ΣΥΝΕΔΡΙΑ ΤΗΣ 10ΗΣ ΔΕΚΕΜΒΡΙΟΥ 1998

ΠΡΟΕΔΡΙΑ ΑΓΑΠΗΤΟΥ Γ. ΤΣΟΠΑΝΑΚΗ

ΦΥΣΙΚΗ. – **Neutrino Masses in Flipped** $SU(5)$, by *D. V. Nanopoulos,**
of the Academy of Athens.

Abstract

Motivated by the Super-Kamiokande atmospheric neutrino data, we discuss possible textures for Majorana and Dirac neutrino masses within the see-saw framework. We consider the possible pattern of neutrino masses in a 'realistic' flipped $SU(5)$ model derived from string theory, illustrating how a desirable pattern of mixing may emerge. Both small –or large– angle MSW solutions are possible, whilst a hierarchy of neutrino masses appears more natural than near-degeneracy. This model contains some unanticipated features that may also be relevant in other models: the neutrino Dirac matrices may not be related closely to the quark mass matrices, and the heavy Majorana states may include extra gauge-singlet fields.

1. Introduction

There have recently been reports from the Super-Kamiokande collaboration [1] and others [2] indicating that the atmospheric neutrino deficit is due to neutrino oscillations. The data on electron events with visible energy greater than 200 MeV are in very good consistency with Standard Model expectations. On the other hand, the number of events with muons is about half of the expected number, and the deficit becomes more acute for larger values of L/E, indicating that neutrino oscillations dilute the abundance of atmospheric ν_μ. The possibility that $\nu_\mu \to \nu_e$ oscillations dominate is disfavoured by both Super-Kamiokande [1] and CHOOZ data [3]. A fit to $\nu_\mu - \nu_\tau$ oscillations, with $\Delta m^2 = 5 - 50 \cdot 10^{-4}$ eV^2 and $\theta \sim \pi/4$ matches the data very well, but an admixture of $\nu_\mu \to \nu_e$ oscillations cannot be excluded.

One intriguing feature of this scenario is the large mixing angle that is required, and the question that arises is how one could achieve this in theoretically motivated models. Large mixing angles in the neutrino sector do arise naturally in a sub-class of GUT models with flavour symmetries, as in [4], where they were used to explain what was then only an "atmospheric neutrino anomaly" [5]. Many models with a single $U(1)$ symmetry predict small mixings [6], principally because of the constrained form of the

* ΔΗΜ. ΝΑΝΟΠΟΥΛΟΣ, Νετρίνα μὲ μάζες σὲ ἀνεστραμμένο $SU(5)$.

Dirac mass matrices. However, this is also not a generic feature, and textures with large $\nu_\mu - \nu_\tau$ mixing have also been presented in [7]. Moreover, string-derived models may well have a richer structure, with three or four $U(1)$ symmetries.

However, models where the large neutrino mixing arises from the Dirac mass matrix may have a problem with quark masses. In many GUTs such as $SO(10)$, the neutrinos and up-type quarks couple to the same Higgs and are in the same multiplets, so their couplings arise from identical GUT terms. Thus, in these cases one would generate simultaneously large mixing in the u-quark sector. Then, in order to obtain small mixing in V_{CKM}, one needs to invoke some cancellation with mixing in the d-quark sector. One way to overcome these difficulties may be to invoke additional symmetries, as arise in string-derived GUT models. In 'realistic' models which also give the correct pattern of quark masses and mixings, one can hope to generate large neutrino mixing, due to the combined form of the Dirac and heavy Majorana mass matrices, even in cases where the off-diagonal elements of the Dirac mass matrix are not large by themselves. A study of phenomenologically viable heavy Majorana mass matrices leading to a large mixing angle, for different choices of the Dirac mass matrix, has previously been presented in [8].

Realistic string models have been in particular constructed in the free-fermionic superstring formulation, with encouraging results. Recently, due to better understanding of non-perturbative string effects, which may remove the previous apparent discrepancy between the string and gauge unification scales, interest in string-motivated GUT symmetries has been revived. In this framework, we have looked recently [9] at the predictions for quark masses in the context of a flipped $SU(5) \times U(1)$ model [10], which is one of the three-generation supersting models described in the free-fermion formulation. The extension to lepton fields are not unique. Moreover, the model contains many singlet fields, and which of them develop non-zero vacuum exprectation values (vev's) depends on the choice of flat direction.

GUT and string models form the motivation for the analysis contained in this paper. Some novel features appear: flipped $SU(5)$ avoids the tight relation between u-quark and neutrino Dirac mass matrices, and gauge-singles fields may be candidates for ν_R fields [11]. Within this model, we prefer a hierarchy of neutrino masses, and may obtain either the small —or the large— angle MSW solution to the solar neutrino problem.

The layout of this paper is as follows. After a brief review in Section 2 of the data and their implications, Section 3 studies neutrino mass matrices in the string model of [9] (which is reviewed in the Appendix), and Section 4 summarizes our conclusions, where we point to features that may be generalizable to other models.

2. Neutrino Data and their Implications

The atmospheric neutrino data reported by Super-Kamiokande and other experiments [1,2] are explicable by

(a) $\nu_\mu - \nu_\tau$ oscillations with

$$\delta m^2_{\nu_\mu \nu_\tau} \approx (10^{-2} \text{ to } 10^{-3}) \text{ eV}^2 \quad (1)$$

$$sin^2 2\theta_{\mu\tau} \geq 0.8 \quad (2)$$

A description in terms of $\nu_\mu - \nu_e$ oscillations alone fits the data less well, and is in any case largely exluded by the CHOOZ experiment [3]. However, there may be some admixture of $\nu_\mu - \nu_e$ oscillations.

The solar neutrino data may be explicable in terms of $\nu_e - \nu_\alpha$ oscillations with either (b_1) a small-angle MSW solution [12]

$$\delta m^2_{\nu_e \nu_\alpha} \approx (3-10) \times 10^{-6}\, eV^2 \quad (3)$$

$$sin^2 2\theta_{\alpha e} \approx (0.4-1.3) \times 10^{-2} \quad (4)$$

or (b_2) a large-angle MSW solution

$$\delta m^2_{\nu_e \nu_\alpha} \approx (1-20) \times 10^{-5}\, eV^2 \quad (5)$$

$$sin^2 2\theta_{\alpha e} \approx (0.5-0.9) \quad (6)$$

or (b_3) vacuum oscillations

$$\delta m^2_{\nu_e \nu_\alpha} \approx (0.5-1.1) \times 10^{-10}\, eV^2 \quad (7)$$

$$sin^2 2\theta_{\alpha e} \geq 0.67 \quad (8)$$

where α is μ or τ.

One may also consider the possibility (c) that there is a significant neutrino contribution to the mass density of the Universe in the form of hot dark matter, which would require $\Sigma i\ m_{\nu i} \geq 3$ e V. If this was to be the case, the atmospheric and solar neutrino data would enforce $m_{\nu_e} \approx m_{\nu_\mu} \approx m_{\nu_\tau} \geq 1$ eV. This would be only marginally compatible with $(\beta\beta)_{0\nu}$ limits, which might require some cancellations in the event of large mixing, as required in scenarios (b_2, b_3) above. Motivation for a significant hot dark matter component was provided some years ago by the need for some epicycle in the standard cold dark matter model for structure formation, in order to reconcile the COBE data on fluctuations in the cosmic microwave background radiation with other astrophysical structure data [13]. Alternative epicycles included a tilted spectrum of primordial fluctuations and a cosmological constant. In recent years, the case for mixed hot and cold dark matter has not strengthened, whilst recent data on large red-shift supernovae favour a non-zero cosmological constant [14].

