ΑΝΑΚΟΙΝΩΣΕΙΣ ΜΕΛΩΝ

ΑΣΤΡΟΝΟΜΙΑ.— **Probable Values of the Time of Rise for the Forthcoming Sunspot Cycles,** *by John Xanthakis* *.

It has been shown[1] that the values of the time of rise corresponding to the successive sunspot cycles do not seem to show periodicity. If we consider, however, the values of the time of rise for the cycles of the same polarity i. e. the cycles corresponding to the odd and even values of the current number N separately, we find a more or less clear tendency for the minima of the time of rise to recur every 8 cycles for cycles with $N = 2K + 1$ and every 10 cycles for cycles with $N = 2K$, $K = 0, \pm 1, \pm 2, \ldots$

It will be shown in the following that each of these two groups of cycles, $N = 2K + 1$ and $N = 2K$, can be subdivided into two subgroups, each of which presents an individual periodic variation. The periods of these variations are very probably multiples of 8 sunspot cycles for cycles with $N = 2K + 1$ and 10 sunspot cycles for cycles with $N = 2K$. In fact, if we consider the values of the time of rise $T_R^*$ corresponding to the sunspot cycles $N = -12$ (1610,8 — 1619,0) to $N = 3$ (1775,5 — 1784,7) determined on the basis of the smoothed mean monthly relative sunspot numbers[2] as well as the values of the time of rise $T_R$ corresponding to the latter sunspot cycles, $N = 4$ to $N = 19$, determined on the basis of

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* ΙΩΑΝΝΟΥ ΞΑΝΘΑΚΗ, Ἔκφρασις τοῦ χρόνου ἀνόδου συναρτήσει τοῦ ἀριθμοῦ τῶν ἡλιακῶν κύκλων.

the observed mean monthly relative sunspot numbers[1], then the existing observational data show that these values can be satisfactorily represented by the help of the following relations :

### a) Odd Cycles

$$(1) \quad T = \left[ a_1 - b\,(3 - \Psi_1)\sin\,(N-1)\frac{2\pi}{64} \right]\cos^2\,(N-1)\,\frac{2\pi}{8} +$$

$$+ \left[ a_2 + 2b\sin\,(N-1)\frac{2\pi}{24} - 4b\cos^2(N-1)\frac{2\pi}{24}\sin^2\frac{N+1}{4}\frac{\pi}{2} + \Psi_2 \right]\sin^2(N-1)\frac{2\pi}{8}$$

### b) Even Cycles

$$(2) \quad T = \left[ a_3 - b\sin(N-5)\frac{2\pi}{40} + \frac{3}{2}b\sin(N-5)\frac{2\pi}{5} - \Psi_3 \right]J + a_4\left[ 1 - \frac{1}{4}\cos(N-4)\frac{2\pi}{20} \right](1-J)$$

where :

$J = 1$ for the sunspot cycles with $N = 10K$, $10K - 2$, $10K - 4$

$J = 0$ » » » » » $N = 10K - 6$, $10K - 8$, $K = 0, \pm 1, \pm 2, \ldots$

$a_1 = 6,4 \quad a_2 = 5,0 \quad a_3 = 4,65 \quad a_4 = 4,4 \quad b = 0,85.$

In the above relations (1) and (2) the quantities $\Psi_1$, $\Psi_2$, and $\Psi_3$ represent additional periodic terms acting only during certain sunspot cycles, that is

$$\Psi_1 = 2\left[ 1 - \frac{3}{4}\sin\,(N-7)\frac{2\pi}{40} \right]\sin^2\frac{N-1}{4}\frac{\pi}{2}$$

$$\Psi_2 = \frac{2}{3}\,b\cos N\,\frac{2\pi}{24}\cos K\pi, \quad N = 4K - 1, \quad K = 0,1 \qquad 6,7 \qquad 12,13 \ldots.$$

$$\Psi_3 = \frac{3}{2}\,b\cos N\,\frac{2\pi}{40}\cos K\pi, \quad N = 2K - 2, \quad K = -1,0 \qquad 19,20 \qquad 39,40 \ldots.$$

The continuous lines (a) and (b) in the upper part of fig. (1) and (2) represent, respectively, the first (a) and the second (b) term of the relations (1) and (2). In these figures the small circles represent the observed values of the time of rise while the dots connected with straight lines illustrate the values of T computed by the help of relations (1) and (2).

