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ΜΑΘΗΜΑΤΙΚΑ.— **A Study of the Emission of Stress Waves During Impact and Fracture,** *by* ***P. S. Theocaris,*** *Fellow of the Academy* *.

Ἀνεκοινώθη ὑπὸ τοῦ Ἀκαδημαϊκοῦ κ. Περικλῆ Θεοχάρη.

ABSTRACT

Emission of body and surface waves during impact and fracture of plates under conditions of plane-stress and plane-strain were studied. When a load is dynamically applied at a point on the edge of a semi-infinite plate two waves are produced, a longitudinal wave and a shear wave.

On the other hand, the non-equilibrated tensile stress in the fractured section of a plate creates an elastic wave, which travels radially along the plate at the sound velocity. Moreover, the high surface deformation around the crack tip, when conditions of plane stress are prevailing, propagates as a surface wave, following fracture of this zone, at the respective Rayleigh-wave velocity. All these waves travel with a circular wavefront, if the plate is uniformly loaded. In the case of a fast running crack emitting Rayleigh waves, the moving singularity of the crack tip causes a highly dynamic stress field of varying intensity with time allover the specimen. This dynamic stress field results in a significant change of the mechanical properties of a strain-rate dependent material and therefore it influences the velocity of propagation and the scheme of fracture-Rayleigh wavefronts.

All the above-mentioned phenomena were studied experimentally and qualitative and quantitative analyses are given.

* Π. Σ. ΘΕΟΧΑΡΗ, **Μελέτη τῶν κατὰ τὴν κροῦσιν καὶ θραῦσιν ἐκπεμπομένων ἐλαστικῶν κυμάτων.**

1. INTRODUCTION

Several investigators have considered the propagation of stress waves in infinite plates, due to various forms of loading. Mathematical treatments dealing with some phases of the problem have been presented among others by Kolsky [1], Sherwood [2], Cagniard [3] and Ewing, Jardetsky and Press [4].

Roesler [5] and Christie [6] have employed photoelastic methods to study phenomena associated with wave propagation and especially reflection at glancing angle. Of special interest is also the interaction of stress waves with the interface between two dissimilar media, with the resulting generation of reflected, refracted and head waves [7].

The theoretical study of the waveguide effect in plates has been studied by Redwood [8], while Schardin [9] and Beinert [10] treated this problem experimentally by the Schlieren-optical method.

The phenomenon of wave emission during fracture has concentrated the interest of a large number of scientists, after the work of Miklowitz [11] on the afterfailure fracture in bars. However, the case of fractured bars is simpler than the wave emission, created by the two-dimensional crack propagation in plates.

When fracture is occurred in a tensile rod, the unbalanced tensile and moment stresses in the fractured section create compressional (unloading) and flexural waves, respectively [12]. The crack initiated at a single point of a section of the bar extends radially and no singularity of stresses is developed. Contrary to this case, the crack propagation in a plate, creating a two-dimensional stress field, may generate such singular stress states.

Indeed, the motion of a fast-running crack with stable velocity is a dynamic effect, which results in a continuous change of the stresses in any given point of the plate. This change takes place with the transversal and longitudinal wave velocities in the medium. When a crack starts to grow, stress discontinuities arise across the radiated longitudinal and shear wavefronts. These waves are generated by the abrupt change of the stress intensity factor from its static to a dynamic value [13]. On the other hand, Rayleigh waves are generated on the surface of the crack lips and on the surface of the plate, because of the dynamic application of stresses and displacements vertically to these surfaces, respectively.

The present study constitutes an overview for describing and explaining wave-propagation phenomena during impact, methods of visualization of fracture waves and velocity aspects of Rayleigh-fracture waves, emitted by a moving crack, in a strain-rate dependent elastic medium.

2. IMPACT WAVES

Elastic half-space theory defines two basic types of waves, body waves and surface waves. The body waves are of two types, compression waves (P-waves) and shear waves (S-waves) and there is one surface wave, the Rayleigh wave (R-wave).

Elastic waves may originate in many ways, from earthquakes, explosion pile-driving operations, or vibrating machine footings. The source of elastic waves may be contained within the half-space, or may be on its surface. The energy, coupled into the soil by a surface source, is transmitted away from the source by a combination of P-, S-, and R-waves.

Fig. 1 indicates that body waves propagate radially outward from the source along hemispherical wave fronts, and that the Rayleigh waves propagate outward on a cylindrical wave front. All waves encounter an increasingly larger volume of material, as they travel outward, and, therefore, the energy density in each wave decreases with its distance from the source. This decrease in energy-density, or decrease in displacement amplitude is called *geometrical damping.*

The geometrical-damping law, governing body waves, is expressed by r^{-1}, except along the surface where it is r^{-2}, and the geometrical-damping law for the Rayleigh wave is given by $r^{-1/2}$.

For a vertically oscillating circular energy source on the surface of a homogeneous, isotropic, elastic half-space, Miller and Pursey [**14**] have determined the distribution of total input energy among the three elastic waves to be 67% Rayleigh wave, 26% shear wave, and 7% compression wave. The fact that two-thirds of the total input energy is transmitted away from a surface energy-source by the Rayleigh wave, and that the Rayleigh wave decays much more slowly with distance than the body waves, indicate that the Rayleigh wave is of primary concern for foundation isolation problems.

The velocities of the elastic waves are [**1**] :

$$C_d = \left[\frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)}\right]^{1/2} \tag{1}$$

Region	Wave present
1	P
2	P+SP
3	P+SP+S
4	P+S

$$\frac{\sin a_d}{C_d} = \frac{\sin a_s}{C_s}$$

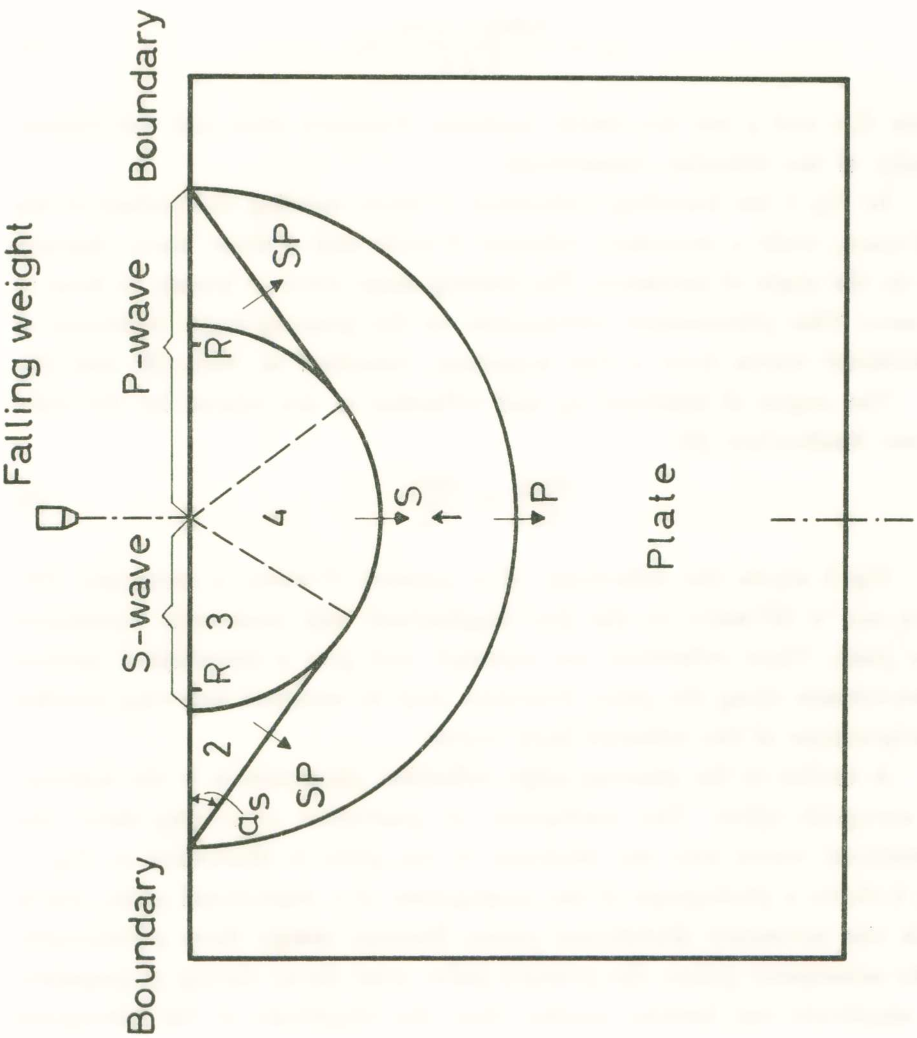

Fig. 1. Body and surface waves emitted by impact in a plate.

where C_d is the velocity of dilatational wave (P-wave) in an unbounded medium, whereas :

$$C_d = \left[\frac{E}{\rho\,(1-\nu^2)}\right]^{1/2} \tag{2}$$

the velocity of dilatational wave in an infinite plate. The velocities of shear and Rayleigh waves respectively, both for unbounded media and infinite plates are expressed by :

$$C_s = \left[\frac{E}{2\,(1+\nu)\,\rho}\right]^{1/2} \tag{3}$$

$$C_R = \frac{0{,}862 + 1{,}14\nu}{1+\nu}\,C_s \tag{4}$$

where E, ν and ρ are the elastic modulus, Poisson's ratio and the volume density of the material, respectively.

In Fig. 1 the travelling cylindrical P-wave, meeting the surface of the half-space, trails a secondary reflected P-wave and a shear wave, depending on the angle of incidence. The trailing shear wave is known as *head* or *SP-wave*. This phenomenon corresponds to the glancing-angle reflection of dilatational waves from a free boundary, described in Refs. [**5**] and [**6**].

The angles of incidence a_d and reflection a_s are related by the well-known Snellius'law [**5**] :

$$\frac{\sin a_d}{C_d} = \frac{\sin a_s}{C_s} \tag{5}$$

Fig. 2 shows the reflections of a primary P-wave, a secondary PP-wave and a SP-wave on the free longitudinal and transwerse boundaries of a plate. These reflections are repeated and give a complicated pattern of wavefronts along the plate. Fractures may be occurred following possible superpositions of the reflected body waves.

