τευμα. Εἶχα τὴν εὐχάριστον εὐκαιρίαν νὰ διατρέξω τὰς σελίδας τοῦ βιβλίου τῶν ἀπομνημονευμάτων του καὶ εἶδα νὰ ἀπεικονίζεται εἰς κάθε σελίδα του ἡ συμπαθὴς μορφὴ τῆς ἠθικῆς του προσωπικότητος, εἶδα νὰ ἀποτυπώνεται πιστῶς ὁ ἰδιότυπος χαρακτὴρ τῆς πολεμικῆς του δράσεως. Εἶμαι καὶ ἐγὼ βέβαιος ὅτι τὸ παρουσιαζόμενον βιβλίον του θὰ συμβάλῃ σοβαρῶς εἰς τὴν διαλεύκανσιν καὶ τὴν ἀκριβῆ τελικὴν διατύπωσιν τῆς γενικῆς Ἱστορίας τοῦ Μικρασιατικοῦ πολεμικοῦ ἄθλου.

ΑΝΑΚΟΙΝΩΣΙΣ ΜΗ ΜΕΛΟΥΣ

ΗΛΕΚΤΡΟΛΟΓΙΑ.— On the Identification Problem in Linear Systems*, *by John E. Diamessis***. Ἀνεκοινώθη ὑπὸ τοῦ Ἀκαδημαϊκοῦ κ. Ἰωάνν. Ξανθάκη.

1. INTRODUCTION

This paper is concerned with the problem of linear system identification, that is the determination of the dynamic characteristics of a system from input and output measurements. The identification of a physical process or system occupies a central place in the general theory of systems since before any study of a physical system is attempted the dynamics of the system must either be given or must be determined in some way. In this paper a solution of the identification problem is obtained by comparing the unknown system to a known adjustable model by means of an error signal [1]. Both system and model are characterized by their transfer functions and the model is the analog of the system. The error signal is obtained by performing certain operations on the impulsive responses of both the system and the model. By varying the parameters of the model the error signal can be made zero. Under the condition of zero error signal the model is an exact representation of the system.

Model methods used so far require measurements at various points in the system [2,3] instead of only the input and the output, or the model is a complex structure and a large number of error signals is required. [4] Other

---

* The Moore School of Electrical Engineering, University of Pennsylvania, Philadelphia, Pennsylvania.

** JOHN E. DIAMESSIS, Ἐπὶ τοῦ προβλήματος τοῦ προσδιορισμοῦ τῶν ἀγνώστων παραμέτρων γραμμικοῦ συστήματος.

model methods require elaborate and expensive equipment for the automatic identification of the system. [5,6] The method of identification proposed here can be easily mechanized; it uses the simplest possible model and only one error signal. The adjustment of the model can be made manually or automatically by means of feedback loops. [1] Except for its simplicity, the essentially new feature of this method is that the zeros, the poles and the multiplying constant of the unknown system can be determined directly and the identification is exact in the absence of noise. Other similar schemes [7] use fixed poles and adjustable gains, and therefore the model can at best be only an approximation. A scheme which also can determine the gain, poles and zeros of the system [8], using analog computer techniques, has the serious drawback that division is necessary. The identification scheme presented here can be realized using an analog or a digital comuter. [1]

2. THE CONCEPT OF IDENTIFICATION- THE PROBLEM PROPOSED

A general formulation of the identification problem presents certain difficulties because of the dependence of the meaning of the term identification on the particular problem under study. One could suggest to define identification as the complete determination of the dynamics of the system. But in many studies of physical systems a complete knowledge of the dynamics of the system is not only extremely difficult but also not necessary. The question is what we really want to know about the system; the initial state, the final state, or some significant parameters which adequately characterize the system for our purposes. We define then identification as the determination of those dynamic characteristics of a system which are necessary for the given problem. Clearly this definition cannot be used directly to suggest a classification of groups of identification problems and to indicate possible methods of carrying out the identification. A more formal definition, following Zaheh [9], would be as follows:

Given 1) a system, H, whose characterization is not given (H here is the celebrated «black box»); 2) a set of systems Y which is known to contain H, on the basis of a priori information; 3) the set of all inputs F on which operation whith H is defined. Determine by observing the responses of H to various inputs, a member of the set Y which is equivalent to H (the equivalence of H being in the sense that the responses of H to any of the inputs in F are identical with those of H). The complexity of the identification problem then depends mainly on the freedom to select the inputs F and the nature of the set Y.

