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ΜΗΧΑΝΙΚΗ. — **Failure Criteria in Fiber Composites,** by *Academician Pericles S. Theocaris**

ABSTRACT

The paraboloidal failure criterion for isotropic materials was extended to transtropic materials by maintaining the direction of its axis of symmetry and the form of its surface. It was shown that the paraboloid of revolution failure surface becomes an elliptic paraboloid surface, it has its axis of symmetry parallelly translated to the hydrostatic axis and its sections normal to this axis become ellipses of ellipticity and orientation depending on the amount of anisotropy of the material. Examples are shown from graphite-epoxy composites.

1. INTRODUCTION

Failure criteria are based on Hill's well known theory for anisotropic metals [1] which is based on Mises' first attempt to formulate failure in anisotropic solids [2]. While Hill's criterion does not take into account the strength differential effect apparent in all solids, Hoffman's criterion presents a further improvement by incorporating this effect in Hill's criterion. This was achieved by adding the linear terms in the quadratic expression of Hill's criterion [3].

The tensor polynomial criterion introduced by Tsai and Wu [4] constituted a flexible and mathematically elegant version of a criterion, formulated by means of the

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Cartesian components of the stress tensor, is represented by hypersurfaces in the six-dimensional stress space, impossible to be readily visualized geometrically in a physical stress space. Only plane sections of this hypersurface were therefore studied representing quadric surfaces in the (σ_x, σ_y, σ_{xy}) parametric space. However, even these subspaces do not yield a direct interrelation with the directions of the externally imposed loading and the material strength directions, a drawback causing the necessity of meticulous and delicate experiments for its definition.

Theocaris [5] has presented recently a paraboloid of revolution failure criterion for isotropic bodies, which, later on, was extended to an elliptic paraboloid, convenient for anisotropic materials [6, 7]. The physical basis and properties of this elliptic paraboloid criterion is presented in this paper suitable for transtropic materials, as they are the fiber-reinforced composites.

2. THE ELLIPTIC PARABOLOID FAILURE SURFACE

Consider a transtropic body with σ_{T1} and σ_{C1} the longitudinal (strong) failure strengths in tension (T) and compression (C) and σ_{T2}, σ_{C2} its respective transverse strengths on the isotropic plane, which is normal to the longitudinal (fiber) axis. When the principal stress axes coincide with the material principal strength directions the failure surface should pass through the points A_1 (σ_{T1}, 0,0), A_2 (0, σ_{T2}, 0), A_3 (0, 0, σ_{T2}) and B_1 ($-\sigma_{C1}$, 0, 0), B_2 (0, $-\sigma_{C2}$, 0) and B_3 (0, 0, $-\sigma_{C2}$). The equation of a quadratic surface passing through these points and having its axis of symmetry parallel to the hydrostatic axis ($\sigma_1 = \sigma_2 = \sigma_3$) is given by [6]:

$$\frac{1}{\sigma_{T1}\,\sigma_{C1}}\sigma_1^2 + \frac{1}{\sigma_{T2}\,\sigma_{C2}}(\sigma_2^2 + \sigma_3^2) - \frac{1}{\sigma_{T1}\,\sigma_{C1}}\sigma_1\sigma_2 + \left(\frac{1}{\sigma_{T1}\,\sigma_{C1}} - \frac{2}{\sigma_{T2}\,\sigma_{C2}}\right)\sigma_2\sigma_3 -$$

$$- \frac{1}{\sigma_{T1}\,\sigma_{C1}}\sigma_3\sigma_1 + \left(\frac{1}{\sigma_{T1}} - \frac{1}{\sigma_{C1}}\right)\sigma_1 + \left(\frac{1}{\sigma_{T2}} - \frac{1}{\sigma_{C2}}\right)(\sigma_2 + \sigma_3) = 1 \qquad (1)$$

Since relation (1) is valid only for coincidence of the principal stress directions with the material principal strength directions, for the case of an arbitrary orientation of these two systems another elliptic paraboloid is associated with these directions. This paraboloid should have its axis of symmetry parallel to the hydrostatic axis and it is expressed by the same relation (1) in which the appropriate failure strengths in these directions were calculated by using the appropriate transformation and Hoffman's criterion.

Therefore, for any orientation of the principal directions of external loading the association of the convenient elliptic paraboloid defined with the appropriate strength properties of the material along the principal stress directions yields a direct, clear and comprehensive view of safe loading paths on the structure and the correct evaluation of its respective load bearing capacities in these directions.

