give two appropriaté forms for r_j :

$$r_1 = 1 / \frac{h}{2\pi} \sum_{n=1}^{N} (\varepsilon_n \cdot t - \mathbf{p}^{(n)} \cdot \mathbf{q}^{(n)}) + \vartheta, \tag{10}$$

$$r_2 = \lambda_n \mathbf{J}_N \cdot \mathbf{q}^{(n)} \wedge \mathbf{p}^{(n)} + \vartheta \tag{10$'$}$$

where $\mathbf{J}_N$ is the total spin and the total energy of the nucleus $\mathscr{E}$ is conserved, i.e., $\mathscr{L}\,\mathscr{E} \equiv 0$. $\{\lambda_n\}$ are parameters.

Clearly we have on the boundary of R_i^{3N} for the distributions f_i and f_0 with respective arguments s_1 and r_1 the compatibility relations

$$\mathbf{p}^{(n)} \cdot \mathbf{q}_0^{(n)} = \mu_n \frac{h}{2\pi} / 2(\mathbf{p}^{(n)})^2;\ \mathbf{q}_0^{(n)} \in S^{3N} \tag{11}$$

and

$$\mathbf{p}^{(n)} = -\mu_n \frac{h}{2\pi} \mathbf{F}^{(n)}, \tag{12}$$

where S^{3N} is the boundary of R_i^{3N}.

Eq. (12) tells us that due to the absence of forces on R_0^{3N} the linear moments of the nucleons must be constant. On the other hand Eq. (11) implies that the nucleus has not the same boundary surface for two nucleons with different momenta (energies). The conditions given by Eqs. (11 - 12) imply the continuity of probability and of current on the boundary S^{3N} of the system. It is important for the existence of the normalisation integrals that s_j and r_j violate the causality principle at most on a subset of R^{3N} with finite measure. This requirement is satisfied, for example, if $(j = 1)$ $|\mathbf{q}^{(n)}| \cdot \cos\omega_n < |\mathbf{p}^{(n)}| \cdot t / 2m$, where $\omega_n = \{\text{angle of } (\mathbf{p}^{(n)}, \mathbf{q}^{(n)})\}$, and m is the mass of the corresponding particle. Similar conclusions can be seen to be true also for other values of j.

Next it is observed that for the above used boundary conditions to be satisfied α must be equal to -1. If however the boundary condition $f \equiv 0$ is applied on S^{3N}, then α may take an imaginary value. In this case the requirement of positivity for the distribution function implies that the product $\varepsilon . t$ occuring in s_1 takes values according to $\varepsilon . t = 2\pi \frac{h}{2\pi} \lambda$, where λ is any integer. From this requirement the Heiseberg uncertainty principle follows. Thus it is shown that, if $\Delta\varepsilon$ and Δt are non vanishing variations of ε and t, then the relations

$$\Delta\varepsilon . \Delta t = 2\pi \frac{h}{2\pi} [l - k - m - n] \quad \text{and} \quad \Delta\varepsilon . \Delta t = 2\pi \frac{h}{2\pi} \frac{m . n}{k}$$

are necessary and sufficient for the distribution function to be positive. Both of them imply the inequality $| \Delta\varepsilon . \Delta t | > 2\pi \frac{h}{2\pi}$.

We turn now our attention to the phase ϑ. To show its physical significance we calculate the conditional probability for a nucleon to be in any state belonging to a given set, while the entire nucleus is in a state belonging to another set of states. This probability is given by

$$\Pi \sim \Big(\sum_{\text{all phases}} \exp(-s_j) \Big) \cdot \Big(\sum_{m=1}^{M} c_m^n \exp\big(-m \cdot (\psi_n^j + \vartheta_n)\big) \Big), \tag{13}$$

where M is a positive integer ($M \leqslant \infty$).