A similar to the glancing-angle reflection phenomenon is the *mechanical waveguide effect*. The mechanism of generation of trailing shear and dilatational waves into the thickness of the plate is illustrated in Fig. 3. Fig. 4 shows a photograph of the propagation of a dilatational pulse, which trails also secondary dilatational pulses. Because energy flows permanently to its subsequent pulses, the primary pulse must decay during propagation. Its amplitude can become smaller than the amplitude of the subsequent

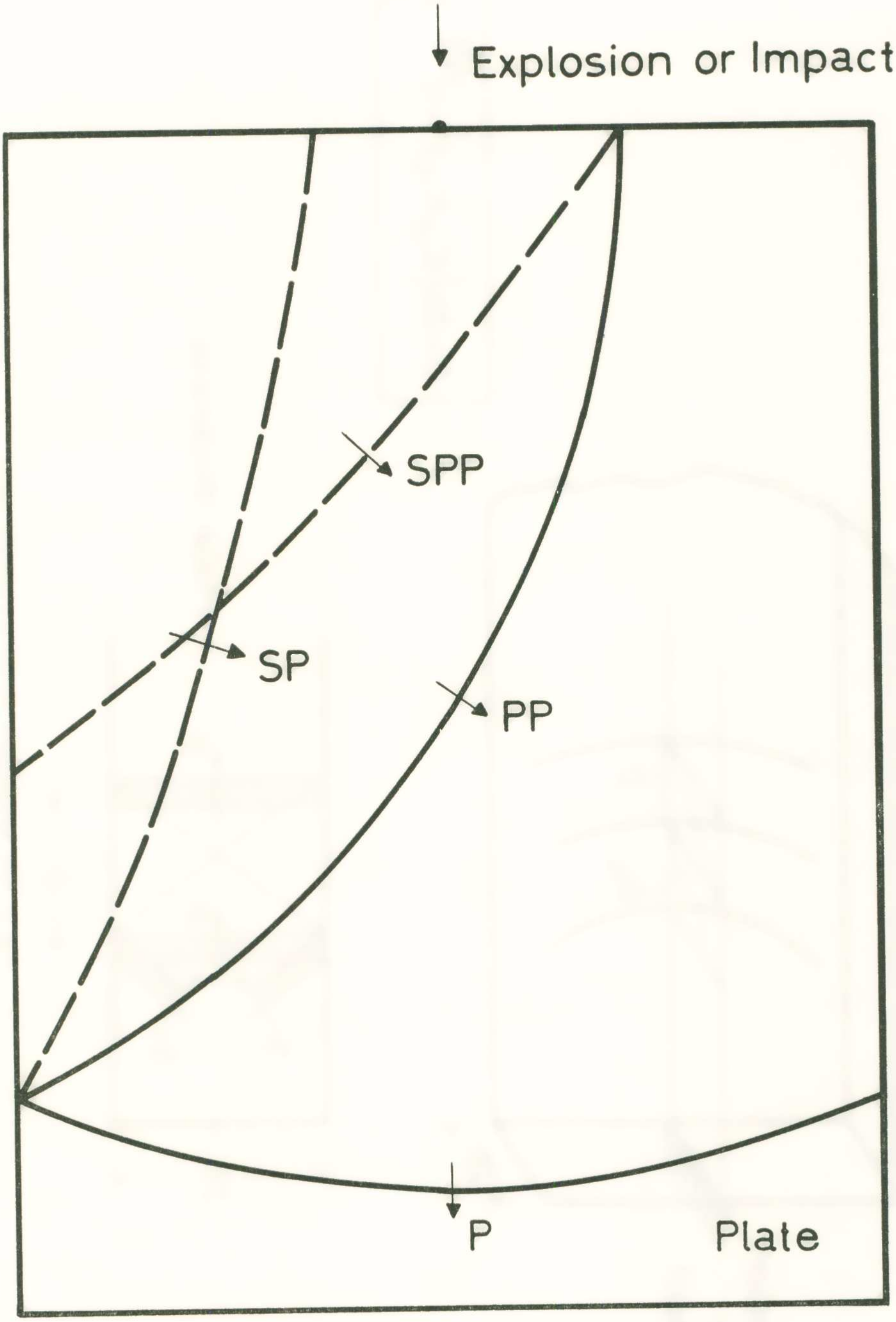

Fig. 2. Reflections of a primary P-wave and the secondary PP-wave and SP-wave on the free boundaries of a plate.

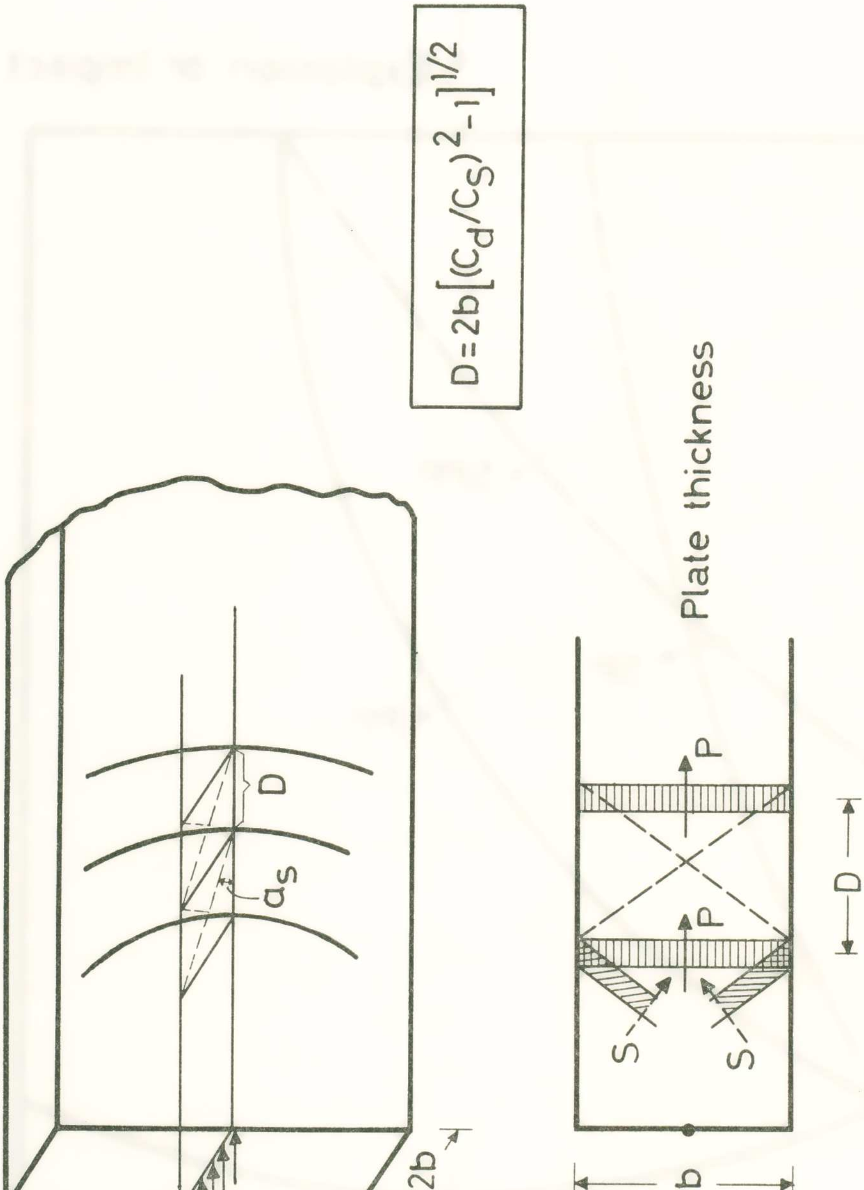

Fig. 3. The mechanism of the waveguide effect.

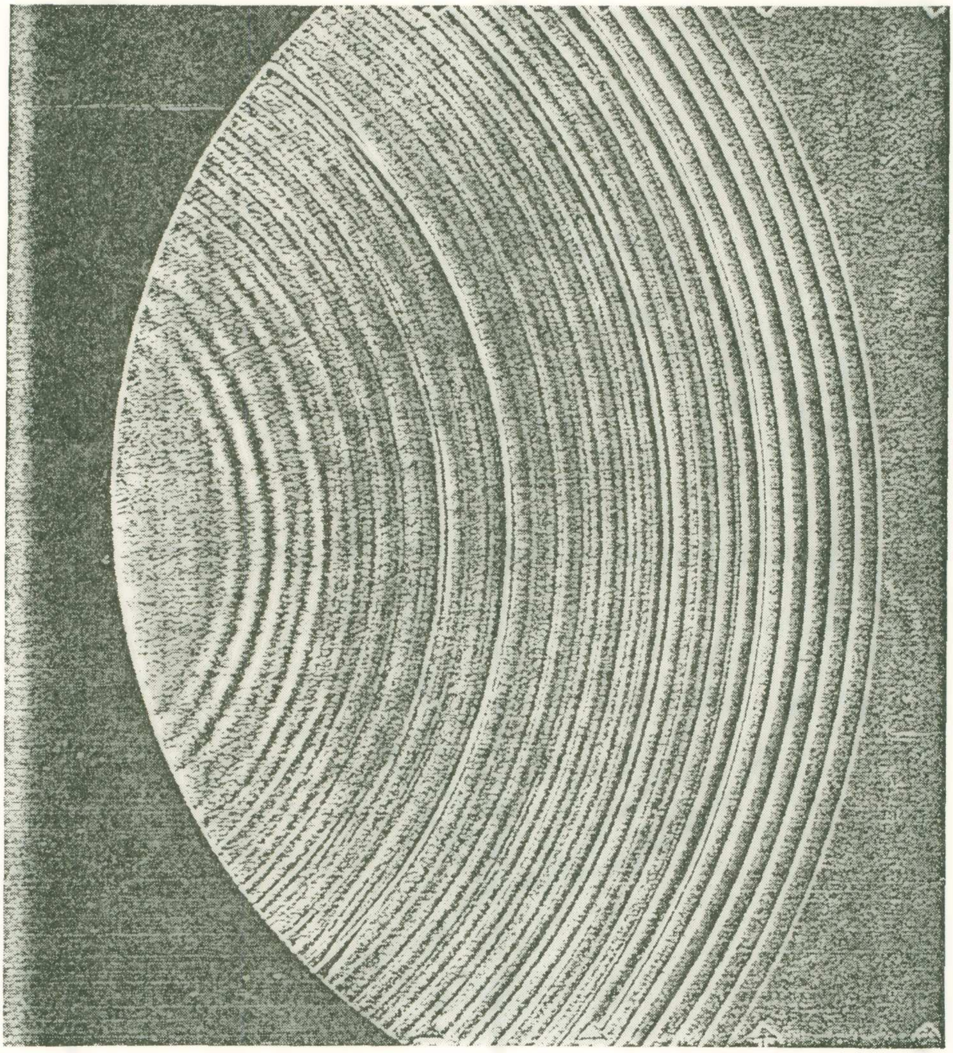

Fig. 4. Propagation of a dilatational pulse and secondary dilatational pulses.