For plane-stress states of transtropic materials along a principal stress plane, say the (σ_1,σ_2)-plane $(\sigma_3=0)$, the failure locus is derived as the intersection of the ellipsoid (1) by the plane $\sigma_3=0$, it yields an ellipse whose equation is given by:

$$\frac{\sigma_1^2}{\sigma_{T1}\,\sigma_{C1}}+\frac{\sigma_2^2}{\sigma_{T2}\,\sigma_{C2}}-\frac{\sigma_1\sigma_2}{\sigma_{T1}\,\sigma_{C1}}+\left(\frac{1}{\sigma_{T1}}-\frac{1}{\sigma_{C1}}\right)\sigma_1+\left(\frac{1}{\sigma_{T2}}-\frac{1}{\sigma_{C2}}\right)\sigma_2-1=0 \qquad (2)$$

For a complete understanding of the topography of the failure surface and especially for the three dimensional states of stress, besides the principal intersections of the elliptic paraboloid with the planes $\sigma_1=0$, $\sigma_2=0$ or $\sigma_3=0$, whose expressions are analogous to relation (2), other interesting sections are worthwhile studying. Thus, the intersections of the paraboloid either with diagonal planes, expressed by $\sigma_1=\sigma_2$, $\sigma_2=\sigma_3$ or $\sigma_3=\sigma_1$, or with planes parallel to the deviatoric plane $(\sigma_1+\sigma_2+\sigma_3)=0$ and expressed by $(\sigma_1+\sigma_2+\sigma_3)=k$, where k is a constant, are necessary.

The intersection of the paraboloid by the diagonal plane $\sigma_2=\sigma_3$, which contains the strong principal σ_1-axis, is a parabola whose axis of symmetry is generally parallel to the projection of the hydrostatic axis on this plane. For this particular diagonal plane, containing the σ_1-strong axis, this plane does contain both the hydrostatic axis and the axis of symmetry of the parabola. In other words, this parabola splits the paraboloid into two equal and symmetric halves.

This parabola, expressed in the plane $(\sigma_1,\bar{\sigma})$, where $\bar{\sigma}=\sqrt{2}\sigma_2=\sqrt{2}\sigma_3$, is given by:

$$\frac{\sigma_1^2}{\sigma_{T1}\,\sigma_{C1}}+\frac{\bar{\sigma}^2}{2\sigma_{T2}\,\sigma_{C2}}-\frac{\sqrt{2}\sigma_1\bar{\sigma}}{\sigma_{T1}\,\sigma_{C1}}+\left(\frac{1}{\sigma_{T1}}-\frac{1}{\sigma_{C1}}\right)\sigma_1+\sqrt{2}\left(\frac{1}{\sigma_{T2}}-\frac{1}{\sigma_{C2}}\right)\bar{\sigma}-1=0 \qquad (3)$$

Introducing the characteristic quantities of the anisotropy of the transtropic material we define the strength differential parameters along the strong direction R_L and the weak plane R_T, as well as the single parameter of anisotropy of the transtropic material R_{LT}. These quantities are expressed by:

$$R_L=\frac{\sigma_{C1}}{\sigma_{T1}},\ R_T=\frac{\sigma_{C2}}{\sigma_{T2}}\ \text{and}\ R_{LT}=\frac{\sigma_{T1}}{\sigma_{T2}} \qquad (4)$$

With these definitions the distance d, normalized to the strong tensile failure stress, σ_{T1}, between the hydrostatic axis and the axis of symmetry of the elliptic paraboloid for the transtropic material is expressed by:

$$\frac{d}{\sigma_{T1}} = \frac{\sqrt{6}R_L}{9}\left\{(1 - \frac{1}{R_L}) - R_{LT}\,(1 - \frac{1}{R_T})\right\} \tag{5}$$

Which, of course, for isotropic materials, where R_L=R_T=R and R_{LT}=1 becomes equal to zero. Indeed, the axis of symmetry of the paraboloid for isotropic materials coincides with the hydrostatic axis.

Finally, the intersections of the symmetric elliptic paraboloid by planes $(\sigma_1 + \sigma_2 + \sigma_3) = k$ along at distance p from the origin of coordinates to the hydrostatic axis are ellipses, which, when projected on the deviatoric plane $(\sigma_1 + \sigma_2 + \sigma_3) = 0$ are expressed by:

$$\sqrt{3}\,(\frac{1}{\sigma_{T1}\,\sigma_{C1}} + \frac{1}{\sigma_{T2}\,\sigma_{C2}})\,x^2 + 3\,(\frac{1}{\sigma_{T1}\,\sigma_{C1}} - \frac{1}{\sigma_{T2}\,\sigma_{C2}})xy + \frac{3\sqrt{3}}{2}(\frac{1}{\sigma_{T1}\sigma_{C1}})y^2 +$$

$$+\frac{\sqrt{6}}{2}\,[(\frac{1}{\sigma_{T2}} - \frac{1}{\sigma_{C2}}) - (\frac{1}{\sigma_{T1}} - \frac{1}{\sigma_{C1}})]\,x + \frac{\sqrt{2}}{2}\,[(\frac{1}{\sigma_{T2}} - \frac{1}{\sigma_{C2}}) - (\frac{1}{\sigma_{T1}} - \frac{1}{\sigma_{C1}})]\,y +$$

$$p\,[(\frac{1}{\sigma_{T1}} - \frac{1}{\sigma_{C1}}) + 2\,(\frac{1}{\sigma_{T2}} - \frac{1}{\sigma_{C2}})] - \sqrt{3} = 0 \tag{6}$$