Under these circumstances, we consider abandoning the cosmological requirement (c). In this case, the atmospheric and solar neutrino onditions (a,b) no longer impose near degeneracy on any pair of neutrinos, though this remains a theoretical possibility.

Thus, one is led to consider the possibility of a hierachy of neutrino masses: $m_{\nu 3} \gg m_{\nu 2}, m_{\nu 1}$, leaving open for the moment the possibility of a second hierachy $m_{\nu 2} \gg m_{\nu 1}$. In either case, condition (a) requires $m_{\nu 3} \approx (10^{-1}$ to $10^{-1\,1/2})$ eV, and if there is a second hierarchy $m_{\nu 1} \gg m_{\nu 2} \approx (10^{-2}$ to $10^{-3})$ eV. One may then wonder about the magnitude of the mixing angles. It is well known that large mixing is generic if off-diagonal entries in the mass matrix are larger than differences between diagonal entries. Can one reverse this argument, i.e., to what extent is a large mixing angle incompatible with a hierarchy of mass eigenstates $m_{\nu 3} \gg m_{\nu 2}$? We study this question and examine whether the necessary mass matrices have any chance of arising in a model derived from string theory.

3. Neutrino Mixing in a Realistic Flipped $SU(5)$ Model

Let us now look at a specific example of the structure generated by $U(1)$ symmetries, namely the Ansatz made in [9] in the context of a 'realistic' flipped $SU(5)$ model derived from string, which is reviewed in the Appendix. This model contains many singlet fields, and the mass matrices depend on the subset of these that get non-zero vev's, which in turn depends on the choice of flat direction in the effective potential, which is ambiguous, so far.

3.A. Charged-Lepton Masses and Mixing

Usually one works in a field basis that is diagonal for the mass eigenstates of the charged leptons. In the context of the flipped $SU(5)$ model, this has to be identified relative to the string states listed in the Appendix, which requires a discussion of the charged-lepton mass matrix. The importance of this discussion lies in the possibility that there might be additional mixing coming from this sector. In this connection, we recall that the mixing angles of relevance to experiment are the combinations given by

$$V_\nu = V_\nu^{m\dagger} V_{\ell_L}^m \tag{9}$$

where the symbols V^m_{ν,l_L} denote the rotation matrices for neutrinos and left-handed charged leptons, respectively, required to diagonalize their mass matrices.

The candidate terms for charged-lepton mass terms at the third-order level are

$$\overline{f}_1 \ell_1^c h_1, \quad \overline{f}_2 \ell_2^c h_2, \quad \overline{f}_5 \ell_5^c h_2. \tag{10}$$

where, here and later, we do not display factors of the gauge coupling. The first term generates the τ mass, but since the last two are proportional to the same Higgs h_2, they cannot yield a mass hierarchy. We therefore assume that the vev of the effective light Higgs has only a small component in the h_2 direction, as also assumed in [9]. Thus, in a first approximation we assign l_1^c and the charged component of $\overline{f}_1$ to the τ, and the corresponding $l^c_{2,5}, \overline{f}_{2,5}$ to the e, μ, with the precise flavour assignments of the latter to be discussed below.

Assuming a very small vev for h_2, the next candidate mass terms appear at fifth order[1] [15]:

$$\overline{f}_2 \ell_2^c h_1 (\overline{\phi}_i^2 + \overline{\phi}^+ \overline{\phi}^-), \quad \overline{f}_5 \ell_5^c h_1 (\overline{\phi}_{1,4}^2 + \overline{\phi}^+ \overline{\phi}^-) \tag{11}$$

Among the fields in parentheses, previous analyses suggest (see the Appendix) that $\overline{\varphi}_{1,2}$ and $\overline{\varphi}^-$ have zero vev's. Therefore the possible mass terms are

$$\overline{f}_2 \ell_2^c h_1 \overline{\phi}_{3,4}^2, \quad \overline{f}_5 \ell_5^c h_1 \overline{\phi}_4^2 \tag{12}$$

1. Here and subsequently, higher-order interactions should always be understood to be scaled by the appropriate inverse power of some relevant dimensional scale M_s. We expect this to be $O(10^{18})$ GeV in conventional string theory, but it might be as low as $\sim 10^{16}$ GeV in M theory. The vev's we quote later for singlet fields are likewise in units of M_s.

It is apparent that, in order to obtain a hierarchy: $m_\mu \gg m_e$, we must assume that either $\overline{\varphi}_3^2 \gg \overline{\varphi}_4^2$ or the invers. As we argue later on the basis of the u-quark masses and mixing that $\overline{\varphi}_3 \gg \overline{\varphi}_4$.

Continuing to seventh order, we find the term:

$$\overline{f}_5 \ell_2^c h_1 \Delta_2 \Delta_5 (\overline{\phi}_i)^2 \tag{13}$$

but, to this order, we still find to term mixing $\overline{f}_1$, l_1^c with the other lepton fields. As mentioned in the previous paragraph, we assume that $\overline{\varphi}_3 \gg \overline{\varphi}_4$, $\overline{\varphi}_{1,2}=0$. The charged lepton mass-mixing problem can therefore be reduced to the following 2×2 matrix in the $\overline{f}_{5,2}$, $l_{5,2}^c$ basis:

$$m_\ell(2 \times 2) \propto \begin{pmatrix} \overline{\phi}_4^2 & \Delta_2 \Delta_5 \overline{\phi}_3^2 \\ 0 & \overline{\phi}_3^2 \end{pmatrix} \tag{14}$$

where, again in view of the u-quark mass matrix discussed below, we believe that $\Delta_2\Delta_5$ is not small. Since $\overline{\varphi}_3 \gg \overline{\varphi}_4$, we assign the charged leptons to the eigenvectors of (14) as follows: $(e^c, \mu^c) = (l_5^c, l_2^c)$ and $(e_L, \mu_L) = (\overline{f}_5 - O(\Delta_2\Delta_5)\overline{f}_2, \overline{f}_2 + O(\Delta_2\Delta_5)\overline{f}_5)$, with the ratio of mass eigenvalues

$$\frac{m_\mu}{m_e} \sim \frac{m_{\ell_1}}{m_{\ell_2}} \sim \frac{\overline{\phi}_3^2}{\overline{\phi}_4^2} \tag{15}$$

Thus we see explicitly that we can arrange a hierarchy $m_\mu \gg m_e$, at the price of potentially large mixing angle among the left-handed charged leptons: V_L^m (12) $= O(\Delta_2\Delta_5)$. This would lead us naively to expect correspondingly large $\nu_e - \nu_\mu$ mixing, unless there is some cancellation with V_ν^m in (9).

3.B. Dirac Neutrino Masses

Even with a given choice of a flat direction, the neutrino mass matrix that arises from the string model is rather complicated, because one must consider light Majorana, Dirac and heavy Majorana mass matrices. The first of these could arise from direct effective operators involving two left-handed neutrinos, two light Higgs doublets, and singlet fields. However, we find no candidates for such terms up to fifth order, and shall not discuss them further here. As for the Dirac mass matrix, since the neutrino flavours are in the same representations as the u-type quarks, with the left-handed neutrinos belonging to the representations $\overline{f}_{1,2,5}$, whilst the right-handed neutrinos naively belong to the decuplets $F_{2,3,4}$, one would naively expect the relation

$$m_\nu^D = (m_u)^\dagger \tag{16}$$

However, one should also not forget that there may be Dirac mass couplings of light neutrinos to singlet states not included among the $F_{2,3,4}$, and that these fields may also mix with the singlets via Majorana mass terms, possiblilities that will play important rôles later.