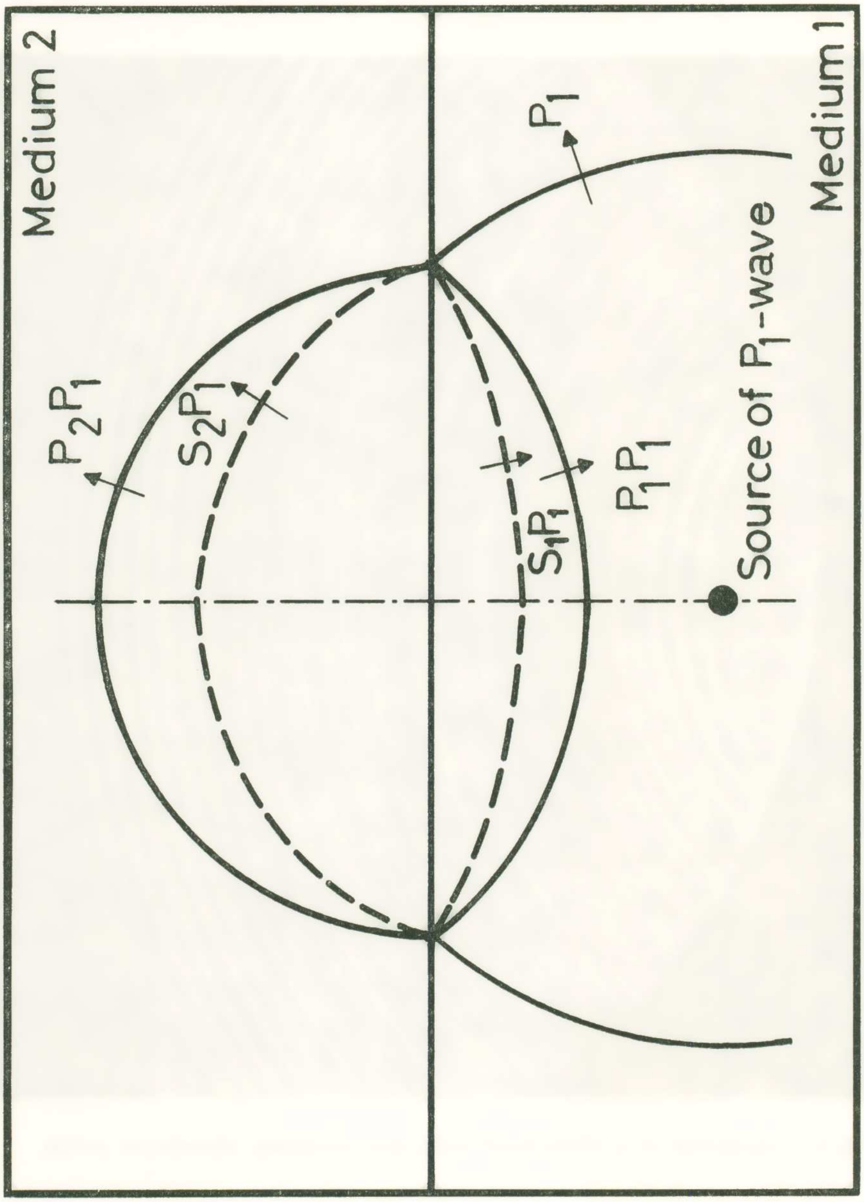

Fig. 5. Reflection and refraction of a P-wave on the interface between dissimilar materials.

pulses and can even approach zero. Then, the next pulse takes over the role of the leading pulse.

The problem of wave propagation in layered media is of great importance in seismology and has been the object of several analytical and experimental investigations. Fig. 5 shows the reflection and refraction of a P-wave on the interface, between dissimilar materials. The medium 2 has a larger value of its elastic modulus than the medium 1.

3. FRACTURE WAVES

Experimental Evidence

The experimental set-up of transmitted caustics [**15**] was used in combination with a Cranz-Schardin high-speed camera. All quasi-static external loadings were applied by a Schenck hydropulse tester with a piston-velocity of 0.0002 ms^{-1}, which creates a strain-rate equal to $\varepsilon = 0.0008\ s^{-1}$.

The specimens were made of transparent polymethyl-methacrylate (PMMA) and polycarbonate (PCBA) plates, $0.30 \times 0.10\ m^2$, with a variable thickness from 0.002 m to 0.010 m.

Fig. 6 shows a photograph of the wave pattern that was produced around a propagating crack in a PMMA-specimen of thickness d = 0.003 m. In this photograph primary waves emitted at different steps of the fracture process, as well as secondary waves produced by them, give a complicated pattern, as a consequence of their mutual interference, Doppler effect and variable stress field around the running crack.

In order to overcome these difficulties in observing the wave patterns appearing around propagating cracks in typical single-edge notched (SEN) specimens, we have devised a special form of specimen.

The types of specimens, selected in our experiments, are illustrated in Fig. 7. In a thin and long plate there is only a narrow ligament that connects the two parts of the specimen, which are fixed in the grips of the testing machine (Fig. 7a). The length of the ligament is less than 0.001 m. With this type of specimen, fracture of the narrow connecting link, by the applied force P, creates only a short-time pulse, which engenders a short-wave train, convenient for studying phenomena of step-propagation of a moving crack, without any existence of Doppler-shift effects, wave interference and deformed wavefronts.

The fractured area of the ligament, in a few microseconds, especially for brittle materials, recovers its unstrained form. Hence the observation of emitted waves during fracture, takes place in an unstrained plate.

As a consequence of the independence of the wave-trains, created by narrow fractured ligaments, the shapes of the wavefronts are now circular, even near the tips of the ligament.

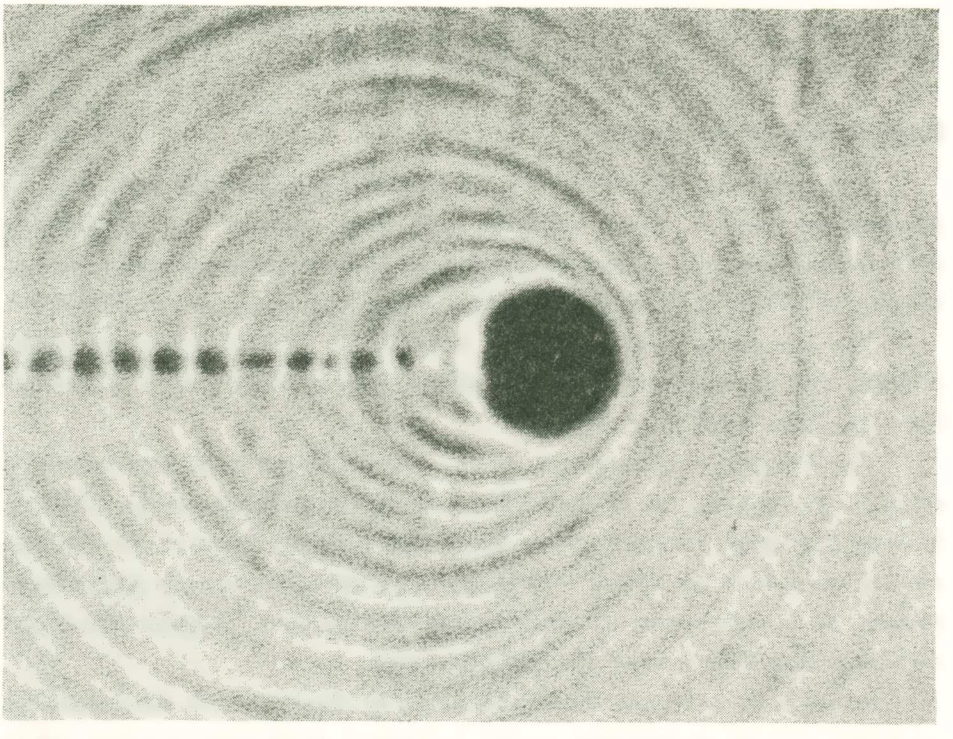

Fig. 6. Wave pattern due to crack propagation in a PMMA-plate of thickness $d = 0.003$ m.

The above-mentioned model constitutes a suitable model for simulating a stepwise extension of a crack and can be useful for the study of phenomena related to crack propagation by the coalescence of voids and microcracks ahead of the moving tip [**16**].

In order to simulate the last step of fracture, close to the opposite longitudinal boundary of the plate, another type of specimen which carried two edge ligaments, see Fig. 7c, was used in some tests.

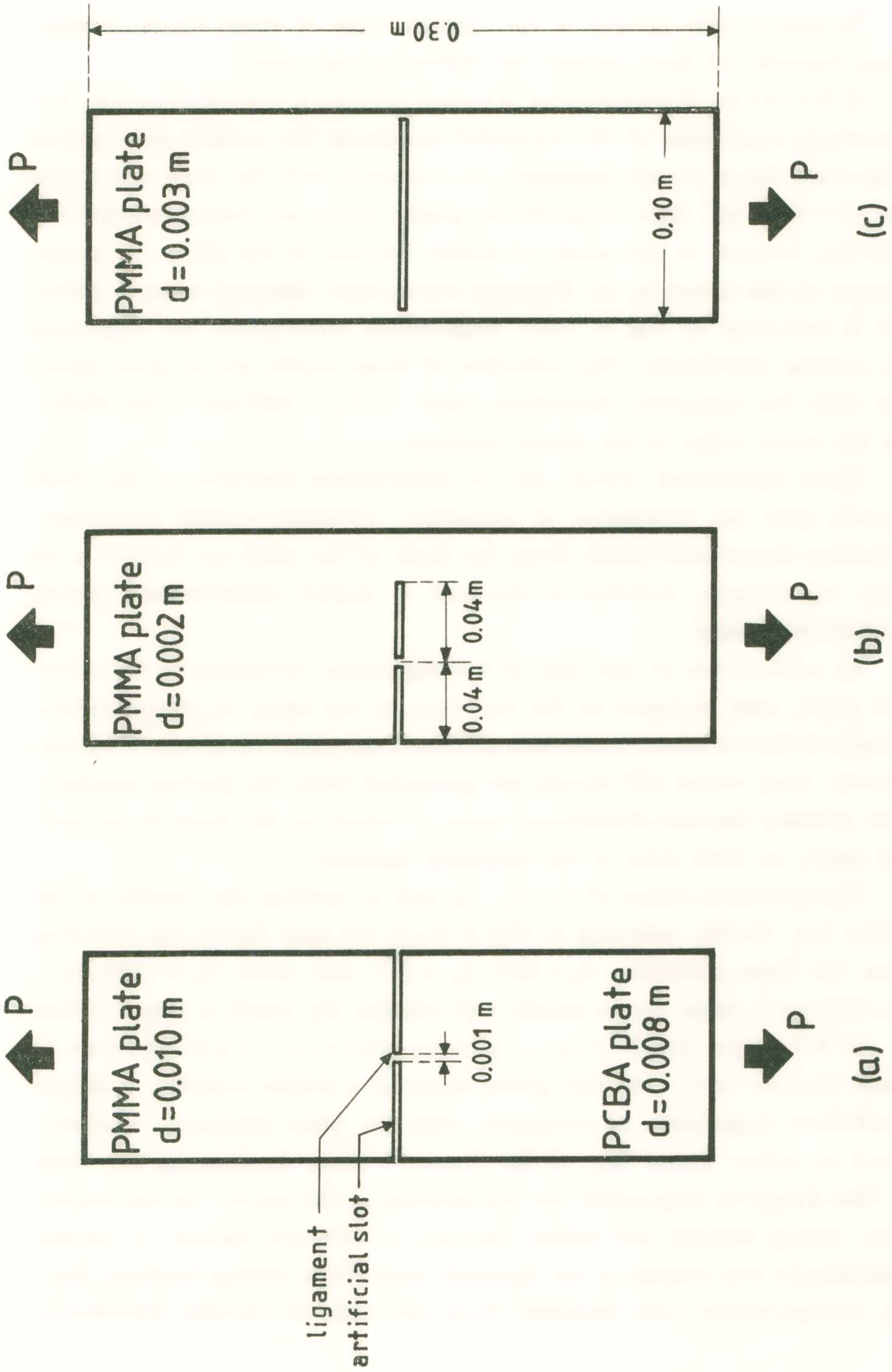

Fig. 7. Geometry of specimens with ligaments.