Then, the distance p of the untersecton of the paraboloid by the hydrostatic axs from the origin of the coordinate system $(\sigma_1,\sigma_2,\sigma_3)$ is expressed by:

$$\frac{p}{\sigma_{T1}} = \frac{\sqrt{3}}{(1 - \frac{1}{R_L}) + 2R_{LT}\,(1 - \frac{1}{R_L})} \tag{7}$$

For an isotropic material relation (7) reduces to the well known relation [5]:

$$\frac{p}{\sigma} = \frac{R}{\sqrt{3}\,(R\text{-}1)} \tag{8}$$

since $\sigma_{T1} = \sigma_{T2} = \sigma_T$, $\sigma_{C1} = \sigma_{C2} = \sigma_C$, $R_L = R$, and $R_{LT} = 1$.

3. APPLICATION TO A FIBER UNIDIRECTIONAL COMPOSITE

As a typical example for applying the elliptic paraboloid criterion we use the data known from a graphite-epoxy composite system, which is a transtropic material with the following failure strengths in its principal directions of anisotropy:

$\sigma_{T1} = 1033.50MPa$, $\sigma_{C1} = 689MPa$, $\sigma_{T2} = 41.34MPa$, $\sigma_{C2} = 117.13MPa$
and:

$$R_L = 0.667, \quad R_T = 2.833 \quad \text{and } R_{LT} = 25.00. \tag{9}$$

Figure 1a presents for the failure surface of this composite the intersections of this elliptic paraboloid by planes parallel to the σ_3=0 plane at distances k from this plane equal to k=-8σ_{T2}, -6σ_{T2}, -4σ_{T2}, -2$_{T2}$,0 and 2σ_{T2}. The projection of the axis of symmetry of the paraboloid, which passes through the centers of the elliptic sections is inclined by an angle of 45° to the σ_1-or σ_2-axes. Fig. 1b presents the intersection of the elliptic paraboloid with the plane σ_3=0.

Exactly the same patterns we derive from intersections of the paraboloid with planes parallel to the σ_2=0 plane for the same parametric values of k, since the section of the elliptic paraboloid by the σ_2=0 plane is expressed by an identical equation.

All these intersections are ellipses with their centers lying on the axis of symmetry of the paraboloid. This axis lies parallel to the diagonal plane (σ, $\bar{\sigma}$) at a distance

$$d=3.025\sigma_{T1}=3126.34MPa$$

from the hydrostatic axis.

The ellipse derived from the intersection of the elliptic paraboloid and the deviatoric plane ($\sigma_1+\sigma_2+\sigma_3$)=0 presents the following characteristics:

The coordinates of its center in the deviatoric plane:

$$\sigma_{1a}/\sigma_{T1}= -\frac{\sqrt{2}R_L}{6}\left\{R_{LT}\left(1-\frac{1}{R_T}\right)-\left(1-\frac{1}{R_L}\right)\right\}$$

$$\bar{\sigma}_a/\sigma_{T1}=\frac{\sqrt{6}R_L}{18}\left\{R_{LT}\left(1-\frac{1}{R_T}\right)-\left(1-\frac{1}{R_L}\right)\right\} \tag{11}$$

The major axis of the ellipse subtends an angle θ_a with the projection of the σ_1-axis on the deviatoric plane, which is independent of the particular mechanical characteristic properties of each transtropic material. This angle is always equal to

$$\theta_a = 30°$$

The coordinate system (σ_{1a},σ_a) is chosen so tha the σ_{1a}-axis corresponds to the projection of the σ_1-principal stress axis on the deviatoric plane, whereas the σ_a-axis corresponds to the straight line coinciding with the projections of the σ_2-and σ_3-axes on the same plane. The positive direction of the σ_a-axis coincides with the positive direction of the projection of the σ_3-axis.

