cryptocrystalline carbonate phosphorite (collophane e.t.c.) as regularly the second happens during the supergenesis.

In continuation, the conditions of the genesis of this apatite, are discussed.

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ΘΕΩΡΗΤΙΚΗ ΦΥΣΙΚΗ.— **On a problem of nonlinear mechanics.** Part I, *by* ***Dem. G. Magiros****. Ἀνεκοινώθη ὑπὸ τοῦ κ. Ἰωάνν. Ξανθάκη.

1. Introduction

The behavior of an oscillatory system, which is linearly damped but in which the restoring force is of cubic type, is investigated. The forced system is governed by a differential equation with coefficients not necessarily small. The usefulness of the auxiliary coefficients has been pointed out in the author's previous publications [1], [2]. It is shown under which conditions the system may oscillate with frequency half of that of the external force, that is with «subharmonics of order $^1/_2$». The amplitudes of the subharmonics and their components, and the bounds for the amplitude of the external force, are found in terms of the coefficients of the basic differential equation. Also the regions are found in the $\frac{c_1}{c_3}$, I-plane, where we have subharmonics with two, one or neither amplitudes.

2. The amplitudes of the subharmonics

The basic differential equation of the system is:

$$\ddot{Q} + \bar{k}\dot{Q} + \bar{c}_1 Q + \bar{c}_2 Q^2 + \bar{c}_3 Q^3 = B \sin 2t. \tag{1}$$

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* ΔΗΜ. Γ. ΜΑΓΕΙΡΟΥ, Ἐπὶ προβλήματος τῆς μὴ γραμμικῆς μηχανικῆς. Μέρος Ι.

By introducing auxiliary coefficients according to:

(2) $$\bar{k} = \varepsilon k\ ,\ 1 - \bar{c}_1 = \varepsilon c_1\ ,\ \bar{c}_2 = \varepsilon c_2\ ,\ \bar{c}_3 = \varepsilon c_3\ ,\ \varepsilon > O$$

the equation (1) can be written as:

(3) $$\ddot{Q} + Q = \varepsilon f(Q,\dot{Q}) + B \sin 2t.$$

The solution of (3) is given by:

(4) $$Q = x \sin t - y \cos t - \frac{1}{3} B \sin 2t,$$

where the components x and y of the amplitude r of the subharmonics are constants in case $\varepsilon = O$, and functions of time, $u_1(t)$ and $u_2(t)$ respectively, in case $\varepsilon \neq O$, but $\lim\limits_{\varepsilon \to 0} u_1(t) = x$, $\lim\limits_{\varepsilon \to 0} u_2(t) = y$.

By applying the formulae (38a) and (38b) of the paper [2] we can find the following two equations which must be satisfied by x and y.

(5) $$kx + c_1 y - \frac{3}{4} c_3 y^3 - \frac{3}{4} c_3 x^2 y - \frac{1}{6} c_3 B^2 y - \frac{1}{3} c_2 B x = O,$$

$$- c_1 x + ky + \frac{3}{4} c_3 x^3 + \frac{3}{4} c_3 x y^2 + \frac{1}{6} c_3 B^2 x + \frac{1}{3} c_2 B y = O.$$

Equations (5), for $c_3 \neq O$, give:

(6) $$(\mu - \frac{1}{3} \nu B)x + (\lambda - A)y = O,$$

$$(\lambda - A)x - (\mu + \frac{1}{3} \nu B)y = O,$$

(6a) $$\lambda = \frac{4}{3} \frac{c_1}{c_3} = \frac{4}{3} \frac{1 - \bar{c}_1}{\bar{c}_3}\ ,\ \mu = \frac{4}{3} \frac{k}{c_3} = \frac{4}{3} \frac{\bar{k}}{\bar{c}_3}\ ,$$

$$\nu = \frac{4}{3} \frac{c_2}{c_3} = \frac{4}{3} \frac{\bar{c}_2}{\bar{c}_3}\ ,\ A = r^2 + \frac{2}{9} B^2\ ,\ r^2 = x^2 + y^2.$$

The condition for non-zero amplitude r of the subharmonics is the same as the condition for non-zero solutions of system (6), and this condition gives:

$$r^2_{1,2} = \lambda - \frac{2}{9} B^2 \mp \sqrt{\frac{1}{9} \nu^2 B^2 - \mu^2}.$$

3. The reality of the amplitude r.

The reality of r, as given by (7), implies the following conditions to be satisfied: (a) the expression under the radical must be non-negative, and (b) the right-hand side of (7) must positive.