From Table I which gives the values of T as well as the observed values of the time of rise $T_R^*$ (for the sunspot cycles $N = -12$ to $N = 3$) and the time of rise $T_R$ (for the sunspot cycles $N = 4$ to $N = 19$), we see that relations (1) and (2) represent the values of $T_R^*$ and $T_R$ with an
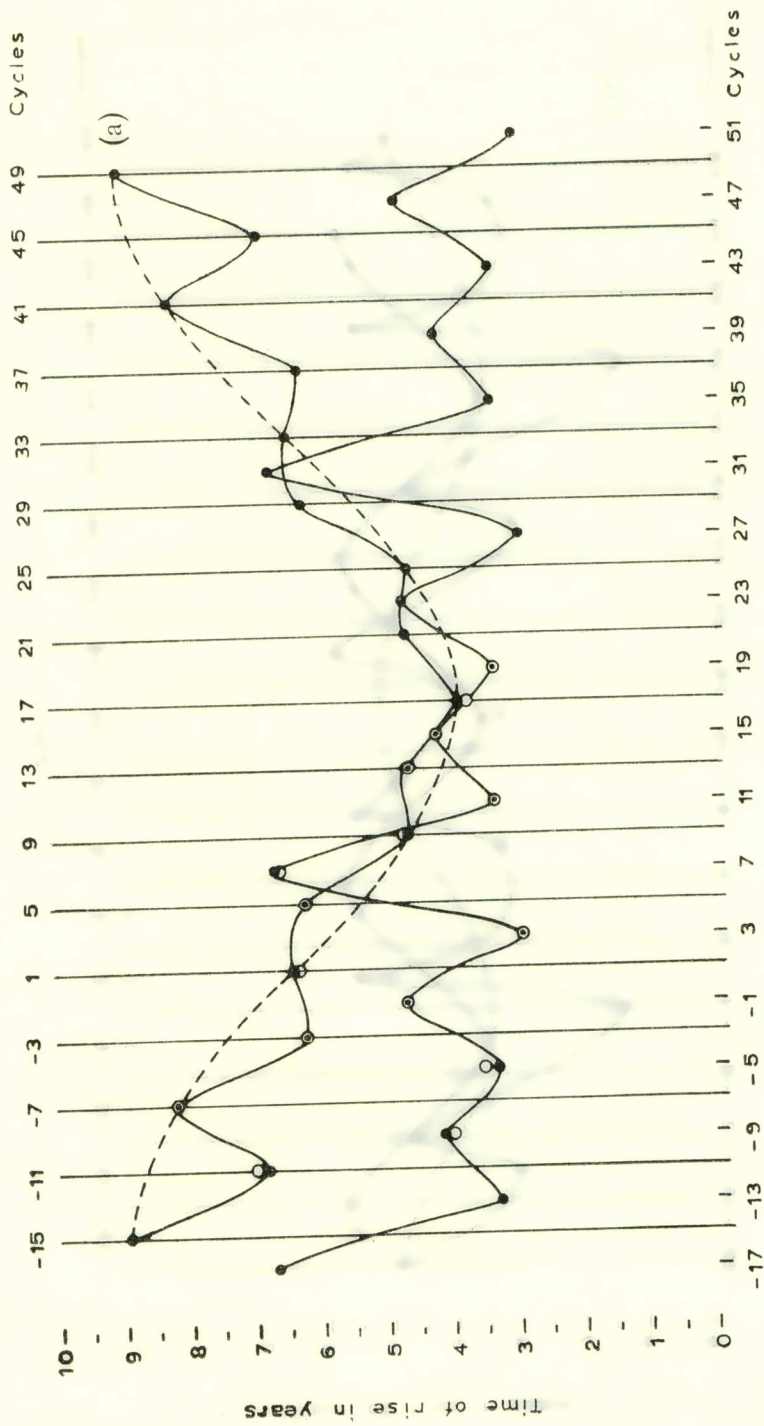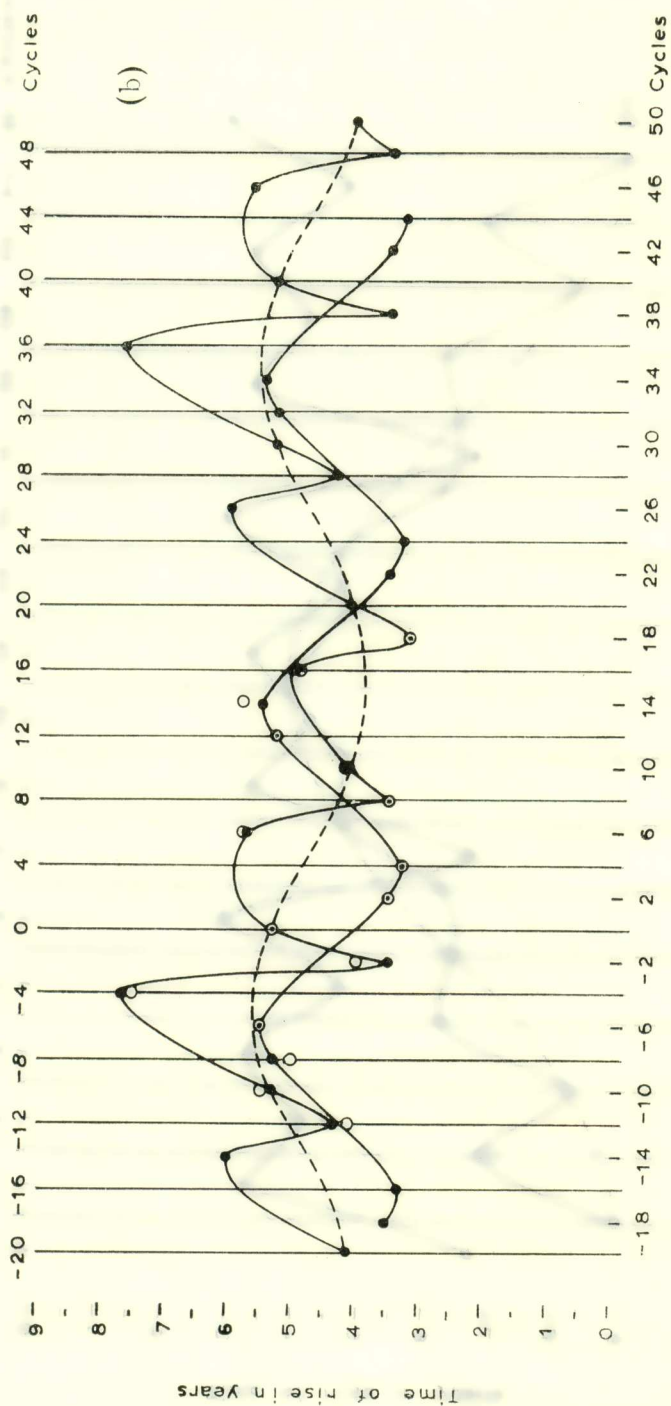
Fig. 1.