Then, from the trihedron formed by the deviatoric plane, the (σ_1,σ_{1a})-plane and the plane defined by the σ_1-axis and the major axis of the elliptic section, it is easy to define the angle θ_a subtended by the major axis of the ellipse and the σ_1-axis which again is a universal constant for all transtropic materials and equal to

$$\theta_d = 52° 12 \tag{13}$$

The coordinates σ_{1a} and σ_a of the ellipse on the deviatoric plane for the composite defined by the quantities (9) are as follows:

$$\begin{aligned} &\sigma_{1a}=-2.618\sigma_{T1}=-2705.70\mathrm{MPa} \\ &\bar{\sigma}_a=1.517\sigma_{T1}=1567.82\mathrm{MPa} \end{aligned} \tag{14}$$

The elliptic intersection indicated in Fig. 1b is expressed by:

$$\frac{1}{\sigma_{T1}\,\sigma_{C1}}\sigma_1^2+\frac{1}{\sigma_{T2}\,\sigma_{C2}}\,\sigma_2^2-\frac{1}{\sigma_{T1}\,\sigma_{C1}}\sigma_1\sigma_2+\left(\frac{1}{\sigma_{T1}}-\frac{1}{\sigma_{C1}}\right)\sigma_1+\left(\frac{1}{\sigma_{T2}}-\frac{1}{\sigma_{C2}}\right)\sigma_2-1=0$$

The coordinates of its center and the inclination of its major axis to the σ_1-axis are expressed by:

$$\frac{1}{\sigma_{T1}}(x_c,y_c)=\left\{-R_LR_{LT}\frac{\left[\frac{1}{R_L}\left(1-\frac{1}{R_L}\right)+\frac{2R_{LT}}{R_T}\left(1-\frac{1}{R_L}\right)\right]}{\left(\frac{4R_{LT}^2}{R_T}-\frac{1}{R_L}\right)},\right.$$

$$\left.-\frac{\left[\left(1\quad\frac{1}{R_L}\right)+2R_{LT}\left(1-\frac{1}{R_T}\right)\right]}{\left(\frac{4R_{LT}^2}{R_T}\quad\frac{1}{R_L}\right)}\right\} \tag{15}$$

and the angle θ_c subtended by the major axis of the ellipse and the σ_1-axis is given by:

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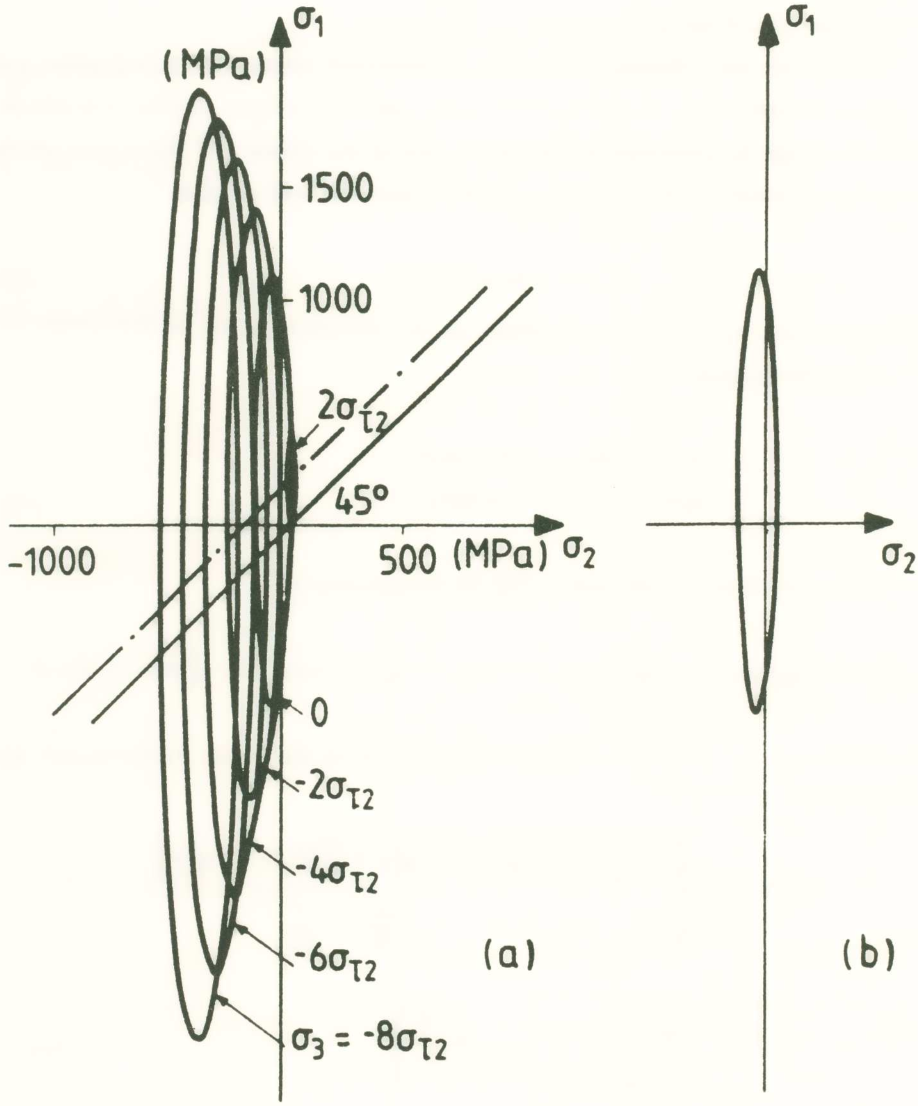

Fig. 1

(a) Intersections of the elliptic paraboloid for a graphite epoxy composite by σ_3=k constant planes.