We next parametrize ϑ_n in the form $\vartheta_n = \chi_n + i\,\varphi_n$ and require that

$$c_m^n = \exp(m \cdot \chi_n) .$$

Summing up the series in Eq. (13) and normalizing appropriatetly we get the new distribution function

$$f_i = \bar{N} \cdot \{\exp(\psi_n^j) - \exp(-i\varphi_n)\}^{-1}, \tag{14}$$

here it has been assumed that $M = \infty$ and the normalisation constant is given by

$$\bar{N} = \int \Pi' d^3 p^{(n)} d^3 q^{(n)} \Big(\sum_{\text{all phases}} e^{-s_j} \Big) . \tag{15}$$

The «′» in the integral of Eq. (15) excepts from the integration the co-ordinates of the n-th particle. Eq. (14) gives the distribution function for a nucleon in the nucleus of which the energy has no upper bound. In order that f_i as given by Eq. (14) be real, φ_n can only take values according to the equation $\varphi_n = 2\pi u_n$, where

$$u_n = \begin{cases} 0, 1, \ldots, \text{ or} & \text{(16a)} \\ 1/2, 3/2, \ldots & \text{(16b)} \end{cases}$$

In the first case, (16a), f_i represents a generalized Bose - Einstein distribution function, while in the case (16b) it represents a generalized Fermi-Dirac distribution function. We infer, therefore, that $u_n = \sigma_n$ and the phase ϑ is straightforwardly related to the particle spin. It is observed in passing that the Maxwell - Boltzmann distribution is also obtained from Eq. (14) if it is assumed that the spin of the particles is purely imaginary ($u_n = -i\infty$).

The non-reduced N-particle distribution is, for systems with energy not bounded from above, given by

$$f_i(\mathbf{p}^{(1)}, \dots \mathbf{p}^{(N)}; \mathbf{q}^{(1)}, \dots \mathbf{q}^{(N)}; t) = \bar{N}_\infty \cdot \left\{e^{s_j} - (-)^{2J_N}\right\}^{-1}, \tag{17}$$

where J_N is the total spin of the system and once more $M = \infty$.

We wish next to give the distribution function for the more realistic case in which the energy of the system is bounded from above by E. By appropriatetly accounting for this condition we obtain the distribution function

$$f_{iE}(\mathbf{p}^{(1)}, \dots \mathbf{p}^{(N)}; \mathbf{q}^{(1)}, \dots \mathbf{q}^{(N)}; t) = \bar{N}_E \cdot \frac{1 - (-)^{2.M.J_N} \cdot e^{-(M-1).s_j}}{e^{s_j} - (-)^{2J_N}}, \tag{18}$$

where $\epsilon \leqslant E$ and M takes values as follows:

$$M = \begin{cases} N_B & \text{Bosons} \\ N_F\,; & \text{Fermions.} \end{cases}$$

In Eq. (18) we have, since the nucleons have spin $\frac{1}{2}$, the equality

$$2J_N = \begin{cases} \text{even}\,; & \text{if } N \text{ is even} \\ \text{odd}\,; & \text{if } N \text{ is odd.} \end{cases} \tag{19}$$

Consequently, the nucleus behaves as a Fermion if A (A = N) is odd and as a Boson if A is even. In this statement which follows directly from Eq. (18) one recognizes the main rule of the nuclear shell model.

It is interesting to note that while Eq. (17) exhibits the Einstein condensation at $s_j = 0$, this is not so in Eq. (18) which for $s_j = 0$ gives ($M < \infty$):

$$f_i(0, \dots 0; 0, \dots 0; 0) = \bar{N}(J_N) \begin{cases} M - 1\,; & \text{Bosons} \\ \left.\begin{cases} 1\,; & M = \text{odd} \\ 0\,; & M = \text{even} \end{cases}\right\}; & \text{Fermions} \end{cases} \tag{20}$$

It follows therefore, that for bound systems with finite number of particles and with bounded energies ($M < \infty$) there is no Einstein condensation no matter whether the system as a whole is a Boson or a Fermion.