Fig. 2.

accuracy equal to $(1 - \dfrac{\sigma}{T_R})$ 100 % = 97 %. Despite this high degree of accuracy, however, the values of the time of rise for the forthcoming sunspot cycles found by extrapolating equations (1) and (2) should be considered with due caution, because in these relations some periodic terms with periods as long as 64 cycles are present, while the observational data on which relations (1) and (2) are based do not refer to more than 32 cycles.

T A B L E   I

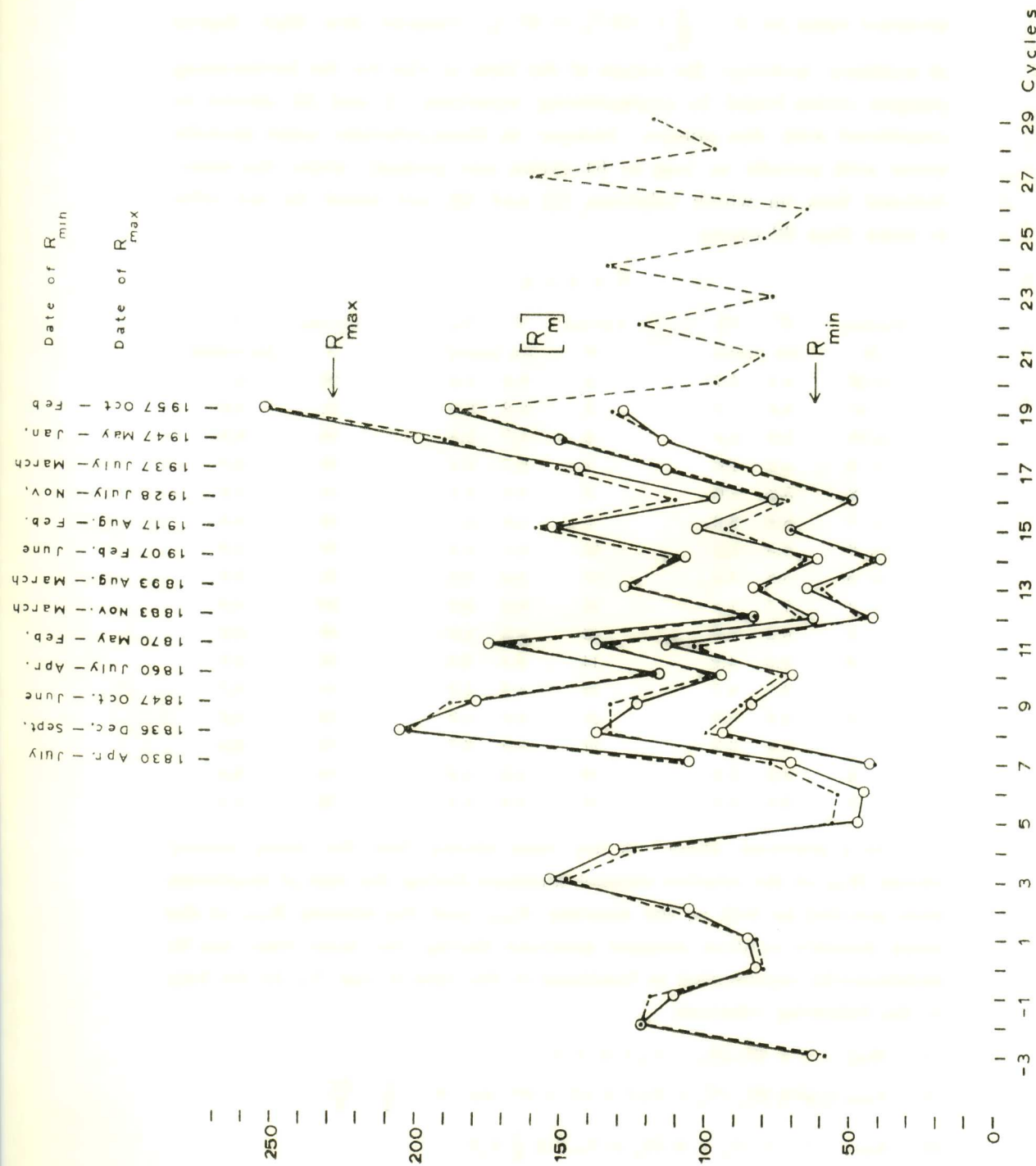| Cycles N | T (in years) | $T_R^*$ | Cycles N | T (in years) | $T_R$ | Cycles N | T (in years) |
|---|---|---|---|---|---|---|---|
| — 12 | 4,3 | 4,7 | 4 | 3,3 | 3,3 | 20 | 4,1 |