(b) The failure locus in the (σ_3=0) plane.

$$\theta_c = \frac{1}{2}\tan^{-1}\left[\frac{1}{(R_T R_{LT}^2/R_T - 1)}\right] \tag{16}$$

The coordinates x_c,y_c and the angle θ_c for the ellipse of Fig. 1b are evaluated as follows:

$$x_c=0.147\sigma_{T1} \qquad y_c=-0.036\sigma_{T1} \qquad \theta_c= \frac{1}{2}\tan^{-1}(0.00684)$$

The angle θ_c is less than 10 minutes, that is the major axis of the ellipse is parallel to the σ_1-axis of the principal stress space.

From Eq. (6) it can be readily also derived that the distance between the mtersection of the elliptic paraboloid and the hydrostatic axis and the origin of the coordinate system (σ_1, σ_2, σ_3) is expressed by:

$$p = \frac{\sqrt{3}}{\left(\frac{1}{\sigma_{T1}} - \frac{1}{\sigma_{C1}}\right) + 2\left(\frac{1}{\sigma_{T2}} - \frac{1}{\sigma_{C2}}\right)} \tag{17}$$

It is obvious from this relation that for transtropic materials, which are more and more anisotropic, and especially as their strength differential parameters R_L and R_T take higher values, the distances of the vertices of their failure surfaces approach closer and closer the deviatoric plane. Then, the elliptic paraboloids become more and more shallow.

On the contrary, for low anisotropies and small strength differential parameters the elliptic paraboloids have their vertices receding to infinity, their distances from the hydrostatic axis are reduced and their elliptic shapes tend to circular ones, having as a general limit the cylindrical surface defined from the Mises yield criterion, valid for isotropic materials without any strength differential effect.

Fig. 1a indicates this phenomenon presenting sections of a strongly anisotropic material, where the values of strength differential parameters R_L and R_T recede strongly from unity, the one being below and the other above this limiting value.

It may be readily derived that, as these values recede from unity and as the coefficient of anisotropy of the material R_{LT} is increasing the shape of the elliptic paraboloid becomes more and more oblong, so that its elliptic sections by the principal stress planes or by the parallel to the deviatoric planes take the shapes of "cigars".

Moreover, from the previously derived relations it may be argued that the term $\{R_{LT}(1 - 1/R_T) - (1 - 1/R_L)\}$, or similar to this term expressions, is critical to the properties of the material. It may be established that the part depending on the strong strength differential parameter $(1 - 1/R_L)$ contributes only a few percent to the characteristic dimensions of the failure surface. For instance, for the graphite-epoxy composite this term contributes only 3 percent and may be eliminated without introducing great errors.

Therefore, it may be concluded that, as the parameter of anisotropy is increasing, the other important characteristic quantity defining the failure surface of a composite is its transverse (weak) strength differential parameter.

Figure 2 presents the intersection of the elliptic paraboloid by the diagonal $(\sigma_1,\bar{\sigma})$-plane for a KEVLAR 49 (u_f=0.60), which at room temperature has the following mechanical properties σ_{T1}=1379MPa R_L=0.20, R_T=4.662 and R_{LT}=46.587 [8].

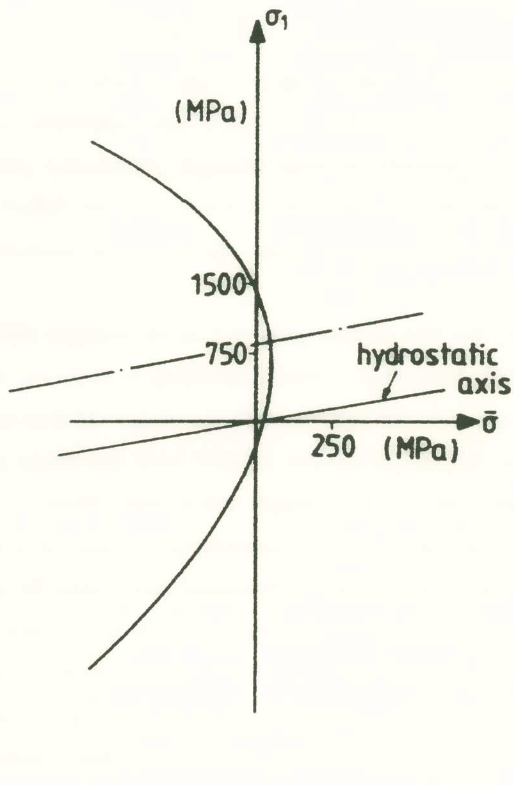

Fig. 2

Intersection of the elliptic paraboloid failure surface on the diagonal (σ_2=σ_3)-plane for KEVLAR 49 (u_f=0.60) transtropic composite.