Due to the fact that s_j is not only energy-dependent as in the original Bose-Einstein distribution function, but also space-and time-dependent, the condensation, when it occurs, is not only in the momentum space, but also in the co-ordinate space and in the time. This fact may give some hint regarding the existence of black holes. In the framework of the present theory with constant forces — no matter how strong they are — no black holes can be formed, if the energies of the particles are bounded from above. Consequently, black holes must exist according to Eq. (17) for $s_j = 0$, only if infinite quantities of energy are available in the universe, and only if the elementary particles are exclusively Bosons. It may, therefore, be concluded that black holes are formed via some process transforming the nucleons into Bosons (gravitons) and they require an infinite amount of energy.

Finally, it is pointed out that a relationship exists between the Einstein condensation and the causality principle. It follows from Eq. (5) and Eq. (17) that the Einstein condensation takes place at the point $\{\mathbf{q}'^{(n)} = 0,\ \mathbf{p}'^{(n)} = \nu_n \cdot \mathbf{F}^{(n)} \mid n = 1, 2, \dots N\}$ at time $t = 0$. At a different point $\{\mathbf{q}''^{(n)}, \mathbf{p}''^{(n)} \mid n = 1, 2, \dots N\}$ the condensation takes place after a time Δt. The sign of this time depends on the relative directions of the forces $\mathbf{F}^{(n)}$ and the corresponding spatial displacements $\mathbf{q}''^{(n)}$. These displacements must have finite values, if the moments of the corresponding particles take finite values. The signs of the time, Δt, correspond to the advanced and to the retarded solutions of the Liouville equation.

ΠΕΡΙΛΗΨΙΣ

Τὸ πρόβλημα τῆς συμπυκνώσεως τῆς ὕλης διερευνᾶται ἀπὸ τῆς ἀπόψεως τῆς στατιστικῆς μηχανικῆς. Bose - Einstein καὶ Fermi - Dirac κατανομαὶ εὑρέθησαν περιγράφουσαι συστήματα N-σωματίων εἰς κατάστασιν μὴ ἰσορροπίας.

Φερμιόνια εἰς συνδεδεμένην κατάστασιν συμπυκνοῦνται μετὰ πεπερασμένης πυκνότητος καὶ συμπεριφέρονται ὡς πυρηνικὴ ὕλη περιγραφομένη ὑπὸ τοῦ μοντέλου τῶν φλοιῶν. Μποζόνια συμπυκνοῦνται εἰς τὴν ἐνέργειαν, εἰς χῶρον καὶ χρόνον μετὰ ἀπείρου πυκνότητος μόνον, ὅταν ὑπάρχῃ διαθέσιμος ἄπειρος ποσότης - ἐνεργείας. Αἱ μελαναὶ ὀπαὶ δύνανται νὰ θεωρηθοῦν ὡς συμπυκνώματα γραβιτονίων κατὰ Einstein.

SUMMARY

The problem of the matter condensation is discussed from the statistical mechanics point of view. From the N-particle Liouville equation non-equilibrium Bose-Einstein and Fermi-Dirac distributions are obtained. Fermions condense, if they are bound, to finite densities and behave according to the nuclear matter described by the shell model. Bosons condense in energy, space and time to give infinite densities only if infinite amounts of energy are available. Black holes may be viewed as Einstein condensates of gravitons.

REFERENCES

1. R. F. Sawyer, Phys. Rev. Lett. 29 (1972), 382.
2. V. Canuto and S. M. Chitre, Phys. Rev. Lett. 30 (1973), 999.
3. A. B. Migdal, Phys. Rev. Lett. 31 (1973), 257.
4. R. F. Sawyer, Phys. Rev. Lett. 31 (1973), 1556.
5. R. F. Sawyer and D J. Scalapino, Phys. Rev. D 7 (1973), 953.
6. C. Syros, Lett. Nuovo Cimento, Vol. 10, N 16 (1974), 718.