Following the formal definition given above the problem treated in the present note can be formulated as follows:

Given a linear, constant, lumped parameter single-input single-output system which belongs to a set of systems Y characterized by the transfer function

$$H(s) = K \frac{s^m + a_{m-1} s^{m-1} + \dots + a_1 s + a_0}{s^n + b_{n-1} s^{n-1} + \dots + b_1 s + b_0}, \quad n \leq m \tag{1}$$

where all the a's and b's are real and positive. Find the values of the a's and the b's assuming that the input and the output of H(s) are only available and that the input is a unit impulse function. Notice that this formulation of the problem is not as general as Zadeh's formulation since it is assumed here that a characterization of the unknown system is given. The method used to solve this problem is described in the next section.

3. METHOD USED

The basic ideas involved in the method of solution are incorporated in the block diagram of figure 1. The system to be identified, the identifying model and the error forming device are the essential elements of the scheme. The system is characterized by its transfer function (1). The model, in a similar manner, is characterized by a transfer function,

$$H'(s) = K' \frac{s^m + a'_{m-1} s^{m-1} + \dots\dots + a'_1 s + a'_0}{s^n + b'_{n-1} s^{n-1} + \dots\dots + b'_1 s + b'_0} \tag{2}$$

of the same form as the system but with different real and positive parameters, a' and b', which can be adjusted at will. The same input signal, for our purposes a unit impulse function, is applied to both the system and the model. The outputs of the system and the model become the inputs to the error forming device. This device operates on its inputs to produce an error signal which is the integral of the square of the difference between the system function and the model function. The error signal is the basis for the comparison of the unknown system to the known and adjustable model. To evaluate the error signal use is made of Parseval's theorem, [1]

$$I_m = \int_0^{\infty} e^2(t)dt \cdot = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} . \ E(s)E(-s)ds \tag{3}$$

where

$$e(t) = h'(t) - h(t) \tag{4}$$

and

$$E(s) = H(s) - H'(s) \tag{5}$$

h(t) and h'(t) are the inverse Laplace transforms of H(s) and H'(s), respectively. The error signal then is a function of both the system and model parameters and it is always positive except in the case where H(s) = H'(s) when it becomes zero. Therefore when the error signal is zero the model is identical with the unknown system. The error signal can be made zero by adjusting the model parameters. Since the error signal is always positive setting the values of the model parameters initially at zero, gives the advantage of making all the adjustments in the same direction.

The scheme was realized on an analog computer and extensive experiments were performed to show the validity of the method.[1] In the process of the experimental work it turned out that, instead of a single impulse, a train of impulses is required to complete the identification. If in certain applications there is objection to the use of impulses, random noise can be used as the input signal. The manner in which the method was presented implied manual adjustment of the model parameters. If the model has only a few parameters the adjustment can be made by trial and error; for the case of many parameters a systematic adjustment procedure is required. The adjustment of the model can also be done automatically by using feedback loops.[1] In the case where the system parameters were varying slowly with time, as in many industrial processes, the method can provide automatic and continuous identification. The method as presented can be used to identify systems of any order. In the next section some illustrative applications of the method in simple cases are discussed.

4. APPLICATIONS

a) The simplest case of identification is when the unknown system is of the first order and has only one parameter to be identified. Figure (2) shows the identification set-up for this case. One has then

$$E_1(s) = L[e(t)] = H'_1(s) - H'(s) = \frac{b' - b}{s^2 + (b + b')s + bb'} \tag{6}$$

and

$$I_1 = \int_0^\infty e_1^2(t)dt = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} E_1(s)E_1(-s)ds \tag{7}$$

which when integrated gives

$$I_1 = \frac{(b' - b)^2}{2bb'(b + b')} \tag{8}$$

It can be seen from (8) that the only way I_1 can be made zero is by maning $b' = b$. Therefore the condition on the error signal to make the parameter of the model b' equal to the parameter of the system b is $I_1 = 0$. Then by observing the value of I_1 on a meter and successively adjusting b', which could be read on a calibrated dial, wa can find the value of b' which corresponds to $I_1 = 0$. Then $b' = b$ and the identification is completed.

b) As a second application one can consider the case where ther are two parameters to be identified. Figure (3) shows the corresponding identification set-up. Then

$$E_2(s) = H_2(s) - H'_2(s) = \frac{(b'_1 - b_1)s + (b'_0 - b_0)}{b_1b'_1s^2 + (b_1b'_0 + b_0b'_1)s + b_0b'_0} \tag{9}$$

and

$$I_2 = \int_0^\infty e_2^2(t)dt = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} E_2(s)E_2(-s)ds \tag{10}$$

which when integrated gives the value of the error signal

$$I_2 = \frac{(b_1 - b'_1)^2 b_0b'_0 + (b'_0 - b_0)^2 b_1b'_1}{2b_0b'_0b_1b'_1b(b_1b'_0 + b_0b'_1)} \tag{11}$$

It can be seen from (11) that to make $I_2 = 0$ we must have $b'_1 = b_1$ and $b'_0 = b_0$. Then by observing I_2 and successively adjusting b_1 and b_0' we can make $I_2 = 0$, in which case $b'_1 = b_1$, and $b'_0 = b_0$ and this completes the iden tification. To realize the se-tup of figure (3), one must realize $H'_2(s)$, and in addition use one adder, one multiplier, and one integrator.