The distance d/σ_{T1} of the axis of symmetry of this elliptic paraboloid from the origin of coordinates is given by

$$d=2.191\sigma_{T1}=3021.39\text{MPa}$$

Again, for this composite, whose strength differential parameters R_L and R_T deviate strongly from unity on both sides, the paraboloid, whose section along the diagonal $(\sigma_1,\bar{\sigma})$-plane is given in Fig. 2, is again a shallow one.

Figure 3 shows another aspect of the shape of the paraboloid by presenting sections of the elliptic paraboloid for the graphite epoxy composite by planes parallel to the deviatoric plane for parametric values of $k=-\sigma_{C2}$, 0 and σ_{C2}. Again, the presented ellipses have the shape of cigars. It is worthwhile indicating again that this paraboloid has its vertex on the tensile-tensile-tensile octant, since the ellipse for $k=\sigma_{C2}$ lies incide the other two ellipses.

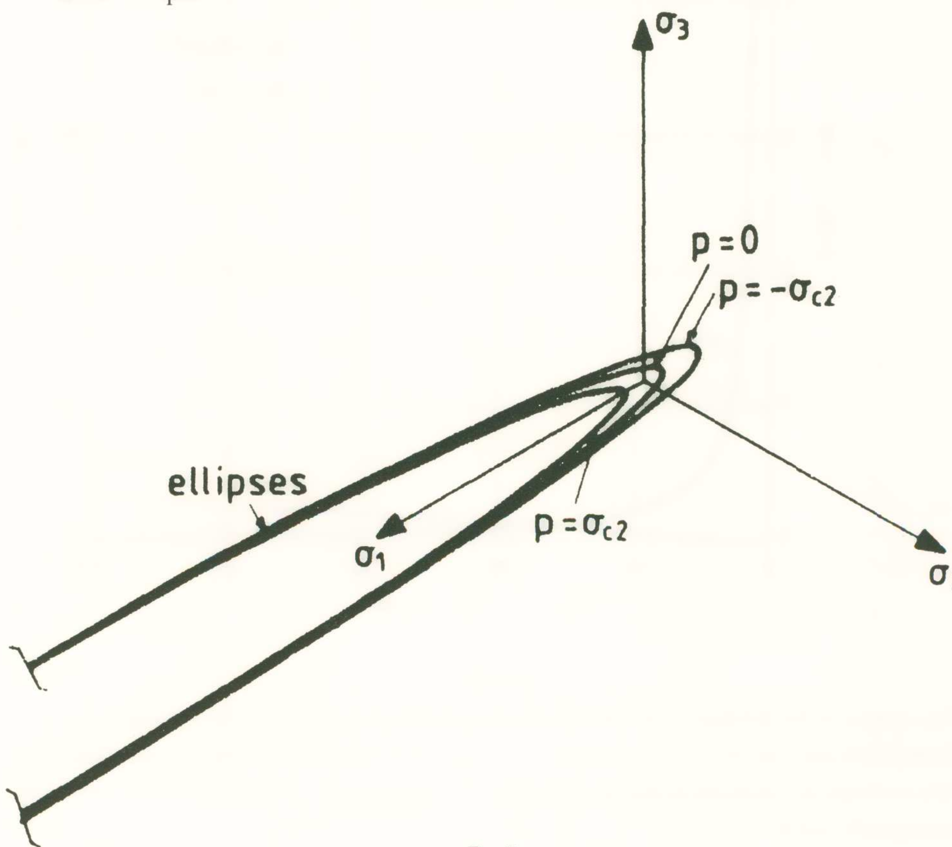

Fig. 3

Intersections of the elliptic paraboloid failure surface for the graphite-epoxy composite by planes parallel to deviatoric plane $(\sigma_1+\sigma_2+\sigma_3)=0$.

Finally, Fig. 4 presents the variation of the distances p of the paraboloid along the hydrostatic axis and the distances d between their axes and the hydrostatic axis, normalized to the failure strength σ_{T1} of the material, versus the parameter of anisotropy R_{LT} for a transtropic material with R_L=0.667 and R_T=1.60. It is clear from this figure, again, that, while the distance d increases linearly with the parameter of anisotropy R_{LT}, the distance p reduces very rapidly as R_{LT} is increasing.