To give a clear picture of the various types of stress waves, created during fracture, we have studied the following four cases :

a) Brittle materials, plane-strain conditions: For plane-strain conditions of the fractured specimens the surface deformation at the crack tip is always negligible, as compared with the thickness of the plate ($d = 0.010$ m). Also, insignificant plastic zones are formed around the crack tips, because of the mode of brittle fracture of the plate. As a consequence of this situation, no Rayleigh waves were observed in such plates, as it is indicated in Fig. 8. Only longitudinal wave-pulses are appearing with circular wavefronts. The velocities of these waves are in good agreement with the respective theoretical value C_d ($C_d = 1800$ ms^{-1}) for PMMA with the static value of the elastic modulus.

These dilatational waves, due to infinitesimal fractures of the short ligament, give the impression of concentric circularly-crested wavefronts. No trailing dilatational pulses along the faces of the plate are formed in all similar experiments, contrary to the case of impact compressional waves, described previously.

An explanation of this lack of trailing pulses, according to the wave-guide effect, may be based on the fact that, in our tests, an almost normal incidence of the wavefront to the lateral faces of the plate was always achieved. However, head waves (SP-waves) are generated from the grazing incidence of the primary fracture-dilatational wave (P-wave) on the faces of the artificial crack, on both sides of the fractured ligament.

The measured values of C_d, C_s, a_d and a_s confirm the validity of the Snellius law. Really, referring to Fig. 8 (7 μs) we may derive the following values for these quantities $a_d = 90^0$, $a_s = 34^0$ and since $C_d = 1800$ ms^{-1}, $C_s = 1030$ ms^{-1}, these values satisfy well relation (5), which is then verified.

b) Brittle materials, plane-stress conditions: In typical fracture tests with thin plates made of a brittle material, a dimple of sufficient magnitude, as compared with the plate thickness, is always formed on either lateral face of the fractured plate surrounding the crack tip. This dimple is responsible for the creation of the *caustic*. In our experiments, during loading and before fracture, a sufficient amount of surface displacements was created in the ligament zone. Thus, during fracture, Rayleigh surface-waves were launched from this area at circular wavefronts.

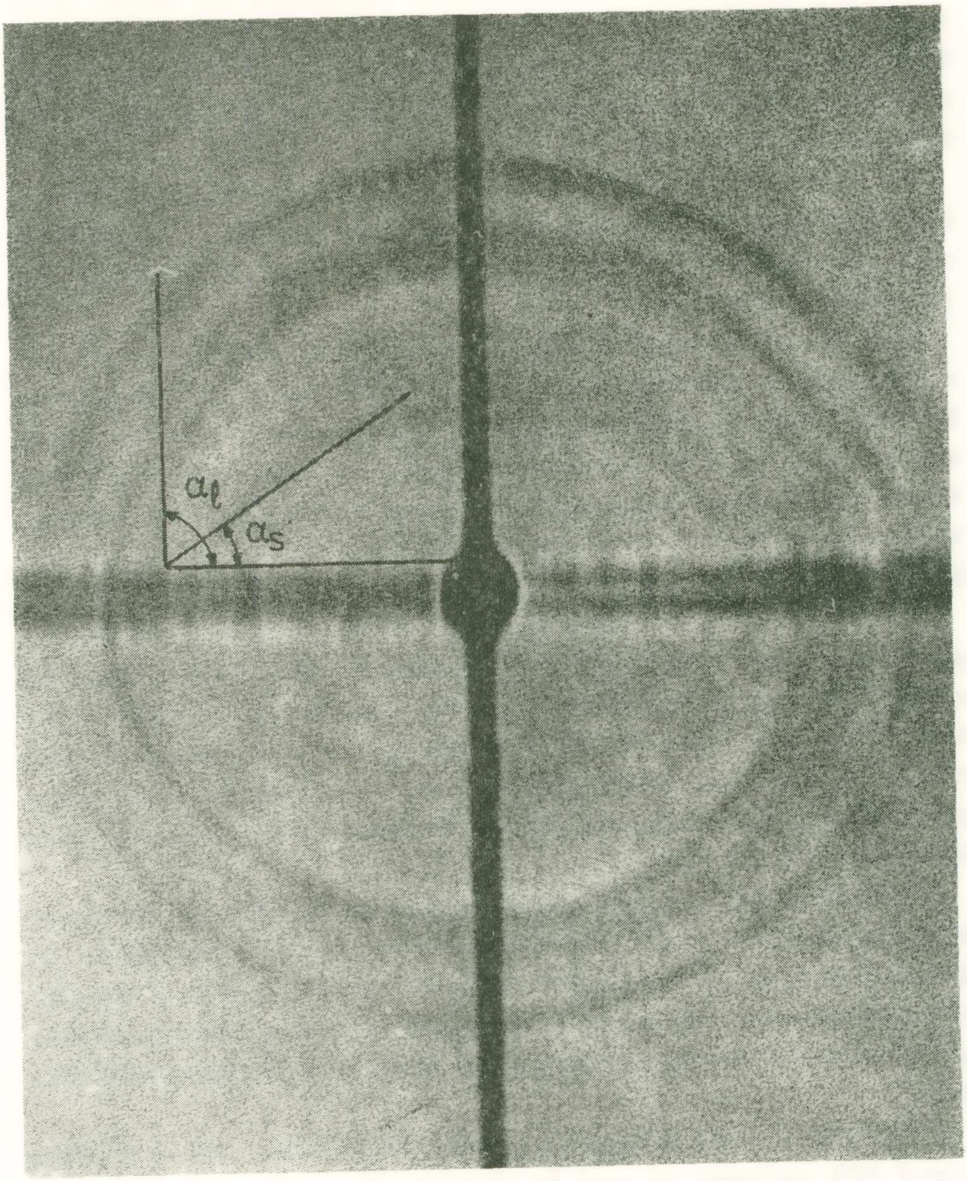

Fig. 8. Photograph of the propagation of primary longitudinal waves and trailing shear waves, in a PMMA specimen of thickness d = 0.010 m.

Fig. 9 indicates the creation of both longitudinal and surface waves during fracture of a PMMA specimen of thickness d = 0.002 m. The Rayleigh waves caused a clear deformation of the lips of the artificial slot. This deformation yields the possibility of distinguishing the Rayleigh waves from the dilatational ones.

Fig. 10 presents a series of photographs showing the propagation of both longitudinal and Rayleigh waves, which are generated during fracture of an edge ligament in a PMMA specimen of thickness d = 0.003 m, with the geometry of Fig. 7c.

c) Ductile materials: The difference already observed previously between thin and thick plates in brittle materials, regarding the launching of the Rayleigh waves has not been detected in these tests. Both dilatational and surface waves were emitted in our experiments with ductile thin and thick plates. The longitudinal waves were less clear than the surface waves in the tests with PCBA specimens, because they are formed by the Schlieren effect, see Fig. 11.

Fracture of PCBA plates is always associated with the creation of plastic zones, due to the ductility of the material, which causes a significant surface deformation around the crack tip. Accordingly, it was expected that the ligament region in our tests will develop plastic zones. In fact, this phenomenon was always observed. The thus developed plastic zones, after fracture of the ligament, created Rayleigh waves propagating along the plate.

Indeed, in the first photographs of Figs. 11 the Rayleigh wavefronts were not circularly crested. This phenomenon may be attributed to the fact that the forms of these fronts are influenced by the shapes of the plastic zones. Therefore, the shapes of the spreading surface disturbances yield, an information about the geometry of the plastic zone formed around crack tips.

d) Wave radiation in the starting phase: When typical SEN specimens are used for observing the crack propagation, body waves are emitted, because the stress intensity factor jumps from his static value to a dynamic one. Then, the dilatational wave is compressive. Similarly, when a crack suddenly stops, the same body waves are generated, but now the dilatational wave is extensive. Fig. 12 shows photographs from the dilatational wave radiation by a starter crack in SEN PMMA- and

PCBA-specimens. After the crack initiation no longitudinal waves were observed during fracture of the specimens. Other investigators [**17**] have observed the emission of a shear wave during the crack initiation.

Rayleigh-wave propagation in a variable stress field

Generally, the fracture process is discontinuous and the crack extension, even at high velocities, is produced by the coalescence of microcracks in the prospective fracture zone. Therefore, the crack motion is taking place in a stepwise manner. As a consequence, the crack velocity and the dynamic stress intensity factor K_I^d vary with time. But, in an elastic-brittle plate without discontinuities (interfaces or other cracks), we may accept that this variation is practically negligible. Indeed, our experiments in SEN PMMA-specimens shows a slight change of the crack velocity and the K_I^d-factor almost at the half of the whole crack-path.

Therefore, it is a good approximation if we consider that the stresses at a fixed point of a plate with a running crack vary only, because of the change of the distance between the point and the crack tip, during crack propagation. The resulting variable stress field produces loaded and unloaded regions of the plate and it is illustrated in Fig. 13, where the optical stress rosette method [**18**] was utilized.

On the other hand, the crack, propagating at infinitesimal steps, emits Rayleigh waves, due to the abrupt application of the near-tip displacements on the surface of the plate.

Fig. 14 presents a photograph of the Rayleigh wave pattern, due to crack propagation at the stable crack velocity interval in SEN PMMA-specimens. Fig. 15 illustrates the distribution of the values of the dynamic elastic modulus E^d around the moving crack tip at two different instants of crack motion, in PMMA. These values resulted from an iterative process [**19**], involving the strain rates $\dot{\varepsilon}(t)$ at a network of points because of the variable stress field, and based on the experimental $E^d - \varepsilon$ curve, taken from tests with dynamically loaded Hopkinson bars.

Since, the Rayleigh wave velocity C_R depends considerably on the dynamic elastic modulus of the region in which the wave is propagating, (see Eqs. (3) and (4)), it is expected that the points at the propagating wavefront will not be equidistant from the position of the wave launching.