| — 11 | 6,8 | 7,0 | 5 | 6,2 | 6,2 | 21 | 4,6 |
| — 10 | 5,3 | 5,5 | 6 | 5,7 | 5,8 | 22 | 3,5 |
| — 9 | 4,2 | 4,0 | 7 | 6,7 | 6,6 | 23 | 4,7 |
| — 8 | 5,3 | 5,0 | 8 | 3,5 | 3,5 | 24 | 3,3 |
| — 7 | 8,2 | 8,2 | 9 | 4,6 | 4,7 | 25 | 4,6 |
| — 6 | 5,5 | 5,5 | 10 | 4,1 | 4,2 | 26 | 6,0 |
| — 5 | 3,3 | 3,5 | 11 | 3,3 | 3,3 | 27 | 2,9 |
| — 4 | 7,7 | 7,5 | 12 | 5,3 | 5,3 | 28 | 4,3 |
| — 3 | 6,2 | 6,2 | 13 | 4,6 | 4,6 | 29 | 6,2 |
| — 2 | 3,5 | 4,0 | 14 | 5,5 | 5,8 | 30 | 5,3 |
| — 1 | 4,7 | 4,7 | 15 | 4,2 | 4,2 | 31 | 6,7 |
| 0 | 5,3 | 5,3 | 16 | 5,0 | 4,9 | 32 | 5,3 |
| 1 | 6,4 | 6,3 | 17 | 3,8 | 3,7 | 33 | 6,4 |
| 2 | 3,5 | 3,5 | 18 | 3,2 | 3,2 | 34 | 5,5 |
| 3 | 2,9 | 2,9 | 19 | 3,3 | 3,3 | 35 | 3,3 |