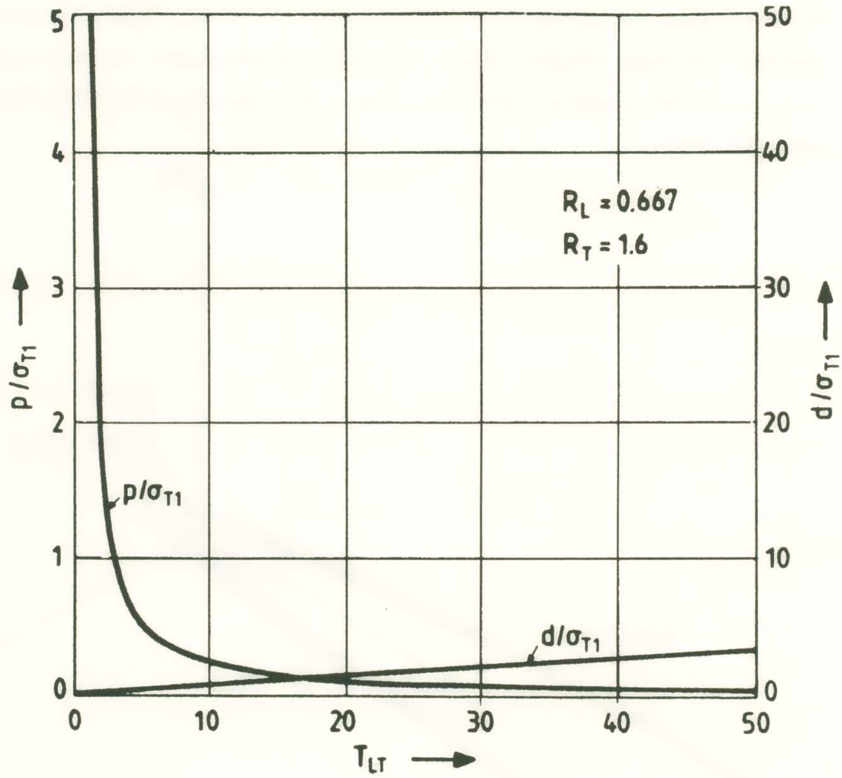

Fig. 4

The variation of the distances p and d of the utersections of the paraboloid wote the hydrostatic axis from the deviatoric plane and the axes of the paraboloids from the hydrostatic axis respectively, normalized to the failure strength σ_{T1}, versus the parameter of anisotropy R_{LT} for particular values of the strength differential parameters R_L and R_T.

4. CONCLUSIONS

In this paper it has been shown that the failure surface for anisotropic materials, and especially for transtropic materials, is an elliptic paraboloid surface whose axis of symmetry is parallel to the hydrostatic axis.

The intersections of this surface by planes normal to the axis of symmetry are ellipses of the same ellipticity and orientation. For the transtropic materials the orientation of the major axes of all such ellipses is constant and equal to $\theta_a=30°$ to the projection of the strong principal axis on the deviatoric plane.

The distance, d, between the hydrostatic and the axis of symmetry of the paraboloids increases with increasing anisotropy. Similarly, the ellipticity of the intersections of the paraboloid increases and they take the form of cigars, as the strength differential parameters recede from unity and the parameter of anisotropy is increasing.

The critical quantities influencing the shape and position of the elliptic paraboloid are mainly the parameter of anisotropy and the transverse strength differential parameter.

Higher anisotropy yields shallow paraboloids, whereas weak anisotropy results in oblong shapes along the hydrostatic direction.