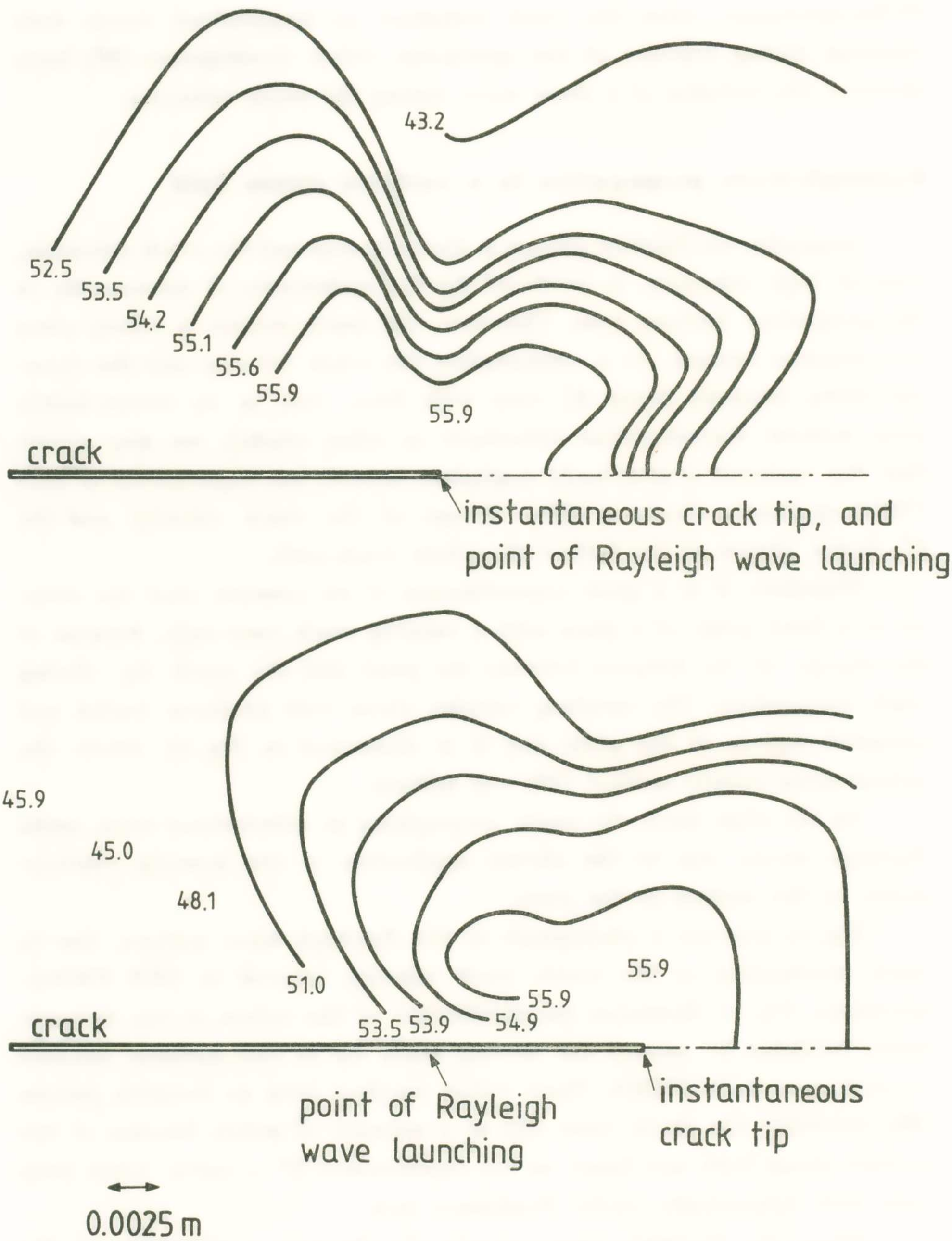

Fig. 15. The values of dynamic elastic modulus ($\times 10^8$ Nm^{-2}) produced by the transient loading of a moving crack, at the instants t = 100 μs and t = 120 μs after crack initation in a PMMA-plate.

Really, an analysis, based on the above-mentioned idea and the method of dynamic caustics [**20**], results to a form of the Rayleigh wavefront illustrated in Fig. 16, which resembles satisfactorily with the experimental non-circular scheme of these fronts.

4. CRACK PROPAGATION AND ARREST IN PLATES CONTAINING DISCONTINUITIES

Arrest phenomena at a bimaterial interface

In order to study the effect of the presence of a discontinuity on the mode of propagation of the various types of waves, bimaterial plates of epoxy polymers with different amounts of plasticizer were tested, in order to have combinations of brittle-ductile and ductile-brittle phases. The cases of a longitudinal interface, normal to the crack trajectory [**21** - **23**], and a slant interface [**24**], showed that the interface behaves as a *decelerator* for the propagation of the crack. The influence of the interface on the propagating crack is related to the abrupt change of the mechanical properties of the plate at the interface. Figs. 17 to 20 present photographs of experiments, where a mode-I crack crosses the interface between dissimilar media. The investigation of these photographs and the study of the caustics formed at the moving tip, shows significant decelerations and increased values of K_I^d - factor, before the *barrier*, crack arrest at the interface and acceleration after the interface crossing. Fig. 21 describes clearly the occurrence of the aforesaid phenomena. The increase of the values of the dynamic stress intensity factor K_I^d is due to the larger initial length of the new starting crack and perhaps due to the reflection of the elastic waves emitted by the crack on the interface wall.

Interaction between a propagating crack with a normal and oblique fault

A similar to the cracked bimaterial plate behaviour was also observed in plates with a running crack against other cracks or holes. Figs. 22 to 25 present photographs of these experiments in thin PMMA-plates. Fig. 26 shows the crack paths in plates containing internal oblique faults at various angles of inclination. From Figs. 23 to 25 it is shown that the angle of

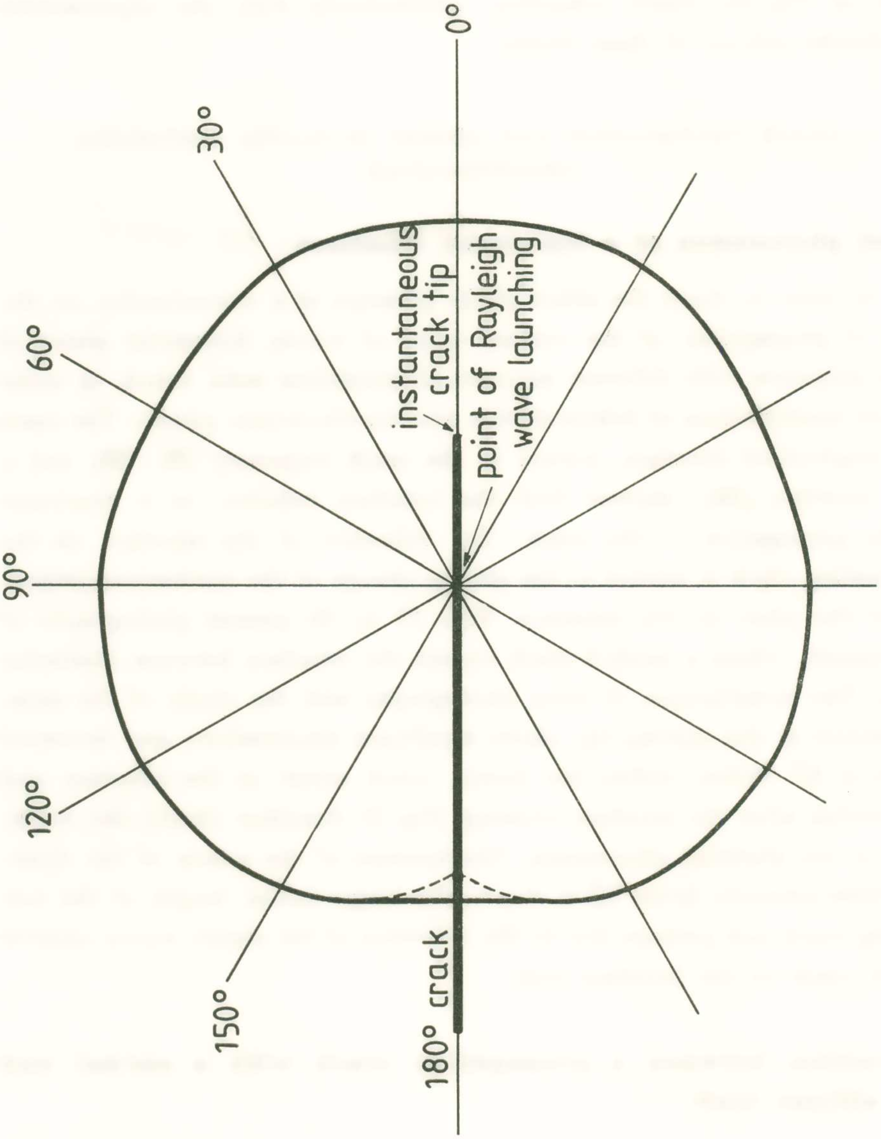

Fig. 16. Rayleigh wavefront emitted by a moving crack tip at the instant t = 10 μs after crack initiation, at the time t = 120 μs.