In a previous paper[3] it has been shown that the mean annual values $[R_m]$ of the relative sunspot numbers during the year of maximum solar activity as well as the maxima $R_{max}$ and the minima $R_{min}$ of the mean monthly relative sunspot numbers during the same year, can be satisfactorily represented as functions of the time of rise $T_R$ by the help of the following relations :

(3)  $[R_m] = C + 2T_0 (T_0 - T_R)^2 + T_0 Y$

(4)  $R_{max} = a^2 + 2T_0' (T_0' - T_R)^2 + aY + 4T_0' \sin \left( N - \dfrac{a}{2} \right) \dfrac{2\pi}{8}$

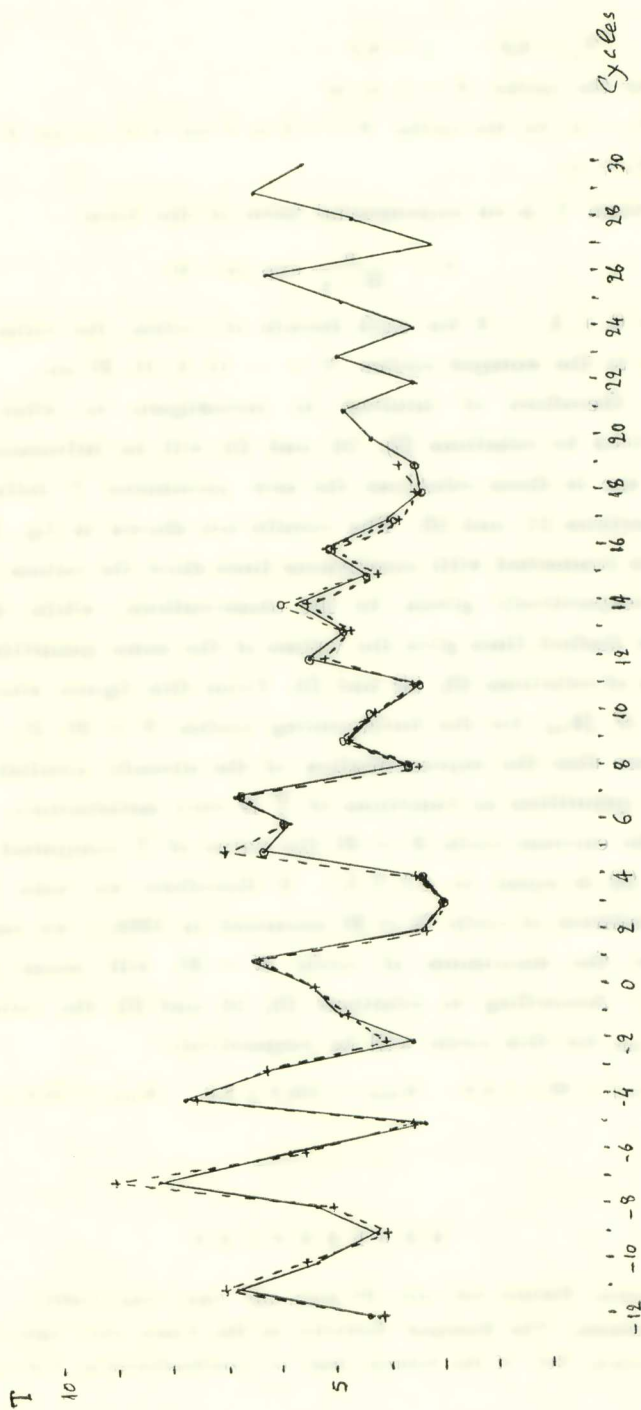(5)  $R_{min} = T_0'^2 + (T_0' + 1) (T_0' - T_R)^2 + \dfrac{1}{2} T_0' Y$

Fig. 4.

where

$T_0 = 5,76$     $T'_0 = 6,3$     $a = 9,0$

$C = 2T_0^2$ for the cycles $N = 7$ to $19$

$C = 2T_0 (T_0 - 1)$ for the cycles $N = -3$ to $6$ but with cycles $N = 0$ and $N = 1$

$C = 2T_0 (T_0 + 1)$.

The term Y is an exponential term of the form

$$Y = \frac{n}{10 - n} \exp [n - 9]$$

where $n = 0, 1, 2, \ldots 9$ for each decade of cycles, the value $n = 0$ corresponding to the sunspot cycles $N = -10, 0, 10, 20$ etc.

It is therefore of interest to investigate to what extent the accuracy given by relations (3), (4) and (5) will be influenced if instead of $T_R$ we use in these relations the new parameter T defined with the help of equations (1) and (2). The results are shown in fig. 3, where the small circles connected with continuous lines show the values of $R_{max}$, $[R_m]$ and $R_{min}$ respectively given by the observations, while the dots connected with dashed lines give the values of the same quantities computed by the help of relations (3), (4) and (5). From this figure where the probable value of $[R_m]$ for the forthcoming cycles $N = 20, 21, \ldots$ are also given, we see that the representation of the already available values of these three quantities as functions of T is very satisfactory.

For the current cycle $N = 20$ the value of T computed by the help of relation (2) is equal to $4,1 \pm 0,1$. If therefore we take into account that the minimum of cycle $N = 20$ occurred in 1964,7, we reach the conclusion that the maximum of cycle $N = 20$ will occur probably in $1968,8 \pm 0,1$. According to relations (3), (4) and (5) the values of $[R_m]$, $R_{max}$ and $R_{min}$ for this cycle will be respectively:

$[R_m] = 98,1 \pm 4,0$     $R_{max} = 132,3 \pm 8,0$     $R_{min} = 75,0 \pm 5,0$

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REFERENCES

1. J. XANTHAKIS, Nature vol. 210, No 5042, pp. 1242 - 1243 (1966).
2. M. WALDMEIER, The Sunspot Activity in the Years 1610 - 1960, Zürich (1961).
3. J. XANTHAKIS, Bul. of the Astron. Inst. of Czechoslovakia, Vol. 17, No 5 (1966).