At third order, we find the following contribution tho the Dirac neutrino mass matrix, which corresponds to the dominant contribution to m_t:

$$F_4\overline{f}_5\overline{h}_{45} \tag{17}$$

Progressing up to sixth order, the following additional terms appear:

$$F_2\overline{f}_2\overline{h}_{45}\overline{\phi}_4, \quad F_4\overline{f}_2\overline{h}_{45}\Delta_2\Delta_5 \tag{18}$$

$$F_2\overline{f}_5\overline{h}_{45}\Delta_2\Delta_5\overline{\phi}_4, \quad F_3\overline{f}_5\overline{h}_{h45}\Delta_3\Delta_5\overline{\phi}_3 \tag{19}$$

We observe that the Dirac matrix again leaves the ν_1 component of $\overline{f}_1$ essentially decoupled from the other light neutrinos, up to sixth order. The most important mixing effects are therefore expected to take place between $\overline{f}_2$ and $\overline{f}_5$, and the problem can be reduced, in a first approximation, to considering only two neutrino species. This is equivalent to the 2×2 mixing matrix for the two heaviest quark generations: $m_u(2 \times 2) = m_\nu^{D\dagger}(2 \times 2)$, and some indications on the values of the vev's appearing in (19) may be obtained from the experimental values of m_c/m_t and the V_{CKM} parameters.

The 2×2 part of the up-quark mass matrix for the two generations is of the following form [9] in the F_2, F_4, $\overline{f}_2$, $\overline{f}_5$ basis:

$$m_u(2 \times 2) = m_\nu^{D\dagger}(2 \times 2) = \begin{pmatrix} \overline{\phi}_4 & \Delta_2\Delta_5\overline{\phi}_4 \\ \Delta_2\Delta_5 & 1 \end{pmatrix} \lambda_t(M_{GUT})\langle\overline{h}_{45}\rangle \tag{20}$$

This implies that the (23) u_L mixing angle, which contributes to V_{CKM}, is given by $\theta^{u_L}_{(23)} = \Delta_2\Delta_5\ \overline{\varphi}_4$, whilst the (23) u_R mixing angle is $\theta^{u_R}_{(23)} = \Delta_2\Delta_5$. The corresponding mass eigenvalues are:

$$m_u^{1,2} \approx \frac{1}{2}\left(1 + \overline{\phi}_4 \pm \sqrt{1 - 2\overline{\phi}_4 + 4(\Delta_2\Delta_5)^2\overline{\phi}_4 + \overline{\phi}_4^2}\right) \tag{21}$$

so we see that the heavier eigenvalue is almost unity, whilst the lighter is suppressed if $\overline{\varphi}_4 \ll 1$:

$$\frac{m_c}{m_t} \sim \overline{\phi}_4 \times \mathcal{O}(1) \tag{22}$$

One should not be too concerned at this stage about the compatibility of this equation with (15), since unknown numerical factors remain to be calculated. More information about the vev's of the fields is provided by the (23) element of V_{CKM}. This also receives a contribution from the (23) element of the down-quark mass matrix, which was also found [9] to be of order $\Delta_2\Delta_5\ \overline{\varphi}_4$. Up to constants of order unity, which we do not keep track of in our analysis of mass matrices, we conclude that

$$\Delta_2\Delta_5\overline{\phi}_4 \approx 0.044 \tag{23}$$

We see from (22) that having $\overline{\varphi}_4$ large and $\Delta_2\Delta_5$ small will not give acceptable solutions. However, the choice of large $\Delta_2\Delta_5$ and smaller $\overline{\varphi}_4$ does lead to acceptable solutions. For example, fixing $\overline{\varphi}_4 \approx 0.044/\Delta_2\Delta_5$, we find for $\Delta_2\Delta_5 \approx 0.8$ that $m_c/m_t = 0.018$, whilst for $\Delta_2\Delta_5 \approx 0.9$ we find $m_c/m_t = 0.008$. However, we should also note that the values of the acceptable field vev's are sensitive to the presence of order unity coefficients. In particular, $\Delta_2\Delta_5$ can become smaller. For example, if the off-diagonal elements in (2) happen to be multiplied by factors of two, we find for $\Delta_2\Delta_5 = 0.47$: $m_c/m_t = 0.009$, and for $\Delta_2\Delta_5 = 0.53$: $m_c/m_t = -0.009$, whilst for $\Delta_2\Delta_5 = 0.5$: $m_c/m_t \approx 0$.

This is why we assumed that $\Delta_2\Delta_5$ is large and $\overline{\varphi}_4 \ll 1$ in our earlier analysis of the charged-lepton mass matrix, which then required $\overline{\varphi}_3 = O(1)$. Analysis of the (13) entry in V_{CKM}, which is $O(\Delta_3\Delta_5\overline{\varphi}_3)$, might then lead one to suspect that $\Delta_3 \ll 1$. However, as can be seen from [9], this would lead to too small a value for the Cabibbo angle. In fact, it is not necessary that $\Delta_3 \ll 1$, since (unlike the (12) entry) the (13) entry in V_{CKM} results from a difference between two terms of the same order originating from u- and d- quark mixing, and there could be a cancellation between them, depending on the precise numerical coefficients.

We have omitted from the above discussion the last term in (19), which includes factors of Δ_3 and $\overline{\varphi}_3$. We have no strong reason to neglect this term, except for the fact that it is of sixth order. Nevertheless, we assume for simplicity that this and other mixing with F_3 can be neglected as a first approximation. Absent from the above discussion has been any Dirac neutrino mass term involving $\overline{f}_1$. There is no such coupling to any of the $F_{2,3,4}$ up to sixth order, but there is such a coupling to φ_1 in fourth order:

$$F_1\overline{f}_1\overline{h}_{45}\phi_1, \tag{24}$$

which may lead to mixing between the ν_1 component $\overline{f}_1$ and the singlet φ_1, if F_1 develops a vev [16]. Since the term (24) is only fourth order, we consider φ_1 as the best candidate for the third ν_R state, rather one of the F_i.

This example serves to warn us that the expected relation (16) may be too naive, the reason being that the u quark is so light that some other effect, such as mixing with additional heavy singlet states, may be important.

3.C. Heavy Majorana Masses

We now discuss the heavy Majorana mass matrix for the fields F_2, F_4, which we parametrize as:

$$\begin{pmatrix} M & M' \\ M' & M'' \end{pmatrix} \tag{25}$$

As we now discuss, the heavy Majorana entries *M, M΄ and M˝* are expected to be generated from higher-order non-renormalizable terms. Their magnitudes play crucial rôles in the mixing of the light neutrinos, as the previous simple 2×2 and 3×3 phenomenological analyses has shown. We find candidate terms for the *M, M΄* contributions at seventh order. Up to this order, a complete catalogue of the operators that could generate heavy Majorana neutrino mass terms involving the fields F_2 and F_4 is given by:

$$
\begin{aligned}
W_{NR} \;=\; & F_2F_2(\overline{F}_5\overline{F}_5\overline{\Phi}_2\phi_1 + \overline{F}_5\overline{F}_5\overline{\phi}_3\phi_4 + \overline{F}_5\overline{F}_5\overline{\phi}_4\phi_3 + \overline{F}_5\overline{F}_5\overline{\phi}_1\phi_2 + \\
& \overline{F}_5\overline{F}_5\phi_{45}\overline{\phi}_{45}\Phi_4 + \overline{F}_5\overline{F}_5\phi^-\overline{\phi}^-\Phi_4 + \overline{F}_5\overline{F}_5\phi^-\overline{\phi}^+\Phi_4 + \overline{F}_5\overline{F}_5D_5\overline{\phi}_3D_4 + \\
& \overline{F}_5\overline{F}_5\overline{\Phi}_2\phi_1\Phi_1 + \overline{F}_5\overline{F}_5\overline{\Phi}_2\phi_1\Phi_3 + \overline{F}_5\overline{F}_5\overline{\Phi}_2\phi_2\Phi_4 + \overline{F}_5\overline{F}_5\overline{\phi}_3\phi_3\Phi_4 + \\
& \overline{F}_5\overline{F}_5\overline{\phi}_3\phi_4\Phi_1 + \overline{F}_5\overline{F}_5\overline{\phi}_3\phi_4\Phi_3 + \overline{F}_5\overline{F}_5\overline{\phi}_3\phi_4\Phi_5 + \overline{F}_5\overline{F}_5\overline{\phi}_4\phi_3\Phi_1 + \\
& \overline{F}_5\overline{F}_5\overline{\phi}_4\phi_3\Phi_3 + \overline{F}_5\overline{F}_5\overline{\phi}_4\phi_3\Phi_5 + \overline{F}_5\overline{F}_5\overline{\phi}_4\phi_4\Phi_4 + \overline{F}_5\overline{F}_5\overline{\phi}_1\phi_1\Phi_4 + \\
& \overline{F}_5\overline{F}_5\overline{\phi}_1\phi_2\Phi_1 + \overline{F}_5\overline{F}_5\overline{\phi}_1\phi_2\Phi_3) + \\
& F_4F_4(\overline{F}_5\overline{F}_5\phi_1\phi_2 + \overline{F}_5\overline{F}_5\phi_3\phi_4 + \\
& \overline{F}_5\overline{F}_5\phi^-\phi^+\Phi_4 + \overline{F}_5\overline{F}_5D_5\phi_3D_4 + \overline{F}_5\overline{F}_5\phi_1\phi_1\Phi_4 + \overline{F}_5\overline{F}_5\phi_1\phi_2\Phi_1 + \\
& \overline{F}_5\overline{F}_5\phi_1\phi_2\Phi_2 + \overline{F}_5\overline{F}_5\phi_1\phi_2\Phi_3 + \overline{F}_5\overline{F}_5\phi_2\phi_2\Phi_4 + \overline{F}_5\overline{F}_5\phi_3\phi_3\Phi_4 + \\
& \overline{F}_5\overline{F}_5\phi_3\phi_4\Phi_1 + \overline{F}_5\overline{F}_5\phi_3\phi_4\Phi_2 + \overline{F}_5\overline{F}_5\phi_3\phi_4\Phi_3 + \overline{F}_5\overline{F}_5\phi_3\phi_4\Phi_5 + \\
& \overline{F}_5\overline{F}_5\phi_4\phi_4\Phi_4) + \\
& F_2F_4\overline{F}_5\overline{F}_5\Delta_2\Delta_5\phi_3
\end{aligned} \tag{26}
$$

Please note that we include at this stage even some combinations involving singlet fields which we had assumed in [9] (see also the Appendix) to have zero vev's. This is done in order to develop a more general picture of the types of terms that are allowed. However, we have dropped combinations of the type D_i^2, since such terms would not allow for two light Higgses.

The only term in (26) that involves the combination F_2F_4 is $F_2F_4\,\overline{F}_5\overline{F}_5\Delta_2\Delta_5\varphi_3$. Previously, in [9], where we studied the implications of this model for the quark mass matrices, we assumed that $\varphi_3 = 0$. However, this restriction may be avoided [16] by a different choice of flat direction. If we adopt the minimal modification of the flat direction chosen in [9] that allows for a non-zero vev for φ_3, none of the additional terms involving F_4F_4 survives. However, there is an effective term $F_2F_2\,\overline{F}_5\overline{F}_5\,\overline{\varphi}_4\,\varphi_3$ that provides F_2F_2 mixing. We therefore conclude that, to seventh order, this model has:

$$M = \overline{F}_5\overline{F}_5\overline{\phi}_4\phi_3, \quad M' = \overline{F}_5\overline{F}_5\Delta_2\Delta_5\phi_3, \quad M'' = 0 \tag{27}$$

Clearly, the form of the heavy Majorana mass matrix depends on the relative magnitudes of the vev's of the $\Delta_2\Delta_5$ and $\overline{\varphi}_4$ field combinations, which we discussed earlier in connection with the matrix $m_u = m_\nu^D$.

This does not complete our discussion of the heavy Majorana mass matrix, since we should also discuss possible mass terms involving φ_1, our candidate for the third ν_R state. To seventh order, the following are the only such candidate Majorana mass terms we find:

$$\phi_1F_4\overline{F}_5\overline{\phi}_{31}\phi_{31}\overline{\phi}_4\phi_2 \rightarrow M_{4\phi}, \quad \phi_1^2\Delta_2\Delta_5\overline{\phi}_{23}T_2T_5 \rightarrow M_{\phi\phi} \tag{28}$$

The first of these mixes φ_1 with F_4, and the latter is a diagonal Majorana mass term. Combining these with (27), we find the following 3×3 heavy Majorana mass matrix in the F_2, F_4, φ_1 basis:

$$\begin{pmatrix} M & M' & 0 \\ M' & 0 & M_{4\phi} \\ 0 & M_{4\phi} & M_{\phi\phi} \end{pmatrix} \tag{29}$$

Since all of these terms arise in seventh order, and the vev's appearing in them are not very tightly constained, diagonalization of the heavy Majorana mass matrix may well require large mixing angles but these cannot be predicted accurately. Nevertheless, it would seem to be a general feature that the characteristic heavy Majorana mass scale $M_N \ll M_s$, since all the entries in (29) are of high order, with several potentially small vev's. This makes the appearance of one or more neutrino masses around 0.1 eV quite natural, as we discuss now.

3.D. Neutrino Mass Textures in Flipped $SU(5)$

As a preliminary to constructing the neutrino mass matrices, we first recall the left-handed charged-lepton assignments motivated earlier: $(e_L, \mu_L, \tau_L) = (\overline{f}_5 - O(\Delta_2\Delta_5)\overline{f}_2, \overline{f}_2 + O(\Delta_2\Delta_5)\overline{f}_5, \overline{f}_1)$. The weak-interaction eigenstates for the light neutrinos must have the same assignments:

$$\nu_e \to \overline{f}_5 - \mathcal{O}(\Delta_2\Delta_5)\overline{f}_2, \quad \nu_\mu \to \overline{f}_2 + \mathcal{O}(\Delta_2\Delta_5)\overline{f}_5 \quad \nu_\tau \to \overline{f}_1 \tag{30}$$

However, it is convenient to work in the basis $(\overline{f}_5, \overline{f}_2, \overline{f}_1)$, which is related to (30) by the rotation

$$V_{\ell_L}^m = \begin{pmatrix} 1 - \frac{1}{2}(\Delta_2\Delta_5)^2 & \Delta_2\Delta_5 & 0 \\ -\Delta_2\Delta_5 & 1 - \frac{1}{2}(\Delta_2\Delta_5)^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{31}$$

As for the massive right-handed neutrinos, the coupling (24) means that ν_{τ_R} has to be assigned to φ_1, since it is the only field to which $\overline{f}_1$ couples at a significant level. In view of the couplings (17,19), we assign $\nu_{\mu R}$ to F_4 and ν_{eR} to F_2.