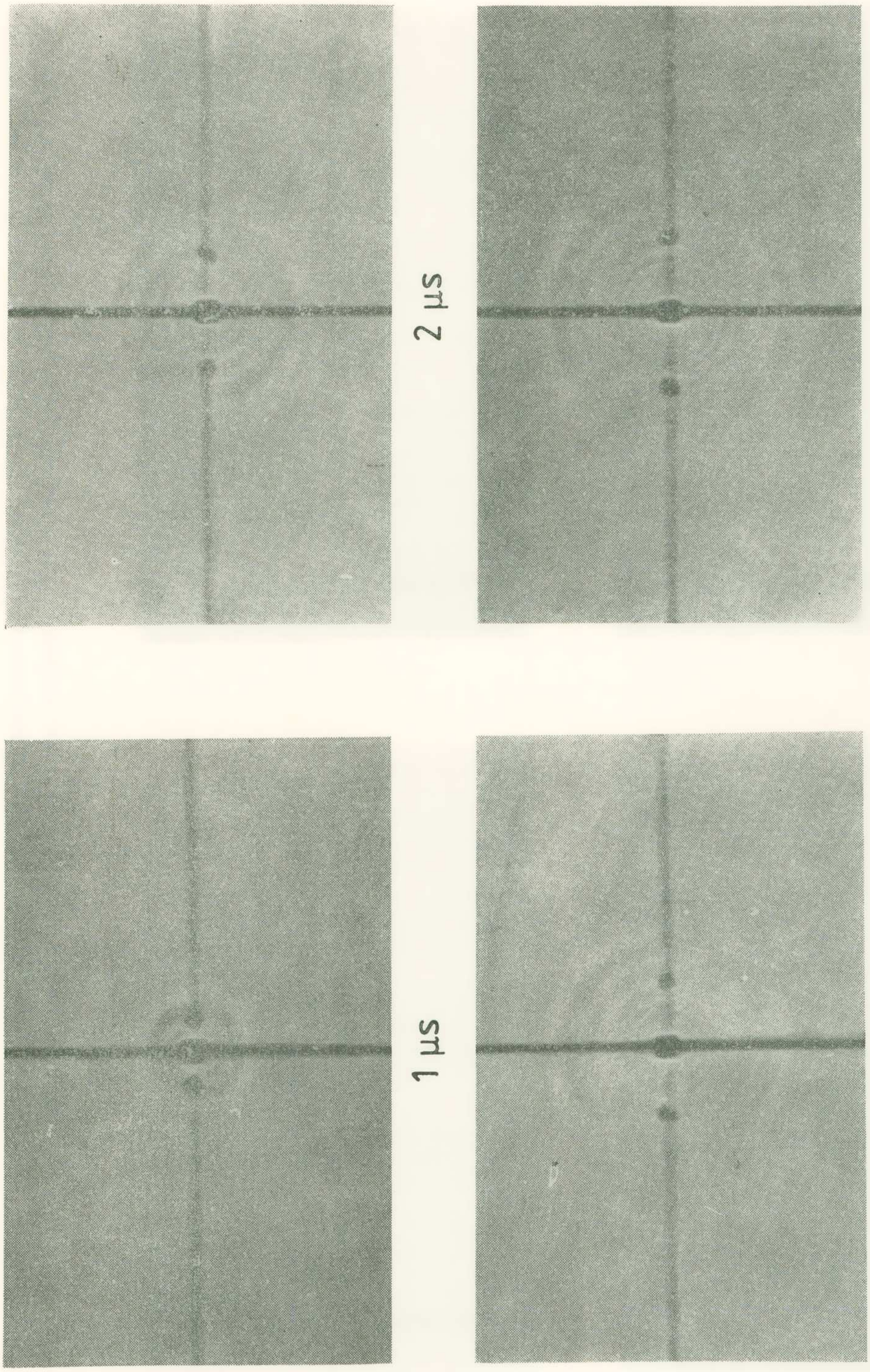

Fig. 9. Series of photographs showing fracture of the ligament and both longitudinal — and Rayleigh — waves propagation, in a PMMA specimen of thickness d = 0.002 m.

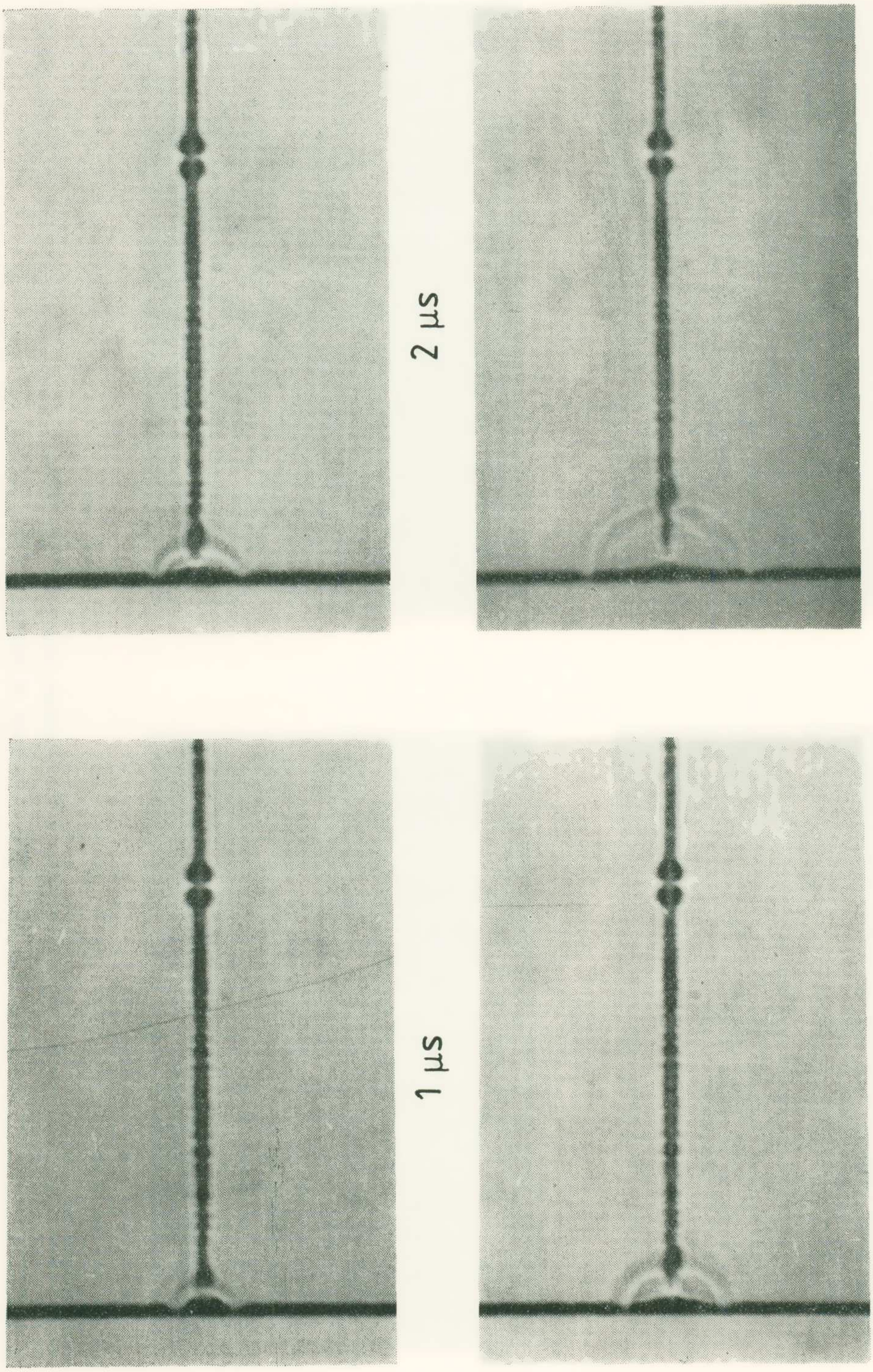

Fig. 10. Series of photographs showing longitudinal — and Rayleigh — waves propagation produced by failure of a PMMA specimen of thickness d = 0.003 m.

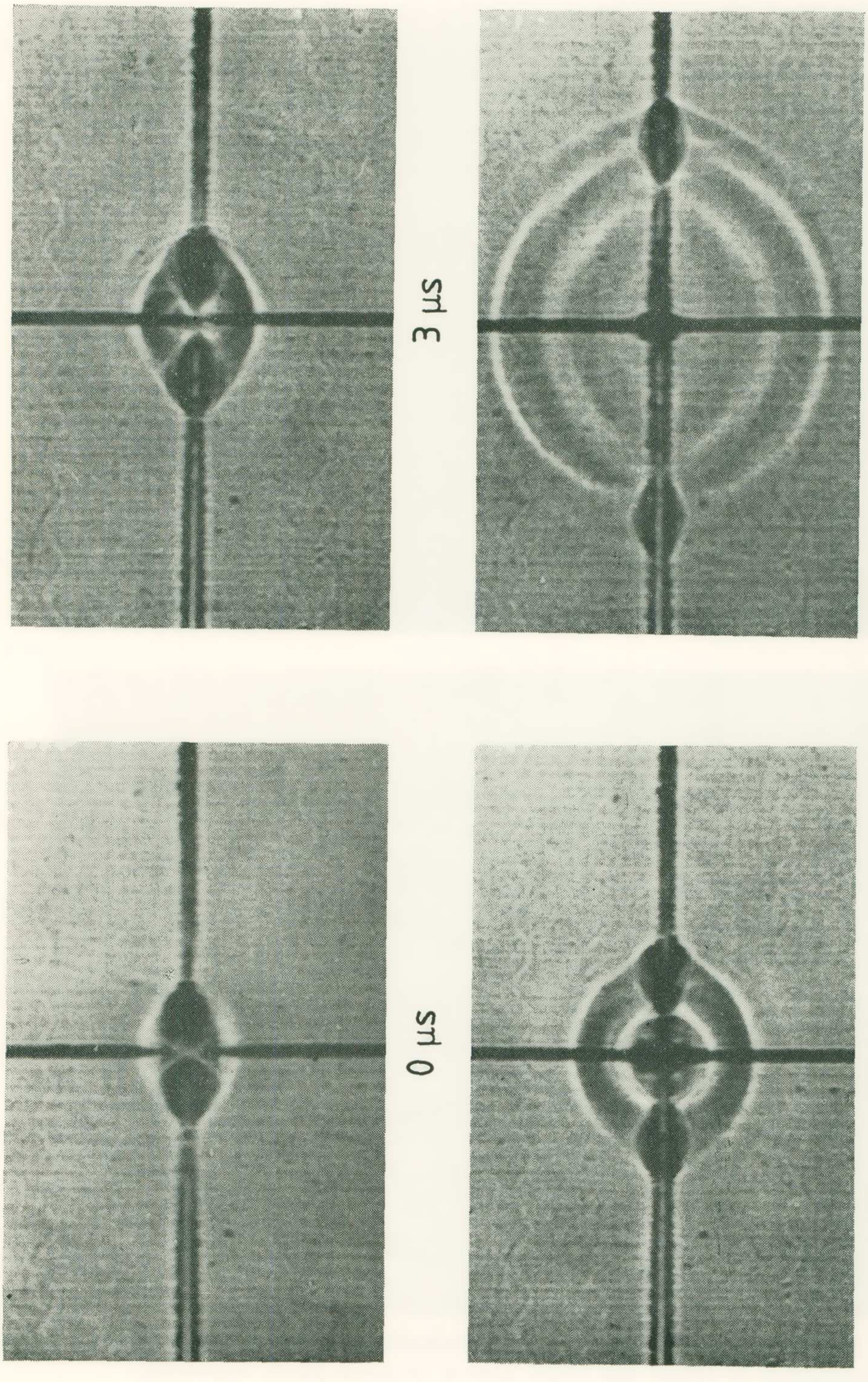

Fig. 11. Series of photographs showing fracture of the ligament and both longitudinal — and Rayleigh — waves propagation, in a PCBA specimen of thickness d = 0.008 m.