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Ὁ Ἀκαδημαϊκὸς κ. **Ἰω. Ξανθάκης** παρουσιάζων τὴν ὡς ἄνω ἀνακοίνωσίν του εἶπε τὰ ἑξῆς :

Εἰς προηγουμένην ἀνακοίνωσίν μας εἰς τὴν Ἀκαδημίαν Ἀθηνῶν ἀνεφέραμεν ὅτι ἡ λύσις τοῦ προβλήματος τῆς προγνώσεως τῆς ἡλιακῆς δραστηριότητος ἐξαρτᾶται πλέον μόνον ἀπὸ τὴν μαθηματικὴν ἔκφρασιν τοῦ χρόνου ἀνόδου τῶν ἡλιακῶν κύκλων συναρτήσει τοῦ ἀριθμοῦ τῶν διαδοχικῶν κύκλων. Συνάμα δὲ διετυπώσαμεν τὴν γνώμην ὅτι ἡ βασικὴ αὕτη παράμετρος, δηλαδὴ ὁ χρόνος ἀνόδου, ἀποτελεῖ τὸν μοναδικὸν ἄγνωστον τοῦ ὅλου προβλήματος.

Ἡ πρόβλεψις ὅμως τῶν ἑκάστοτε τιμῶν τοῦ χρόνου ἀνόδου, ὅταν ὡς δείκτης τῆς ἡλιακῆς δραστηριότητος λαμβάνεται ἡ κλῖμαξ τῶν σχετικῶν ἀριθμῶν ἢ ἀριθμῶν WOLF, ὅπως συνήθως καλοῦνται, εἶναι ἐξαιρετικὰ δυσχερής.

Ἡ δυσχέρεια αὕτη προέρχεται ἀφ' ἑνὸς μὲν ἀπὸ τὴν ἔλλειψιν ἐπαρκῶν δεδομένων ἐκ τῶν παρατηρήσεων, ἀφ' ἑτέρου δὲ ἀπὸ τὸ γεγονὸς ὅτι ἡ μεταβολὴ τῆς παραμέτρου ταύτης συναρτήσει τοῦ αὔξοντος ἀριθμοῦ τῶν ἡλιακῶν κύκλων εἶναι ἐντελῶς ἀνώμαλος. Αἱ γνωσταὶ μαθηματικαὶ μέθοδοι τῆς ἁρμονικῆς ἀναλύσεως καὶ τῶν περιοδοδιαγραμμάτων δὲν δύνανται νὰ ἐφαρμοσθῶσιν ἐπὶ τοῦ προκειμένου. Ἦτο συνεπῶς ἀναγκαῖον τὸ πρόβλημα τοῦτο νὰ ἀντιμετωπισθῇ δι' εἰδικῆς μεθόδου. Τὴν εἰδικὴν ταύτην μέθοδον ὡς καὶ τὸν τρόπον ἐφαρμογῆς της ἐκθέτομεν εἰς τὴν σημερινὴν ἀνακοίνωσίν μας.

Τὰ βασικὰ χαρακτηριστικὰ τῆς μεθόδου ταύτης εἶναι τὰ ἀκόλουθα :