With these choices of bases, m_ν^D takes the form

$$m_\nu^D = \begin{pmatrix} \Delta_2\Delta_5\overline{\phi}_4 & 1 & 0 \\ \overline{\phi}_4 & \Delta_2\Delta_5 & 0 \\ 0 & 0 & F_1 \end{pmatrix} \tag{32}$$

whilst M_{ν_R} is given by

$$M_{\nu_R} = \begin{pmatrix} \overline{F}_5\overline{F}_5\overline{\phi}_4\phi_3 & \overline{F}_5\overline{F}_5\Delta_2\Delta_5\phi_3 & 0 \\ \overline{F}_5\overline{F}_5\Delta_2\Delta_5\phi_3 & 0 & \overline{F}_5\overline{\Phi}_{31}\Phi_{31}\overline{\phi}_4\phi_2 \\ 0 & \overline{F}_5\overline{\Phi}_{31}\Phi_{31}\overline{\phi}_4\phi_2 & \Delta_2\Delta_5\overline{\Phi}_{23}T_2T_5 \end{pmatrix} \tag{33}$$

The resulting m_{eff} is given by

$$m_{eff} = m_\nu^D \cdot (M_{\nu_R})^{-1} \cdot m_\nu^{D\dagger}, \tag{34}$$

and the neutrino mixing angles in the weak-eigenstate basis (30) are given by (9).

Clearly, the forms of the mass matrices depend on the various field vev's. For these, we have some information from analysis of the flat directions and the rest of the fermion masses, but there is still some arbitrariness. For example, in the cases of the decuplets that break the gauge group down to the Standard Model, we know that the vev's should be $\approx M_{GUT}/M_s$. In weakly-coupled string constructions, this ratio is ≈ 0.02. However, the strong-coupling limit of M theory offers the possibility that the GUT and the string scales can coincide, in which case the vev's could be of order unity.

What about the other fields? The analysis of quark masses suggested that $\Delta_2\Delta_5$ should be of order unity, whilst $\overline{\varphi}_4$ should be suppressed. The analysis of flat directions in [9] indicate that if $\overline{\varphi}_3^2$ is large, as we have suggested in order to get the correct m_e / m_μ ratio, then $\overline{\Phi}_{31}\,\overline{\Phi}_{23}$ is also large. The flatness conditions [9] relate $\overline{\Phi}_{31}$, Φ_{31} and φ_2, and can be satisfied even if all the vev's are large, as long as $\overline{\Phi}_{31}\,\Phi_{31}$ and $\overline{\Phi}_{23}\,\Phi_{23}$ are not very cloce to unity. Finally, we note that nothing yet fixes the value of T_2T_5.

Despite these uncertainties, the following features of the mass matrices are apparent. (i) The heavy Majorana matrix M_{ν_R} is likely to have many entries that may be of comparable magnitudes. In particular, (ii) there are potentially large off-diagonal entries that could yield large $\nu_\mu - \nu_\tau$ and / or $\nu_\mu - \nu_e$ mixing. (iii) The neutrino Dirac matrix is *not* equivalent to m_u, and (iv) is also a potential source of large $\nu_\mu - \nu_e$ mixing. We recall (v) that charged lepton mixing is potentially significant and note that, in general, (vi) the mass matrices (32,33) correspond to the *mismatched mixing* case. Finally, we recall (vii) that there is significant mixing of candidate ν_R states with singlet fields.

A complete analysis of the available parameter space goes beyond the scope of this paper, and would perhaps involve placing more credence in the details of this model than it deserves. Accordingly, we limit ourselves to some general comments on the likelihood of mass degeneracies relative to hierarchies in m_{eff}, and on the plausibility of large mixing in the $\nu_\mu - \nu_\tau$ and $\nu_\mu - \nu_e$ sectors.

To this end, we first consider the following simplified forms for the matrices (14,20,29):

$$M_{\nu_R} = \begin{pmatrix} M & 0 & 0 \\ 0 & 0 & M_{4\phi} \\ 0 & M_{4\phi} & M_{\phi\phi} \end{pmatrix}, \quad V_{\ell_L}^{m\dagger} m_\nu^D = \begin{pmatrix} 0 & \alpha s_\psi & 0 \\ 0 & \alpha c_\psi & 0 \\ 0 & 0 & \gamma \end{pmatrix} \tag{35}$$

where our approximations are to neglect M' —but not to make any other *a priori* assumption about the relative magnitudes of entries in M_{ν_R}— and to neglect terms in $V_{lL}^{m\dagger}$ m_ν^D that are $O\,(\overline{\varphi}_4)$ - again with no *a priori* assumption about the relative magnitudes of other entries. These are parametrized by a, γ and an angle ψ, and we denote $\sin\psi$ by s_ψ, etc. The first approximation could be motivated if φ_3 is negligible [9], and M is eventually generated by some other effect: as we shall see, the magnitude of M is not essential for this simplified analysis. On the other hand, its consistency would require $\overline{F}_5$ to be quite large, as could occur in the strong-coupling limit of M theory, whilst the unknown combination $T_2T_5 \sim \overline{\varphi}_4$.

The inputs (35) yield the following effective light-neutrino mass matrix in the weak interaction basis for the neutrinos

$$m_{eff} = \begin{pmatrix} -\frac{M_{\phi\phi}\alpha^2 s_\psi^2}{M_{4\phi}^2} & -\frac{M_{\phi\phi}s_\psi c_\psi}{M_{4\phi}^2} & \frac{\alpha\gamma s_\psi}{M_{4\phi}} \\ -\frac{M_{\phi\phi}s_\psi c_\psi}{M_{4\phi}^2} & -\frac{M_{\phi\phi}c_\psi^2}{M_{4\phi}^2} & \frac{\alpha\gamma c_\psi}{M_{4\phi}} \\ \frac{\alpha\gamma s_\psi}{M_{4\phi}} & \frac{\alpha\gamma c_\psi}{M_{4\phi}} & 0 \end{pmatrix} \tag{36}$$

Transforming to the basis $(c_\psi\nu_e - s_\psi\nu_\mu)$, $(s_\psi\nu_e + c_\psi\nu_\mu)$, ν_τ, m_{eff} in (36) is easily seen to have the form:

$$m_{eff} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\frac{M_{\phi\phi}\alpha^2}{M_{4\phi}^2} & \frac{\alpha\gamma}{M_{4\phi}} \\ 0 & \frac{\alpha\gamma}{M_{4\phi}} & 0 \end{pmatrix} \tag{37}$$

We therefore see that the three mass eigenstates are:

$$\nu_1 \equiv c_\psi\nu_e - s_\psi\nu_\mu \quad : \quad m_1 = 0 \tag{38}$$

$$\nu_2 \equiv c_\eta(s_\psi\nu_e + c_\psi\nu_\mu) - s_\eta\nu_\tau \quad : \quad m_2 = \frac{2\gamma^2}{M_{\phi\phi} + \sqrt{M_{\phi\phi}^2 + 4M_{4\phi}^2(\gamma/\alpha)^2}} \tag{39}$$

$$\nu_3 \equiv s_\eta(s_\psi\nu_e + c_\psi\nu_\mu) + c_\eta\nu_\tau \quad : \quad m_3 = \frac{2\gamma^2}{M_{\phi\phi} - \sqrt{M_{\phi\phi}^2 + 4M_{4\phi}^2(\gamma/\alpha)^2}} \tag{40}$$

where

$$\sin^2 2\eta = \frac{4(M_{4\phi}\gamma/\alpha)^2}{M_{\phi\phi}^2 + 4(M_{4\phi}\gamma/\alpha)^2} \tag{41}$$

These simple results equip us to answer of the questions raised by the phenomenological analysis of the data.