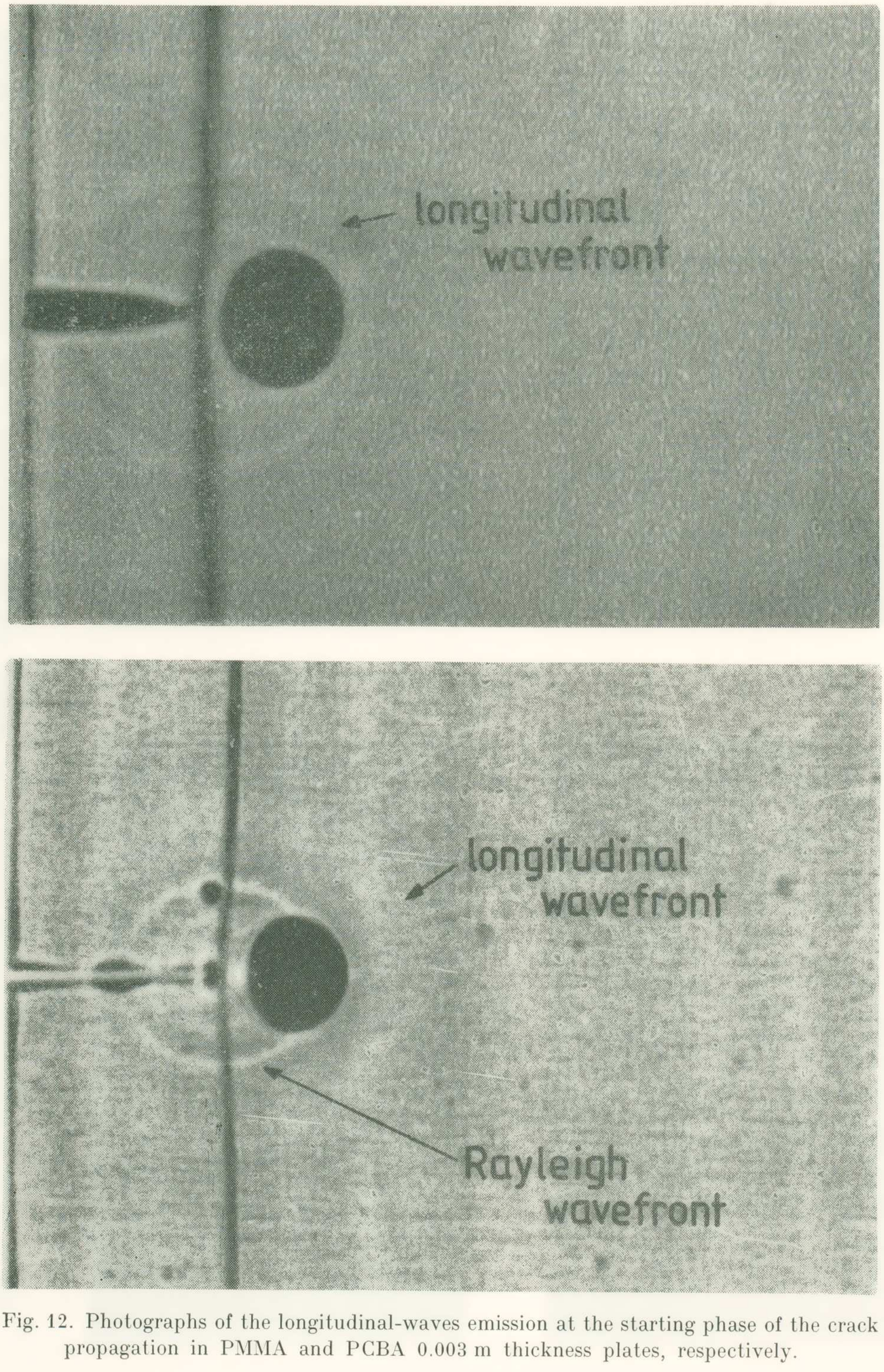

Fig. 12. Photographs of the longitudinal-waves emission at the starting phase of the crack propagation in PMMA and PCBA 0.003 m thickness plates, respectively.

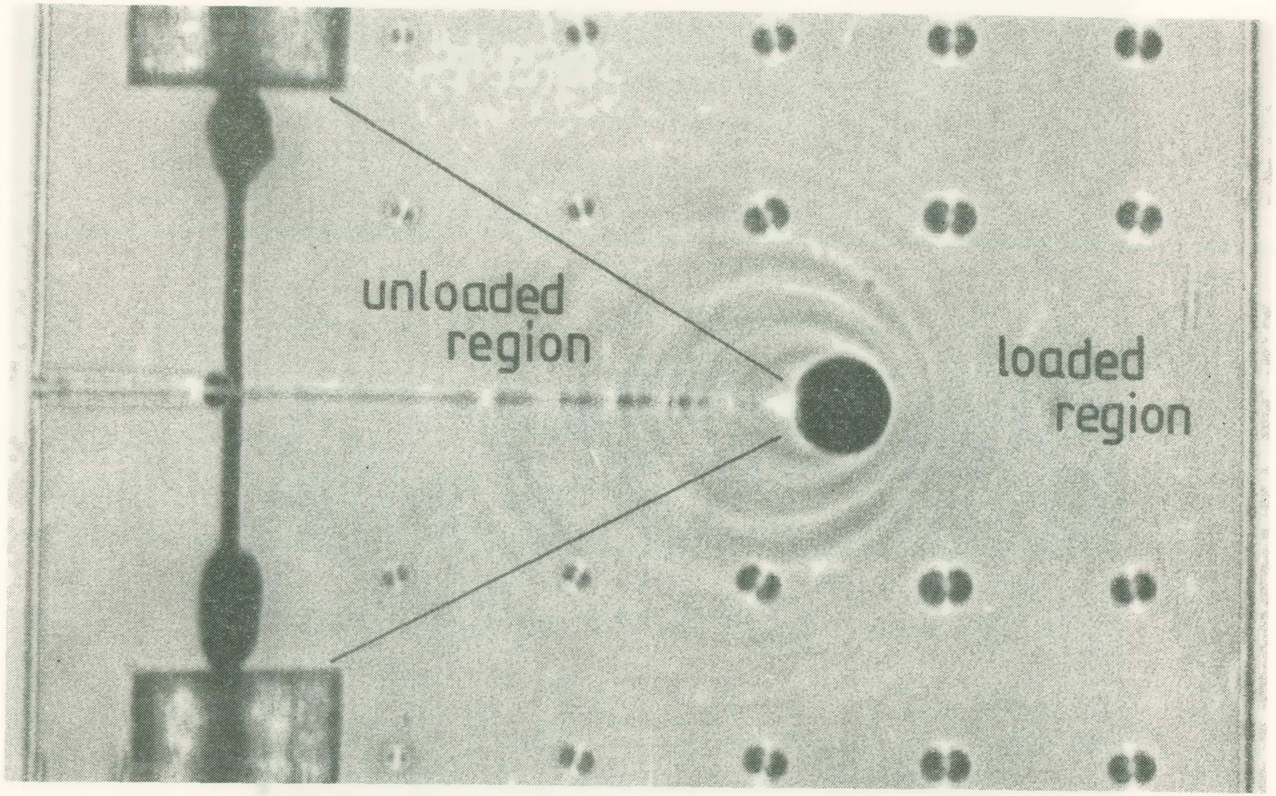

Fig. 13. Photograph of the loaded and unloaded regions produced by a fast running crack in a SEN PMMA plate of 0.003 m thickness. The method of stress optical rosette was used.

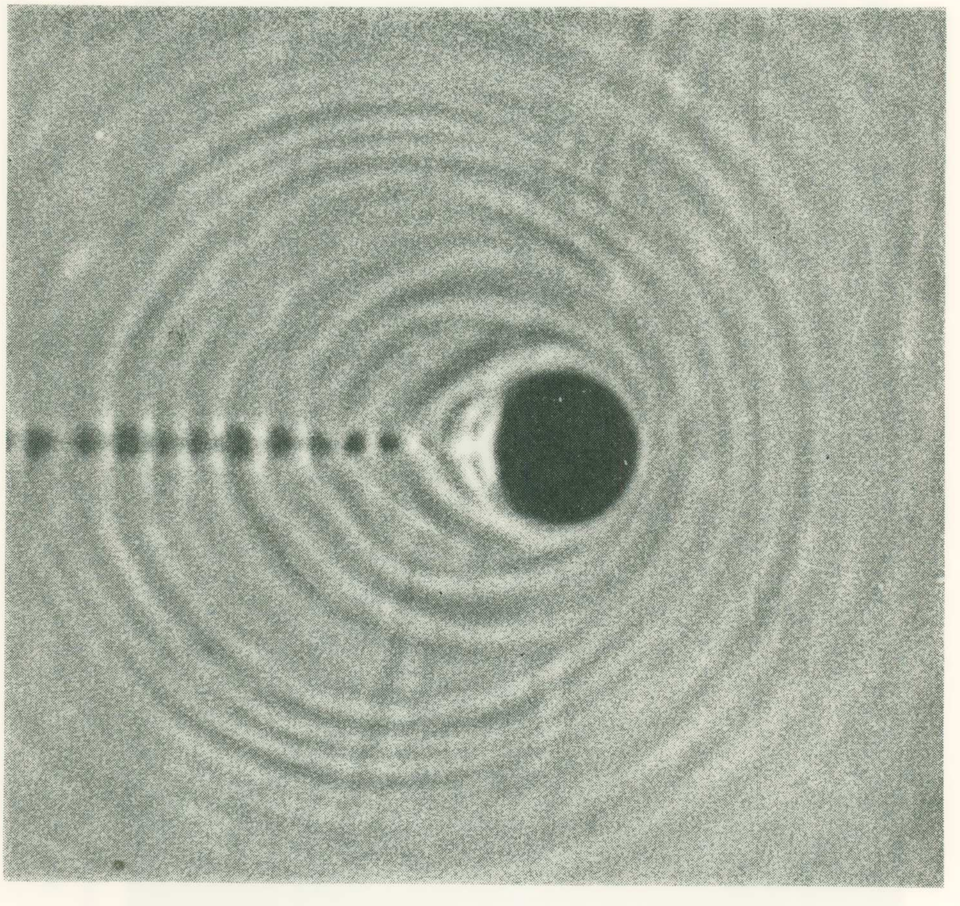

Fig. 14. Photograph of the Rayleigh fracture waves pattern, taken at the stable crack-velocity interval.