The condition (a) gives the restrictions:

$$\text{(8)} \qquad \text{i)} \quad \left| \, B \, \right| \geq 3 \left| \frac{\mu}{\nu} \right| = 3 \left| \frac{k}{c_2} \right| ,$$

$$\text{ii)} \quad \left(B - 3 \frac{k}{c_2} \right) \left(B + 3 \frac{k}{c_2} \right) \geq O .$$

The condition (8i) gives bounds for B, and the shaded regions in Fig. 1 are the permissible ones, according to (8ii).

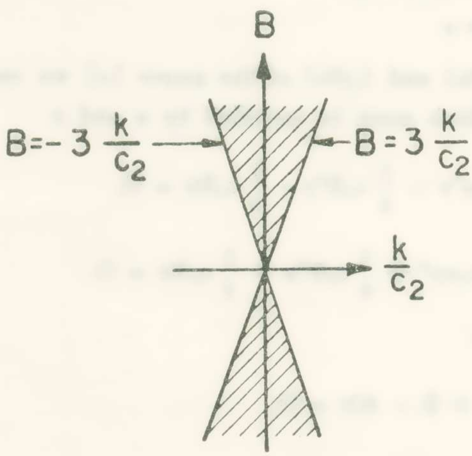

Fig. 1.

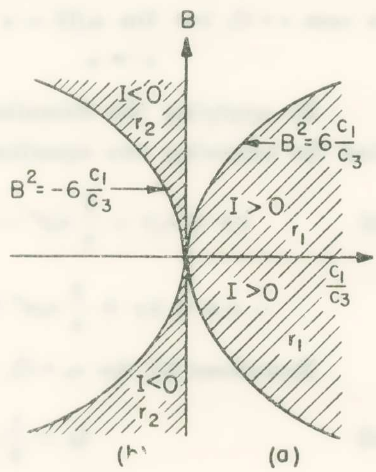

Fig. 2.

The condition (b) gives the following restrictions:

1) The existence of non-zero r_1 implies:

$$\text{(9)} \qquad \sqrt{\frac{1}{9} \nu^2 B^2 - \mu^2} < \lambda - \frac{2}{9} B^2 ,$$

then: $\lambda - \frac{2}{9} B^2 > o$ and $\lambda > o$, or:

$$\text{(10)} \qquad B^2 < 6 \frac{c_1}{c_3} , \; O < \frac{c_1}{c_3} .$$

Squaring (9) we can get: $I > O$ with:

$$\text{(11)} \qquad I \equiv \frac{4}{81} B^4 - \frac{1}{9} (\nu^2 + 4 \lambda) B^2 + \lambda^2 + \mu^2 ,$$

and r_1 exists only for points of the shaded region Fig. 2(a) with $I < O$.

II) The existence of non-zero r_2 implies:

$$(12) \qquad \sqrt{\frac{1}{9}\nu^2 B^2 - \mu^2} > \frac{2}{9} B^2 - \lambda ,$$

and we have three cases for the existence of r_2:

i) For $\lambda < O$, we have $\frac{2}{9} B^2 - \lambda > O$, and squaring (12) we get $I < O$

Then r_2 always exists for points of the shaded region 2(b) with $I \geq O$.

ii) For $\lambda > O$, if $\left|\frac{2}{9} B^2 - \lambda\right|^2 < \frac{1}{9}\nu^2 B^2 - \mu^2$, we have $I \leq O$.

iii) For $\lambda > O$, if $\left|\frac{2}{9} B^2 - \lambda\right|^2 > \frac{1}{9}\nu^2 B^2 - \mu^2$, we have $I > O$.