We see that one neutrino is massless in this simplified picture, but we expect it to acquire a small mass when some of the other mixing effects in (14,20,29) are taken into account. The ratio $|m_3/m_2|$ may be $\gg 1$ if $|M_{4\varphi}\gamma| \ll |M_{\varphi\varphi}\alpha|$, or ≈ 1 if $|M_{4\varphi}\gamma| \ll |M_{\varphi\varphi}\alpha|$. However, obtaining a large hierarchy $|m_3/m_2| \sim 10$, as would be required if $m_3 \sim 10^{-3/2}$ eV and $m_2 \sim 10^{-5/2}$ eV, seems to require less fine tuning than obtaining near-degeneracy: $(m_3^2 - m_2^2)\, m_3^2 \sim 1/100$, as would be required if the neutrino masses were to be cosmologically significant: $m_{2,3} \sim 1$ eV. Moreover, any such degeneracy would be very sensitive to higher-order corrections, and there is no apparent mechanism for making ν_1 approximately degenerate with $\nu_{2,3}$, as would also be required in this scenario.

Large mixing appears naturally in the $\nu_\mu - \nu_e$ sector for generic values of ψ, but its magnitude is model-dependent. In particular, there is the logical possibility of a cancellation between the mixing in $(V_L^m)^\dagger$ and m_ν^D that could suppress it significantly: $\sin\psi \ll 1$. Nevertheless, the large-angle MSW solution seems quite plausible. Large mixing in the $\nu_\mu - \nu_\tau$ sector is also quite generic. The simplified parametrization above might indicate an apparent conflict with a large hierarchy: $|m_3/m_2| \gg 1$. We expect large mixing and a large hierarchy to be quite compatible when the full parameter space of (14,20,29) is explored. Moreover, we should also remember that the effective neutrino mixing angle may be amplified by renormalisation group effects in the case of large $\tan\beta$, so we need not require that the maximal mixing be present already at the GUT scale.

We now consider the complementary possibility, where the field φ_3 develops a large vev. The larger is φ_3, the smaller are m_{ν_μ} and m_{ν_e} with respect to m_{ν_τ}. At this stage, we assume for simplicity that $\varphi_3 \approx 1$ and we define coefficients that keep track of the relation between the various entries of M_{ν_R}. Then, we write M_{ν_R} in (33) as

$$M_{\nu_R} = \begin{pmatrix} \overline{F}_5\overline{F}_5\overline{\phi}_4 & \overline{F}_5\overline{F}_5\Delta_2\Delta_5 & 0 \\ \overline{F}_5\overline{F}_5\Delta_2\Delta_5 & 0 & \overline{F}_5\overline{\phi}_4 \\ 0 & \overline{F}_5\overline{\phi}_4 & \Delta_2\Delta_5\overline{\Phi}_{23}T_2T_5 \end{pmatrix} \equiv \begin{pmatrix} fy^2 & xy^2 & 0 \\ xy^2 & 0 & fy \\ 0 & fy & tx \end{pmatrix} \quad (42)$$

where $\Delta_2\Delta_5 \equiv x$, $T_2T_5 \equiv t$, $\overline{\varphi}_4 \equiv f$ and $\overline{F}_5 \equiv y$. For the Dirac mass matrix, as in the previous case, we have the possibility of cancellations between the charged lepton and neutrino mixing matrices. To simplify the presentation in terms of the mass matrices, we describe two cases separately.

In the absence of a cancellation, the Dirac mass matrix in the weak-eigenstate basis in of the form

$$V_{\ell_L}^{m\dagger}.m_\nu^D \approx \begin{pmatrix} 1 & -gx & 0 \\ gx & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} fx & 1 & 0 \\ f & x & 0 \\ 0 & 0 & y \end{pmatrix} \quad (43)$$

where we have dropped terms of order x^2 in $(V_{lL})^{m\dagger}$. Then,

$$m_{eff} \propto \begin{pmatrix} \begin{matrix} -f^4(-1+g^2)^2x^2 \\ +ftx(1+...) \end{matrix} & \begin{matrix} f^4(-1+g)x(1+gx^2) \\ +ftx^2(1+...) \end{matrix} & -f^2(1+(-2+g)x^2)y^2 \\ \begin{matrix} f^4(-1+g)x(1+gx^2) \\ +ftx^2(1+...) \end{matrix} & \begin{matrix} -f^4(1+gx^2)^2 \\ -ftx^3(1+...) \end{matrix} & f^2x(1+g(-1+2x^2))y^2 \\ -f^2(1+(-2+g)x^2)y^2 & f^2x(1+g(-1+2x^2))y^2 & -4x^2y^4 \end{pmatrix} \quad (44)$$

while

$$\sin^2 2\theta_{23} = \frac{4f^4(x-gx+2gx^3)^2y^4}{(4f^4(x-gx+2gx^3)^2y^4) + (f^4(1+gx^2)^2 + ftx^3(3+...) - 4x^2y^4)} \quad (45)$$

We see therefore that if $\overline{\varphi}_4 \approx \overline{F}_5, F_1$, as would be expected in weak-coupling unification schemes, the entries of m_{eff} are all of the same order of magnitude. In this case, as we discussed in the previous phenomenological analysis, large $\nu_\mu - \nu_e$ and $\nu_\mu - \nu_\tau$ mixings are both generated, whilst cancellations between the various terms can lead to large hierarchies between the neutrino masses.

Suppose now that a cancellation between the charged lepton and the neutrino mixing matrices takes place. In this case, we write

$$V_{\ell_L}^{m} = \begin{pmatrix} 1 & gx & 0 \\ -gx & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (46)$$

where $g \approx (1 - a\overline{\varphi}_4)/x^2$: this leads to (1,2) and (2,1) entries in the Dirac mass matrix of the order of $\overline{\varphi}_4$. In this case,

$$m_{eff} \propto \begin{pmatrix} -2af^2tx(-1+x^2) & -ft(1+x^2)(-1+x^2) & f^2(-1+x^2)y^2 \\ -ft(1+x^2)(-1+x^2) & ft(1-2x^2)/x & f^2(-1+x^2)y^2/x \\ f^2(-1+x^2)y^2 & f^2(-1+x^2)y^2/x & -x^2y^4 \end{pmatrix} \quad (47)$$

and we see a diffrerence from the previous example, in that now all the entries of the (1,2) sector are multiplied by t, and therefore may be suppressed if T_2T_5 is small. The entries for the (2,3) sector are similar to the previous case, with the modification that the (2,2) entry can be nery small. Large (2,3) mixing is again generated for $\overline{\varphi}_4 \approx \overline{F}_5, F_1$.

We conclude this Section by commenting on the possible order of magnitude of neutrino masses in this model, using (40) as our guide. The factor γ appearing in the numerator and denominator is expectd to be $O(1) \times M_W$, since it comes from a trhird-order coupling. The same estimate applies to the factor a appearing in part of the denominator. The factors $M_{4\varphi}$, $M_{\varphi\varphi}$ that also appear there originate from seventh-order couplings, and hence are expected to be considerably smaller, with a typical estimate being $O(10^{-4\pm1}) \times M_s$. Taking $M_s \sim 10^{16}$ to 10^{18} GeV, we might guess that $M_{\varphi\varphi}, M_{4\varphi} \sim 10^{13\pm2}$ GeV. Our final estimate is therefore that

$$m_3 \sim 10^{0\pm2} \text{ eV} \quad (48)$$

which is consistent (within our uncertainties) with the indication provided by the super-Kamiokande data [1] that $m_3^2 \geq 10^{-3}$ eV^2.