1) Θεωροῦμεν κεχωρισμένως τοὺς ἡλιακοὺς κύκλους μὲ ἄρτιον καὶ περιττὸν αὔξοντα ἀριθμόν. Τοῦτο ἀπὸ φυσικῆς ἀπόψεως σημαίνει ὅτι λαμβάνομεν ὑπ' ὄψιν τὸν νόμον τοῦ HALE, δηλαδὴ τὸν νόμον τῆς ἀντιστροφῆς τῆς πολικότητος τῶν ἡλιακῶν κηλίδων ἀπὸ κύκλου εἰς κύκλον. Ὁ διαχωρισμὸς αὐτὸς δεικνύει σαφῶς ὅτι τὰ ἐλάχιστα τῶν τιμῶν τοῦ χρόνου ἀνόδου ἐπαναλαμβάνονται περιοδικῶς ἀνὰ 8 ἡλιακοὺς κύκλους διὰ τοὺς ἔχοντας περιττὸν αὔξοντα ἀριθμὸν καὶ ἀνὰ 10 ἡλιακοὺς κύκλους διὰ τοὺς ἔχοντας ἄρτιον τοιοῦτον.

2) Χωρίζομεν τόσον τοὺς ἀρτίους ὅσον καὶ τοὺς περιττοὺς ἡλιακοὺς κύκλους εἰς δύο ὁμάδας, δι' ἑκάστην τῶν ὁποίων εὑρίσκομεν ἰδίαν μαθηματικὴν ἔκφρασιν τοῦ χρόνου ἀνόδου συναρτήσει τοῦ ἀριθμοῦ τῶν κύκλων. Εἰς ἑκάστην δὲ τῶν μαθηματικῶν τούτων ἐκφράσεων ὑπεισέρχονται περιοδικοὶ ὅροι τῶν ὁποίων αἱ περίοδοι εἶναι πολλαπλάσια τῶν 8 καὶ τῶν 10 ἡλιακῶν κύκλων ἀντιστοίχως. Ἡ σύγκρισις τῶν τιμῶν τοῦ χρόνου ἀνόδου τῶν ὑπολογιζομένων ἐκ τῶν εὑρεθεισῶν μαθηματικῶν σχέσεων μετὰ τῶν τιμῶν τῆς παραμέτρου ταύτης τῶν παρεχομένων ὑπὸ τοῦ συνόλου τῶν παρατηρήσεων ἀπὸ τοῦ 1610 μέχρι σήμερον εἶναι ἐξαιρετικῶς ἱκανοποιητική. Ὁ βαθμὸς ἀκριβείας τῆς παραστάσεως τῶν δεδομένων

ἀνέρχεται εἰς 97 °/₀ περίπου, ὅσος εἶναι δηλαδὴ καὶ ὁ βαθμὸς ἀκριβείας τῶν νεωτέρων παρατηρήσεων.

Πλὴν ὅμως, παρὰ τὸν ὑψηλὸν τοῦτον βαθμὸν ἀκριβείας, ὡρισμέναι ἐπιφυλάξεις εἶναι ἀναγκαῖαι, δεδομένου ὅτι τὸ μέχρι τοῦδε ὑλικὸν τῶν παρατηρήσεων δὲν καλύπτει ἐξ ὁλοκλήρου τὰς μεταβολὰς μακρᾶς περιόδου.

Διὰ τὸν τρέχοντα ἡλιακὸν κύκλον, τοῦ ὁποίου τὸ μέγιστον προβλέπεται κατὰ Σεπτέμβριον ἢ Ὀκτώβριον 1968, αἱ ἀντίστοιχοι τιμαὶ τῶν ἀριθμῶν WOLF αἱ ἐξαγόμεναι ἐκ τῶν θεωρητικῶν μας σχέσεων εἶναι αἱ κάτωθι:

     Διὰ τὸν μέσον ἐτήσιον τῶν ἀριθμῶν WOLF    98,1 ± 4.0

     Διὰ τὸν μέγιστον τῶν μηνιαίων ἀριθμῶν WOLF   132,3 ± 8.0

καὶ  Διὰ τὸν ἐλάχιστον . . . . . . . . . .    75,0 ± 5.0

     Κατὰ τὸ τέλος τοῦ προσεχοῦς ἔτους θὰ εἶναι πᾶς τις εἰς θέσιν νὰ ἐλέγξῃ κατὰ πόσον αἱ προβλέψεις μας αὗται εἶναι ἐπιτυχεῖς.