To show the simplicity of the method, a comparison is made with the method of reference (4) when the system to be identified is given by $H_2(s)$. To realize the set-up required to identify $H_2(s)$ using the method of reference (4), one must realize the transfer functions

$$\frac{\dfrac{b'_0}{b'_1} - S}{\left(\dfrac{b'_0}{b'_1} + S\right)(k + S)}, \qquad \frac{\dfrac{1}{b'_1}}{S + \dfrac{b'_0}{b'_1}}$$

for the model and in addition use one adder, three multipliers, and two integrators for the error forming device. Even in this simple example the ratio of the number of components required for each method is more than two to one. For higher order systems this ratio becomes much bigger. The error signals, using the method of reference (4), can be both positive and negative, a fact which complicates the adjustment of the model.

5. CONCLUSIONS

The scheme presented here is a simple and straightforward identification method and can be easily mechanized. It is general in the sense that it determines the transfer function of the physical process no matter what is its nature (electrical, mechanical, biological, etc.). The model is the analog of the system and it is an optimum as far as complexity is concerned, since for complete identification the model should be at least as complex as the system. The error forming device is also simple, only one error signal being used no matter how many parameters are to be identified; other model schemes require one error signal per parameter.

The nature of the error signal is such that every system parameter, both in the numerator and denominator, contributes in a similar manner to the value of the error signal. To perform the identification access only to the input and output of the system is required. Experiments performed, using the method presented here, showed that in most cases using a simple model and a simple adjustment procedure the dynamics of industrial processes can be determined without having to use the complicated and expensive devices reported in references (5), (6), and elswhere. Other interesting and important problems, not treated here, are the event when external noise corrupts the error signal, as well as the extension of the method for the identification of non-linear systems. These problems will be the subjects of future papers.

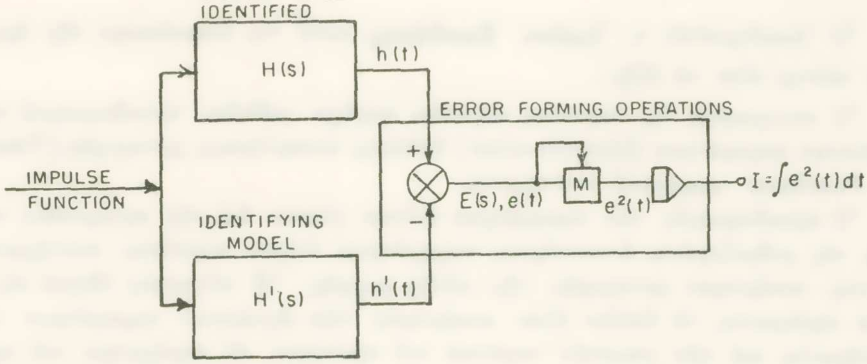

FIGURE 1 THE GENERAL ISE IDENTIFICATION SCHEME

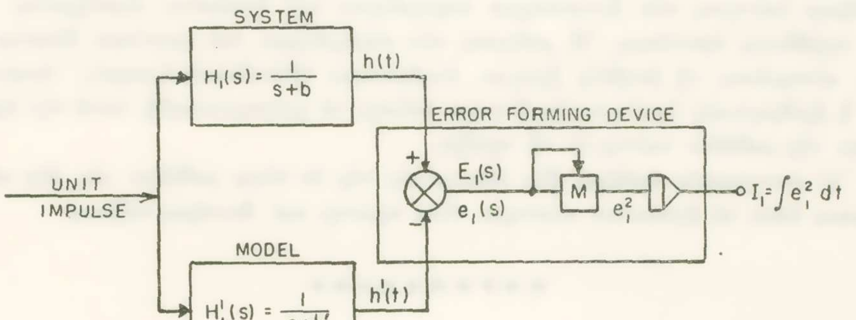

FIGURE 2: BLOCK DIAGRAM FOR THE IDENTIFICATION OF ONE-PARAMETER SYSTEM

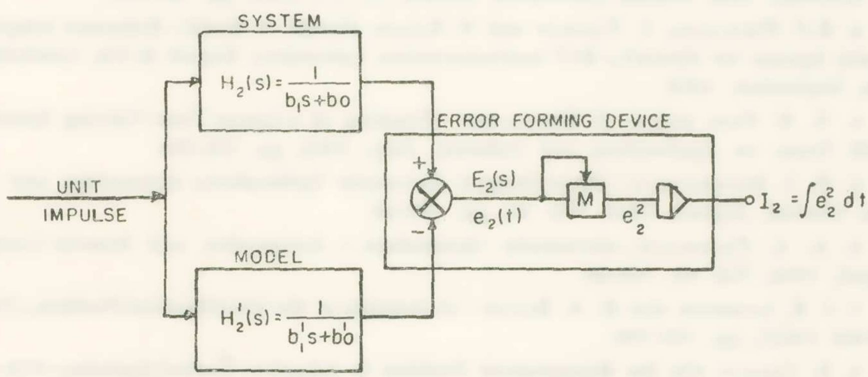

FIGURE 3: BLOCK DIAGRAM FOR THE IDENTIFICATION OF THE TWO-PARAMETER SYSTEM.