The above results can be summarized in Fig. 3

Only points of the first quadrant of the $\frac{c_1}{c_3}$, I-plane can give subharmonics with two different non-zero amplitudes r_1, r_2. Points of the third and fourth quadrant give subharmonics with one amplitude r_2, and points of the second quadrant correspond to no subharmonics.

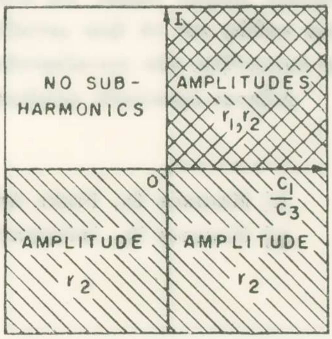

Fig. 3.

4. The components of the amplitudes r.

The system (6) gives:

$$(13) \qquad \frac{y}{x} = \frac{\mu - \frac{1}{3}\nu B}{A - \lambda} = \frac{\lambda - A}{\mu + \frac{1}{3}\nu B} = \theta ,$$

when the components x and y are given as solutions of the system:

$$(14) \qquad y = \theta x , \; x^2 + y^2 = r^2 ,$$

then:

$$x_1 = \frac{r}{\sqrt{1+\theta^2}}, \quad y_1 = \frac{r\theta}{\sqrt{1+\theta^2}}, \tag{15}$$

$$x_2 = -\frac{r}{\sqrt{1+\theta^2}}, \quad y_2 = -\frac{r\theta}{\sqrt{1+\theta^2}}.$$

These solutions correspond to two symmetric points with respect to the origin in the x, y-plane. Since we have two, in general, values of r, and θ depends on r, (15) give four points, then the singularities of the equation (1) are, in general, five, the origin included.

ΠΕΡΙΛΗΨΙΣ

Ἐρευνῶνται ἐνταῦθα αἱ συνθῆκαι ὑπὸ τὰς ὁποίας ἀναπτύσσονται ὑποαρμονικαὶ ταλαντώσεις δευτέρας τάξεως εἰς μὴ γραμμικὸν σύστημα, ὅπου ἡ μὴ γραμμικότης εἰσέρχεται εἰς τὴν ἐλαστικὴν δύναμιν καὶ ἡ ἐξωτερικὴ δύναμις εἶναι ἡμιτονικοῦ τύπου. Οἱ συντελεσταὶ τῆς διαφορικῆς ἐξισώσεως τοῦ συστήματος εἶναι οὐχὶ κατ' ἀνάγκην μικρῶν τιμῶν. Τὰ πλάτη καὶ αἱ συνιστῶσαι τῶν ὑποαρμονικῶν ταλαντώσεων καθὼς καὶ τὰ ὅρια μεταβολῆς τοῦ πλάτους τῆς ἐξωτερικῆς δυνάμεως εὑρίσκονται συναρτήσει τῶν συντελεστῶν τῆς ἐξισώσεως τοῦ συστήματος.

Δίδεται πρακτικὸν κριτήριον ἀναγνωρίσεως ὑπάρξεως τῶν ὑποαρμονικῶν.

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I. ΓΕΝΙΚΑ.

Εἰς δύο προηγουμένας μελέτας μου [1,2] ἀπέδειξα ὅτι εἰς χῶρον X^1, X^4 τὸ «Παράδοξον Ὡρολόγιον» τῆς Γενικῆς Θεωρίας τῆς σχετικότητος δὲν ὑφίσταται. Ταξιδιώτης ταξιδεύων ἀνὰ τὸ διάστημα καὶ ἐπιστρέφων εἰς τὴν Γῆν δὲν θὰ ἔχῃ ζήσει ὀλιγώτερον τοῦ παραμείναντος παρατηρητοῦ.

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