We conclude that the flipped $SU(5)$ model appears capable, within its considerable uncertainties, of proving to be consistent with the magnitudes of the neutrino masses and mixing angles suggested by experiment.

4. Conclusions

In this paper we have first analyzed possible patterns of neutrino masses and mixing compatible with the atmospheric and solar neutrino deficits from a purely phenomenological point of view. In particular, we have emphasized that large neutrino mixing as suggested by the super-Kamiokande atmospheric neutrino data [1] does not necessarily require near-degeneracy between a pair of neutrino masses. Equipped with this phenomenological background, we have gone on to discuss neutrino masses and mixing in a 'realistic' flipped $SU(5) \times U(1)$ model derived from string.

The discussion of this part of our paper serves to reinforce the message that, whilst the string selection rules restrict the forms of terms that one may obtain from a specific string-derived model, it is nevertheless possible to obtain realistic patterns of fermion masses and mixings. We had demonstrated this previously for quarks and charged leptons, and have extended that discussion to neutrinos in this paper. In particular, we have shown that it is possible to have contributions which lead to plausible hierarchical magnitudes of neutrino masses, a large mixing angle that could explain the atmospheric neutrino deficit, and either the large —or the small— angle MSW solution to the solar neutrino deficit.

The higher-dimensional operations that we obtain depend only on the choice of string model, but the detailed forms of the mass matrices clearly depend on the choice of flat direction. This introduces some ambiguity, and work remains to be done to demonstrate that the choice made in this paper remains valid to higher orders in the effective superpotential derived from the string model. Despite this apparent freedom in the choice of vev's, the room for manoeuvre in such a string-derived model is quite restricted, and we find it interesting that it is nevertheless possible to obtain a realistic shceme for fermion masses and mixing and even obtain solutions with large neutrino oscillations.

We conclude by stressing again some aspects of our specific model analysis that might be of general interest to model-builders. (i) Once outside the framework of $SO(10)$ -like models, there is no general expectation that the neutrino Dirac mass matrix should be equivalent to the u-quark mass matrix, in particular because (ii) charged-lepton mixing may also be significant, and different from that of d-type quarks. Moreover, (iii) mixing in the heavy Majorana mass matrix is in general mismatched relative to the other mass matrices, leading to a generic expectation of large mixing angles for the light neutrinos. Specifically, this can occur because (iv) the effective ν_R states may include gauge-singlet fields that are not related by GUT symmetries to any Standard Model particles. Finally, we note that, because the heavy Majorana mass matrix elements typically arise from higher-order non-renormalizable terms, (v) it is quite natural that the mass eigenvalues be much smaller than M_s or M_{GUT}, possibly with values $O(10^{13})$ GeV, as would be required to generate a light neutrino mass $O(0.1)$ eV.

This work has been done in collaboration with J. Ellis, G. Leontaris and S. Lola.

Appendix

In this appendix we tabulate for completenesss the field assignment of the 'realistic' flipped $SU(5)$ string model [10], as well as the basic conditions used in [9] to obtain consistent flatness conditions and acceptable Higgs masses.

$F_1(10,\frac{1}{2},-\frac{1}{2},0,0,0)$	$\bar{f}_1(\bar{5},-\frac{3}{2},-\frac{1}{2},0,0,0)$	$\ell_1^c(1,\frac{5}{2},-\frac{1}{2},0,0,0)$
$F_2(10,\frac{1}{2},0,-\frac{1}{2},0,0)$	$\bar{f}_2(\bar{5},-\frac{3}{2},0,-\frac{1}{2},0,0)$	$\ell_2^c(1,\frac{5}{2},0,-\frac{1}{2},0,0)$
$F_3(10,\frac{1}{2},0,0,\frac{1}{2},-\frac{1}{2})$	$\bar{f}_3(\bar{5},-\frac{3}{2},0,0,\frac{1}{2},\frac{1}{2})$	$\ell_3^c(1,\frac{5}{2},0,0,\frac{1}{2},\frac{1}{2})$
$F_4(10,\frac{1}{2},-\frac{1}{2},0,0,0)$	$f_4(5,\frac{3}{2},\frac{1}{2},0,0,0)$	$\bar{\ell}_4^c(1,-\frac{5}{2},\frac{1}{2},0,0,0)$
$\bar{F}_5(\overline{10},-\frac{1}{2},0,\frac{1}{2},0,0)$	$\bar{f}_5(\bar{5},-\frac{3}{2},0,-\frac{1}{2},0,0)$	$\ell_5^c(1,\frac{5}{2},0,-\frac{1}{2},0,0)$

$h_1(5,-1,1,0,0,0)$	$h_2(5,-1,0,1,0,0)$	$h_3(5,-1,0,0,1,0)$
$h_{45}(5,-1,-\frac{1}{2},-\frac{1}{2},0,0)$		

$\phi_{45}(1,0,\frac{1}{2},\frac{1}{2},1,0)$	$\phi_+(1,0,\frac{1}{2},-\frac{1}{2},0,1)$	$\phi_-(1,0,\frac{1}{2},-\frac{1}{2},0,-1)$
$\Phi_{23}(1,0,0,-1,1,0)$	$\Phi_{31}(1,0,1,0,-1,0)$	$\Phi_{12}(1,0,-1,1,0,0)$
$\phi_i(1,0,\frac{1}{2},-\frac{1}{2},0,0)$	$\Phi_i(1,0,0,0,0,0)$	

$\Delta_1(0,1,6,0,-\frac{1}{2},\frac{1}{2},0)$	$\Delta_2(0,1,6,-\frac{1}{2},0,\frac{1}{2},0)$	$\Delta_3(0,1,6,-\frac{1}{2},-\frac{1}{2},0,\frac{1}{2})$
$\Delta_4(0,1,6,0,-\frac{1}{2},\frac{1}{2},0)$	$\Delta_5(0,1,6,\frac{1}{2},0,-\frac{1}{2},0)$	

$T_1(0,10,1,0,-\frac{1}{2},\frac{1}{2},0)$	$T_2(0,10,1,-\frac{1}{2},0,\frac{1}{2},0)$	$T_3(0,10,1,-\frac{1}{2},-\frac{1}{2},0,\frac{1}{2})$
$T_4(0,10,1,0,\frac{1}{2},-\frac{1}{2},0)$	$T_5(0,10,1,-\frac{1}{2},0,\frac{1}{2},0)$	

The chiral superfields are listed with their quantum numbers [10]. The $F_I, \overline{f}_I, l_I^c$, as well as the h_i, h_{ij}, fields and the singlets are listed with their $SU(5) \times U(1)' \times U(1)^4$ quantum numbers. Conjugate fields have opposite $U(1)' \times U(1)^4$ quantum mubers. The fields Δ_i and T_i are tabulated in terms of their $U(1)' \times SO(10) \times SO(6) \times U(1)^4$ quantum numbers.