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Ἀκολούθως, λαβὼν τὸν λόγον ὁ Ἀκαδημαϊκὸς κ. **Κ. Ἀλεξόπουλος,** εἶπε τὰ ἑξῆς :

Κατὰ τὰ τελευταῖα ἔτη πολὺς λόγος γίνεται μεταξὺ τῶν Ἀστρονόμων καὶ τῶν Κοσμολόγων περὶ τῶν μελανῶν ὀπῶν εἰς τὸν κοσμικὸν χῶρον τοῦ διαστήματος.

Ἀπὸ φυσικῆς πλευρᾶς πρόκειται περὶ ἀντικειμένων πολὺ μεγάλης μάζης συγκεντρωμένης εἰς πρακτικῶς πολὺ μικρὸν ὄγκον. Ἐὰν ἡ ὕλη τοῦ ἀστρικοῦ διαστήματος πορεύεται πρὸς μεγάλας πυκνότητας, τότε ἡ μελανὴ ὀπὴ θὰ πρέπει νὰ ἀποτελῇ μίαν ἀπὸ τὰς τελευταίας βαθμίδας τῆς ἐξελίξεως τῆς ὕλης.

Αἱ μελαναὶ ὀπαὶ ἔχουν τὴν ἰδιότητα νὰ ἀπορροφοῦν πλήρως πᾶσαν φωτεινὴν ἀκτῖνα διερχομένην δι' αὐτῶν, ἐξ οὗ καὶ παρέχεται ἡ δυνατότης πιστοποιήσεως τῆς ὑπάρξεώς των.

Ἡ μεγάλη κοσμολογικὴ αὐτῶν σημασία ἔγκειται εἰς τὸ ὅτι ἐκτὸς τοῦ φωτὸς ἀπορροφοῦν καὶ πᾶν ὑλικὸν σωμάτιον, εἰσερχόμενον εἰς τὸ πεδίον ἕλξεως αὐτῶν.

Ὁ κ. Κωνσταντῖνος Σῦρος, θεωρητικὸς φυσικός, καὶ νῦν καθηγητὴς τοῦ Πανεπιστημίου Πατρῶν, ἀσχολεῖται ἀπὸ ἐτῶν μὲ προβλήματα στατιστικῆς συστημάτων ἀποτελουμένων ἀπὸ πολλὰ σωμάτια, πολλὰς δὲ ἐργασίας αὐτοῦ εἶχα τὴν τιμὴν νὰ ἀνακοινώσω εἰς τὴν Ἀκαδημίαν Ἀθηνῶν. Εἰς τὴν παροῦσαν ἐργασίαν ἀσχολεῖται μὲ ἀνωμαλίας, αἱ ὁποῖαι ἐμφανίζονται εἰς τὰς λύσεις τῶν ἐξισώσεων τῆς γενικῆς θεωρίας τῆς σχετικότητος, ὅταν αὕτη ἐφαρμόζεται εἰς τὰς μελανὰς ὀπάς.

Προφανῶς πρόκειται περὶ συστήματος ἐκ μεγάλου ἀριθμοῦ σωματίων, μὴ εὑρισκομένων ἐν ἰσορροπίᾳ, ἀλλὰ ἡ φύσις τῶν σωματίων δὲν εἶναι γνωστή. Ὁ κ. Σῦρος μελετᾷ δύο εἰδῶν σωμάτια, σωμάτια μὲ ἰδίαν περιστροφὴν — spin ὡς λέγονται — καὶ σωμάτια ἄνευ ἰδίας περιστροφῆς. Εἰς τὴν πρώτην περίπτωσιν ἡ μελέτη καταλήγει εἰς συσσωμάτωμα πυκνότητος ἴσης πρὸς τὴν πυκνότητα τοῦ ἀτομικοῦ πυρῆνος, ἐνῷ εἰς τὴν δευτέραν περίπτωσιν δίδει τὸ περίεργον ἀποτέλεσμα ὅτι ἡ τελικὴ μορφὴ θὰ ἔχει ἄπειρον πυκνότητα.