ΠΕΡΙΛΗΨΙΣ

Ὁ Ἀκαδημαϊκὸς κ. **Ἰωάνν. Ξανθάκης** κατὰ τὴν ἀνακοίνωσιν τῆς ἐργασίας ταύτης εἶπε τὰ ἑξῆς:

Ὁ συγγραφεὺς τῆς παρούσης ἐργασίας παρέχει μέθοδον προσδιορισμοῦ τῶν ἀγνώστων παραμέτρων (Identification) δοθείσης συναρτήσεως μεταφορᾶς (Transfer Function) γραμμικοῦ συστήματος.

Ὁ προσδιορισμὸς τῶν παραμέτρων τούτων γίνεται διὰ τῆς συγκρίσεώς των πρὸς τὰς ρυθμιζομένας ἀντιστοίχους παραμέτρους ἑτέρου προτύπου συστήματος, ἔχοντος συνάρτησιν μεταφορᾶς τῆς αὐτῆς μορφῆς. Ἡ σύγκρισις ὁδηγεῖ εἰς ἓν σῆμα σφάλματος, τὸ ὁποῖον εἶναι συνάρτησις τῶν ἀγνώστων παραμέτρων τοῦ συστήματος καὶ τῶν γνωστῶν τοιούτων τοῦ προτύπου. Αἱ παράμετροι τοῦ προτύπου ρυθμίζονται κατὰ τρόπον τοιοῦτον ὥστε τὸ σῆμα σφάλματος νὰ λάβῃ τὴν τιμὴν μηδέν. Ἡ μηδενικὴ δὲ τιμὴ τοῦ σήματος σφάλματος ἀποτελεῖ καὶ τὴν συνθήκην ἰσότητος τῶν ἀντιστοίχων παραμέτρων τοῦ ἀγνώστου συστήματος καὶ τοῦ ληφθέντος προτύπου. Ἡ ρύθμισις τῶν παραμέτρων τοῦ προτύπου δύναται νὰ γίνῃ αὐτομάτως τῇ βοηθείᾳ βρόχων ἀναδράσεως (Feedback Loops). Ἀναλογικὸς ἢ ἀριθμητικὸς ὑπολογιστὴς δύναται ἐπίσης νὰ χρησιμοποιηθῇ κατὰ τὴν ἐφαρμογὴν τῆς μεθόδου ταύτης ἐν τῇ πράξει.

Ὁ συγγραφεὺς ἐκθέτει δύο ἐφαρμογὰς τῆς ἐν λόγῳ μεθόδου εἰς δύο περιπτώσεις ὅπου τὸ ἄγνωστον σύστημα εἶναι πρώτης καὶ δευτέρας τάξεως.

REFERENCES

1. J. E. Diamessis : «On the Continuous Identification of System dynamics,» Doctoral Dissertation, University of Pennsylvania, Phila., Penna., May, 1963.

2. M. Margolis and C.T. Leondes: «A Parameter Tracking Servo for Adaptive Control Systems,» IRE Wescon Convention Record, Pt. IV, 1959, pp. 104-116.

3. H.P. Whitaker, J. Yarmon and H. Kezer: «Design of Model - Reference Adaptive Control System for Aircraft,» MIT Instrumentation Laboratory Report R-164, Cambridge, Mass, September, 1958.

4. N. N. Puri and C. N. Weygandt: «Tracking of a Linear Time-Varying System,» IEEE Trans. on Applications and Industry, July, 1963, pp. 234-238.

5. R. I. Stakhovskii : «Twin-Channel Automatic Optimalizer,» Automation and Remote Control, August, 1958, Vol. 19, pp. 729-40.

6. A. A. Feldbaum : «Automatic Optimalizer, » Automation and Remote Control, August, 1958, Vol. 19, 718-28.

7. J. K. Lubbock and H. A. Baker : «A Solution of the Identification Problem,» Proc. of 1963 JACC, pp. 191-199.

8. R. Corbin: «On the Measurement Problem in Adaptive Control Systems,» PIB-627 Research Report R-669-58, 1958.

9. L. A. Jadeh: «On the Identification Problem», IRE Trans. PGCT, Vol. CT-3, No. 4 December, 1956.