ΠΕΡΙΛΗΨΗ

Χαρακτηριστικαὶ ἰδιότητες τῶν κριτηρίων ἀστοχίας ἰνωδῶν συνθέτων ὑλικῶν

Τὸ εἰς τὸν χῶρον τῶν κυρίων τάσεων παραβολοειδὲς κριτήριον ἀστοχίας τῶν ἰσοτρόπων ὑλικῶν, τὸ ὁποῖον περιεγράφη εἰς προγενεστέραν μου ἀνακοίνωσιν, ἔχει τὰς βασικὰς ἰδιότητας νὰ εἶναι παραβολοειδὲς ἐκ περιστροφῆς μὲ ἄξονα συμμετρίας τὸν ὑδροστατικὸν ἄξονα $\sigma_1 = \sigma_2 = \sigma_3$. Τὸ κριτήριον αὐτὸ ἐπεκτείνεται εἰς τὴν παροῦσαν ἀνακοίνωσιν διὰ τὰ ἀνισότροπα ὑλικὰ καὶ ἰδιαιτέρως διὰ τὰ ἐγκαρσίως ἰσότροπα τοιαῦτα, τὰ ὁποῖα ἀποτελοῦν μεγάλην κατηγορίαν συνθέτων ὑλικῶν μὲ ἐνισχύσεις, λεπτὰς ἶνας. Τὰ ὑλικὰ αὐτὰ συνέβαλαν μεγάλως εἰς τὴν σύγχρονον βιομηχανίαν τῶν κατασκευῶν, χρησιμοποιούμενα σήμερον εὐρέως εἰς τὰς κατασκευὰς αὐτοκινήτων, ἀεροπλάνων, πλευστῶν μέσων, διαστημοπλοίων ἀλλὰ καὶ εἰς πᾶσαν ἄλλην σύγχρονον κατασκευήν.

Εἰς τὴν ἀνακοίνωσιν αὐτὴν ἀποδεικνύεται ὅτι τὸ παραβολοειδὲς ἐκ περιστροφῆς, τὸ ἰσχύον διὰ τὰ ἰσότροπα ὑλικά, μετατρέπεται εἰς ἐλλειπτικὸν παραβολοειδές, τοῦ ὁποίου ὁ ἄξων συμμετρίας εἶναι πλέον παράλληλος πρὸς τὸν ὑδροστατικὸν ἄξονα. Ἡ ἀπόστασις τῶν ἀξόνων αὐτῶν αὐξάνει γραμμικῶς μὲ τὸν βαθμὸν ἀνισοτροπίας τοῦ ὑλικοῦ, καὶ ἡ κορυφὴ τοῦ παραβολοειδοῦς πλησιάζει πρὸς τὸ ἀποκλίνον ἐπίπεδον τῶν τάσεων καὶ ἑπομένως τὴν ἀρχὴν τῶν ἀξόνων, ὅταν ἡ ἀνισοτροπία αὐτὴ αὐξάνει. Περαιτέρω, διὰ τὴν αὐτὴν αὔξησιν τῆς ἀνισοτροπίας τὸ παραβολοειδὲς καθίσταται ἀνοικτότερον καὶ ρηχότερον.

Εἰδικῶς, διὰ τὰ ἐλλειπτικὰ παραβολοειδῆ τὰ περιγράφοντα τὰ ἐγκαρσίως ἰσότροπα ὑλικά, οἱ μεγάλοι ἄξονες τῶν ἐλλειπτικῶν τομῶν τοῦ παραβολοειδοῦς ὑπὸ ἐπιπέδων καθέτων πρὸς τὸ ὑδροστατικὸν ἄξονα συμμετρίας των εἶναι κεκλιμένοι ὑπὸ γωνίαν 30 μοιρῶν ὡς πρὸς τὸ διαγώνιον ἐπίπεδον τὸ περιλαμβάνον τὸν σ_1 —κύριον ἄξονα.

Διὰ τὴν πληρεστέραν μελέτην τῆς μορφῆς καὶ τῶν ἰδιοτήτων τῶν ἐπιφανειῶν αὐτῶν αἱ κύριαι τομαὶ των ἀπὸ τὰ ἐπίπεδα (σ_1, σ_2), (σ_2, σ_3) καὶ (σ_3, σ_1), καθὼς καὶ τὰ κύρια διαγώνια ἐπίπεδα (σ_1, σ_{23}), (σ_2, σ_{13}) καὶ (σ_3, σ_{12}), μελετῶνται. Κατ' αὐτὸν τὸν τρόπον συμπληρώνονται ὅλαι αἱ δυναταὶ τομαὶ τῶν ἐπιφανειῶν αὐτῶν εἰς τὸν χῶρον καὶ αἱ ἰδιότητες τῶν ἐλλειπτικῶν αὐτῶν παραβολοειδῶν προβάλλονται καταλλήλως.

Παραδείγματα ἐκ τῶν συνθέτων ἰνωδῶν ὑλικῶν τῆς κατηγορίας συνθέτου ὑλικοῦ ἐξ ἰνῶν ἄνθρακος καὶ ἐποξειδικῶν ρητινῶν, καθὼς καὶ τοῦ εὐρέως διαδεδομένου σήμερον συνθέτου ὑλικοῦ ὑπὸ τὴν ὀνομασίαν KEVLAR 49, δίδονται, καὶ ἐξετάζονται αἱ ἰδιότητές των ἐν συναρτήσει πρὸς τὰ ἀντίστοιχα ἐλλειπτικὰ παραβολοειδῆ, τὰ περιγράφοντα τὸν χῶρον ἀντοχῆς καὶ κατὰ συνέπειαν καὶ τὸν χῶρον ἀστοχίας των.

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