As can be seen, the matter and Higgs fields in this string model carry additional charges under additional $U(1)$ symmetries [10]. There exist various singlet fields, and hidden-sector matter fields which transform non-trivially under the $SU(4)\times SO(10)$ gauge symmetry, some as sextets under $SU(4)$, namely $\Delta_{1,2,3,4,5}$, and some as decuplets under $SO(10)$, namely $T_{1,2,3,4,5}$. There are also quadruplets of the $SU(4)$ hidden symmetry which posses fractional charges. However, these are confined and will not concern us further.

The usual flavour assignments of the light Standart Model particles in this model are as follows:

$$\begin{aligned} &\overline{f}_1 : \overline{u}, \tau, \quad \overline{f}_2 : \overline{c}, e/\mu, \quad \overline{f}_5 : \overline{t}, \mu/e \\ &F_2 : Q_2, \overline{s}, \quad F_3 : Q_1, \overline{d}, \quad F_4 : Q_3, \overline{b} \\ &\ell_1^c : \overline{\tau}, \quad \ell_2^c : \overline{e}, \quad \ell_5^c : \overline{\mu} \end{aligned} \tag{49}$$

up to mixing effects, which are discussed in more detail in Section 3. We chose non-zero vacuum expectation values for the following singlet and hidden-sector fields:

$$\Phi_{31}, \overline{\Phi}_{31}, \Phi_{23}, \overline{\Phi}_{23}, \phi_2, \overline{\phi}_{3,4}, \phi^-, \overline{\phi}^+, \phi_{45}, \overline{\phi}_{45}, \Delta_{2,3,5}, T_{2,4,5} \tag{50}$$

The vacuum expectation values of the hidden-sector fields must satisfy the additional constraints

$$T_{3,4,5}^2 = T_i \cdot T_4 = 0, \ \Delta_{3,5}^2 = 0, \ T_2^2 + \Delta_2^2 = 0 \tag{51}$$

For further discussion, see [9] and references therein.

REFERENCES

[1] Y. Fukuda et al., Super-Kamiokande collaboration, *Phys. Lett.* **B433** (1998) 9, *ibid*, **B436** (1998) 33, *Phys. Rev. Lett.* **81** (1998) 1562.

[2] S. Hatakeyama et al., Kamiokande collaboration, *Phys. Pev. Lett.* **81** (1998) 2016; M. Ambrosio et al., MACRO collaboration, *Phys. Lett.***B434** (1998) 451; M. Spurio, for the MACRO collaboration, hep-ex/9808001.

[3] M. Apollonio et al., CHOOZ collaboration, *Phys. Lett.* **B420** (1998) 397.

[4] G. K. Leontaris and D. V. Nanopoulos, *Phys. Lett.* **B212** (1988) 327.

[5] K. S. Hirata et al., Kamiokande collaboration, *Phys. Lett.* **B205** (1988) 416, *Phys. Lett.* **B280** (1992) 146; E. W. Beier et al., *Phys. Lett.* **B283** (1992) 446; Y. Fukuda et al., Kamiokande collaboration, *Phys. Lett.* **B335** (1994) 237; K. Munakata et al., Kamiokande collaboration, *Phys. Rev.* **D56** (1997) 23; Y. Oyama et al., Kamiokande collaboration, hep-ex/9706008; D. Casper et al., IMB collaboration, *Phys Rev.* **D46** (1992) 3720; W. W. M. Allison et al., Soudan-2 collaboration, *Phys. Lett.* **B391** (1997) 491.

[6] H. Dreiner, G. K. Leontaris, S. Lola, G. G. Ross and C. Scheich, *Nucl. Phys.* **B436** (1995) 461.
[7] G. K. Leontaris, S. Lola and G. G. Ross, *Nucl. Phys.* **B454** (1995) 25.
[8] G. K. Leontaris, S. Lola and G. G. Ross, *Nucl. Phys.* **B454** (1995) 25. C. Scheich and J. Vergados, *Phys Rev.* **D53** (1996) 6381; S. Lola and J. Vergatos, Progress in Particle and Nuchear Physis **D40** (1998) 71.
[9] J. Ellis, G. K. Leontaris, S. Lola and D. V. Nanopoulos, *Phys. Lett.* **B425** (1998) 86.
[10] I. Antoniadis, J. Ellis, J Hagelin and D. V. Nanopoulos, *Phys. Lett.* **B194** (1987) 231; *Phys. Lett.* **B231** (1989) 65.
[11] H. Georgi and D. V. Nanopoulos, *Nucl. Phys.* **B155** (1979) 52.
[12] L. Wolfenstein. *Phys. Rev.***D17** (1978) 20; S. P. Mikheyev and A. Y. Smirnov, *Yad. Fiz.* **42** (1985) 1441; S. P. Mikheyev and A. Y. Smirnov, *Yad. Fiz.* **42** (1986) 1441; S. P. Mikheyev and A. Y. Smirnov, *Sov. J. Nucl. Phys.* **42** (1986) 913.
[13] E. L. Wright et al., *Astron. J.* **396** (1992) L13; M. Davis et al., *Nature* **359** (1992) 393; A. N. Taylor and M. Rowan-Robinson, *ibid*. 359 (1992) 396; J. Primack, J. Holtzman, A. Klypin and D. O. Caldwell, *Phys. Rev. Lett.* **74** (1995) 2160; K. S. Babu, R. K. Schaefer and Q. Shafi, *Phys. Rev.* **D53** (1996) 606.
[14] S. Perlmutter et al., Supernova Cosmology Project, *Nature* **391** (1998) 51; A. G. Kim et al., Fermilabe preprint PUB-98-037 (1998); B. P. Schmidt et al., astro-ph/9805200; A. G. Riess et al., *Astron. J.* **116** (1998) 1009.
[15] J. L. Lopez and D. V. Nanopoulos, *Phys. Lett.* **B251** (1990) 73; *Phys. Lett.* **B268** (1991) 359.
[16] J. Rizos and K. Tamvakis, *Phys. Lett.* **B251** (1990) 369.

ΠΕΡΙΛΗΨΗ

Νετρίνα μὲ μάζες σὲ ἀνεστραμμένο $SU(5)$

Πρόσφατα πειραματικὰ δεδομένα γιὰ τὰ ἀτμοσφαιρικὰ νετρίνα, ἀπὸ τὴν ὁμάδα Super-Kamiokande, ὑποδεικνύουν τὴν ὕπαρξη μαζῶν γιὰ τὰ νετρίνα. Μελετοῦμε ἐδῶ, τὶς προβλέψεις γιὰ τὸ Φάσμα μαζῶν τῶν νετρίνων, σὲ ἕνα ἀπὸ τὸ πιὸ πετυχημένα πρότυπα ἑνωποιημένων θεωριῶν, ὅπως πηγάζει ἀπὸ τὴν Θεωρία Ὑπερχορδῶν, τὸ "ἀνεστραμμένο $SU(5)$". Τὰ ἀποτελέσματά μας εἶναι πολὺ ἐνθαρρυντικά, καθὼς προκύπτει ὅτι οἱ προβλεπόμενες μάζες καὶ γωνίες μείξεως τῶν νετρίνων, φαίνεται νὰ ἐξηγοῦν πλήρως τὰ νέα πειραματικὰ δεδομένα. Ἐπίσης, πρέπει νὰ τονιστεῖ τὸ γεγονὸς ὅτι τὸ "ἀνεστραμμένο $SU(5)$" ἔχει κάνει στὸ παρελθὸν καὶ ἄλλες σημαντικὲς προβλέψεις γιὰ τὶς μάζες τῶν quark καὶ λεπτονίων, ὅπως π.χ. γιὰ τὴν μάζα τοῦ top-quark, $m \approx 175$ GeV.