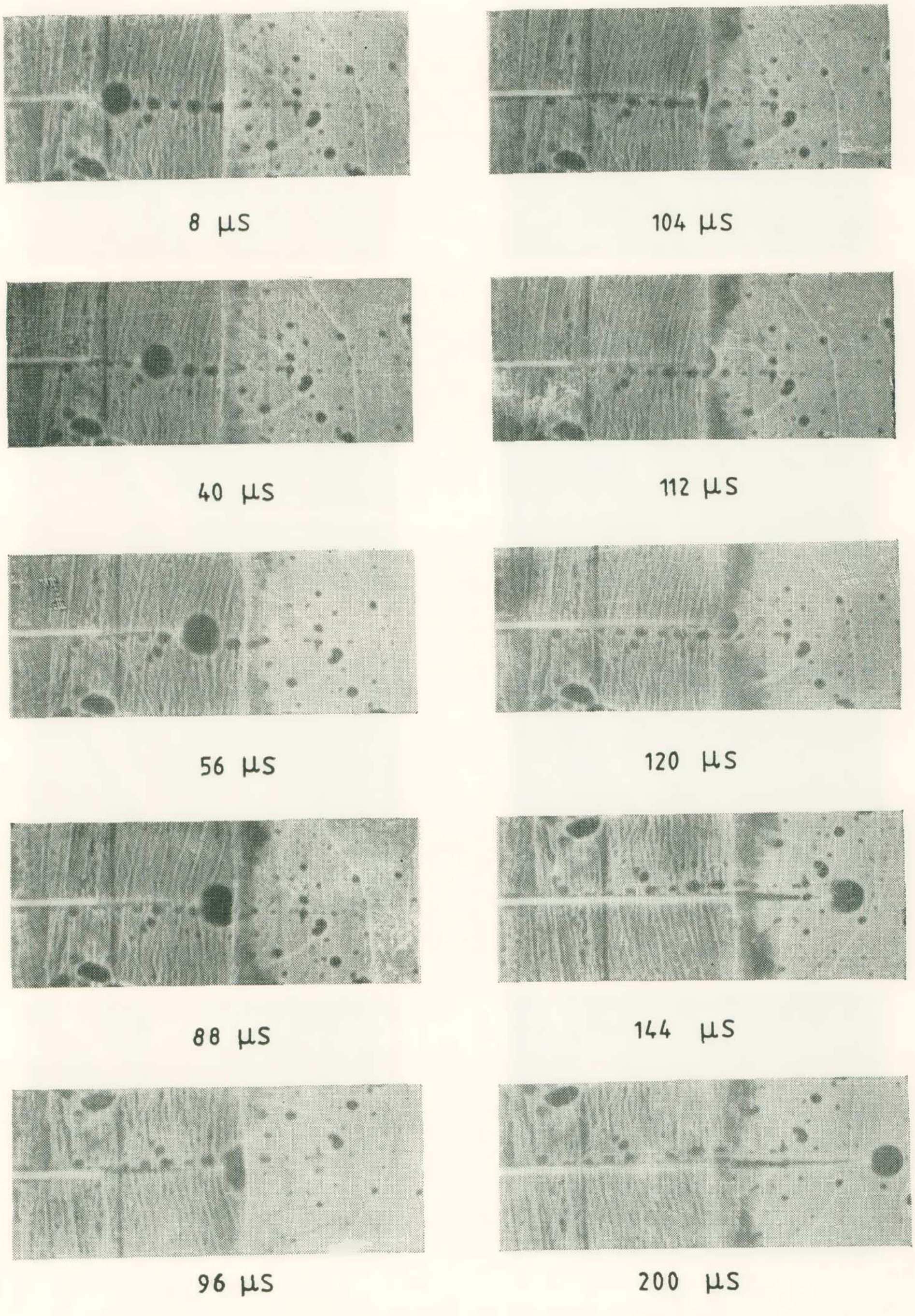

Fig. 17. Series of photographs showing crack arrest and change of the transverse crack trajectory at a bimaterial interface.

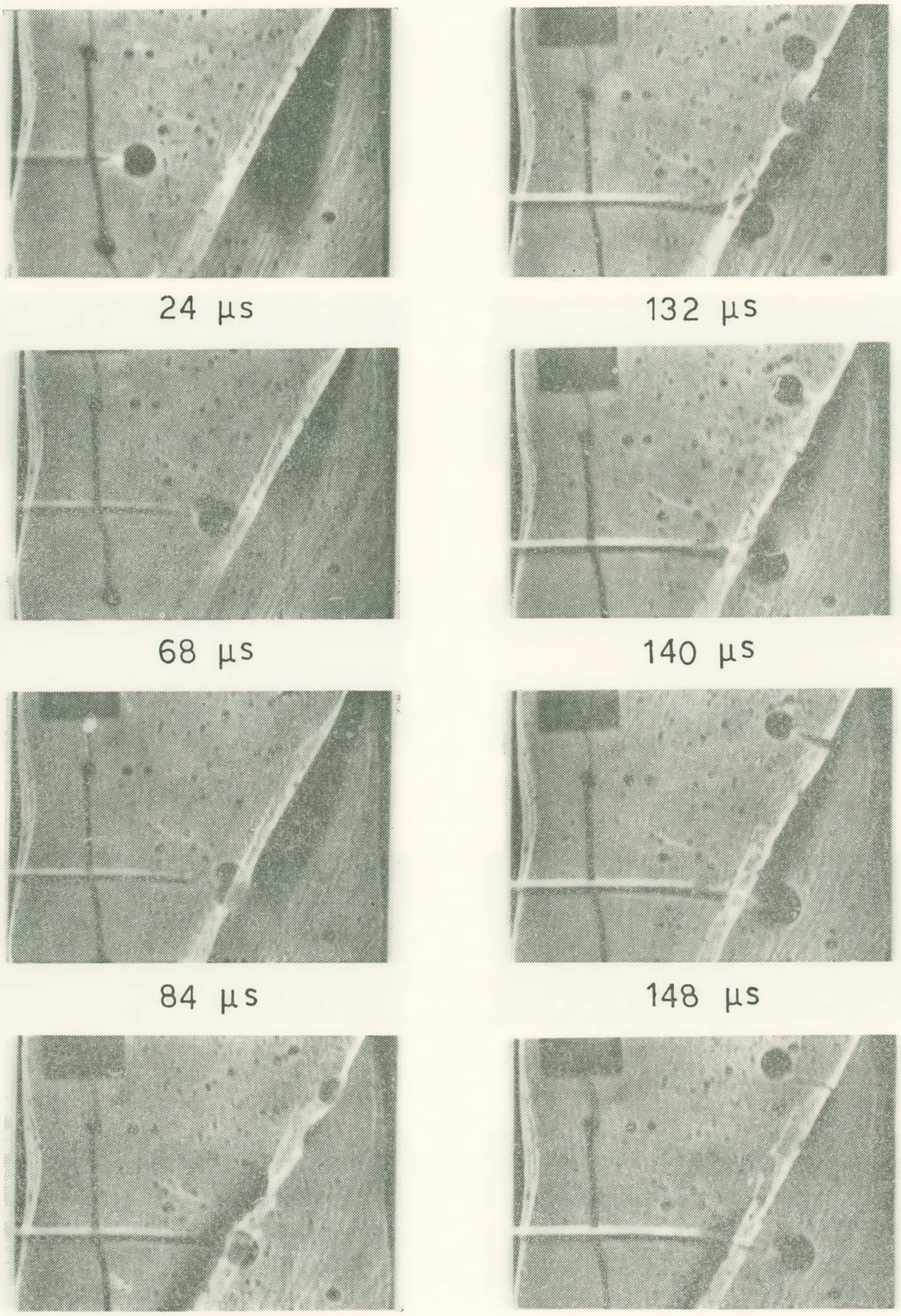

Fig. 18. Series of photographs showing crack arrest and change of the transverse crack trajectory at a bimaterial interface.

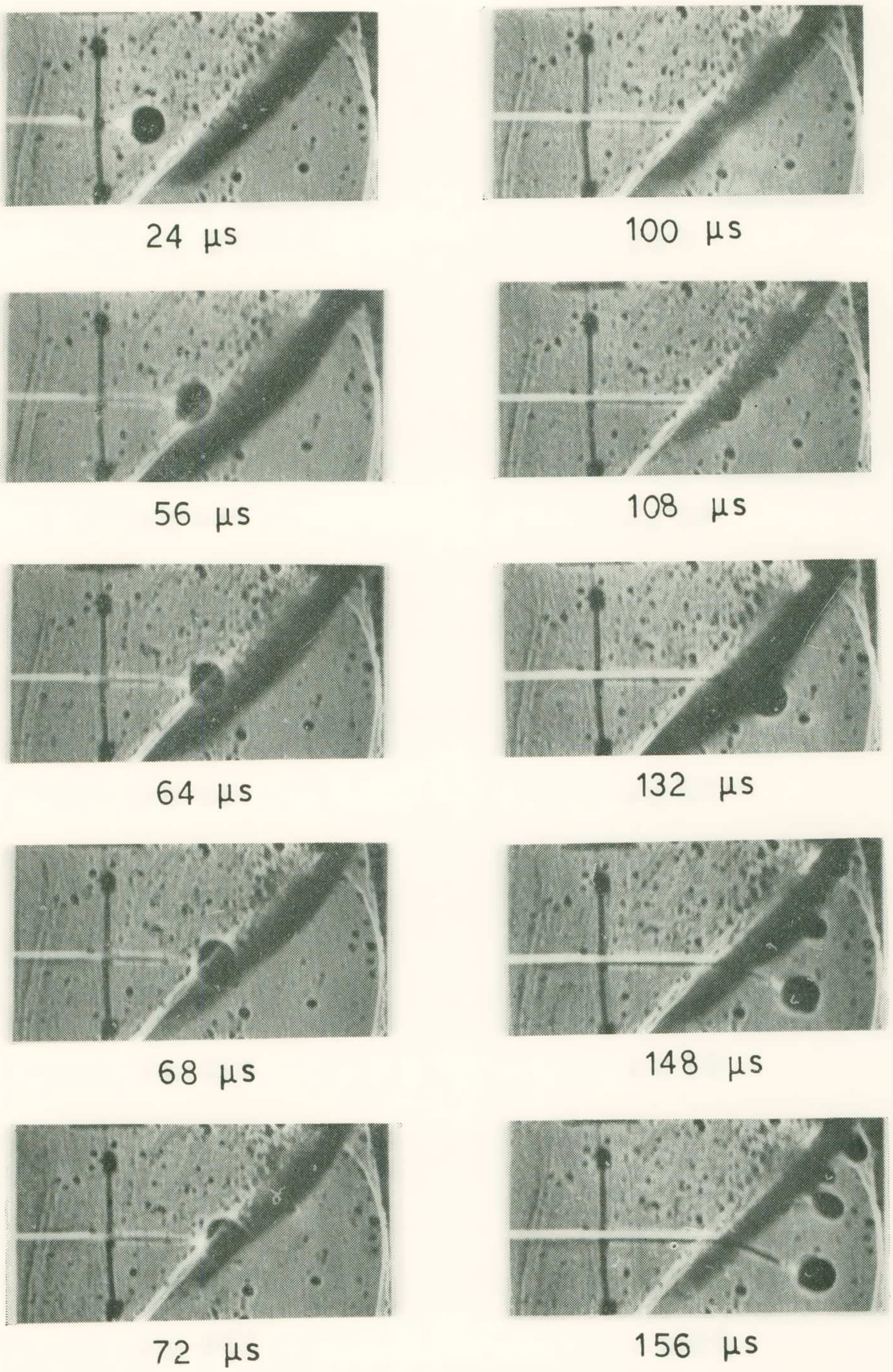

Fig. 19. Series of photographs showing crack arrest and change of the transverse crack trajectory at a bimaterial interface.

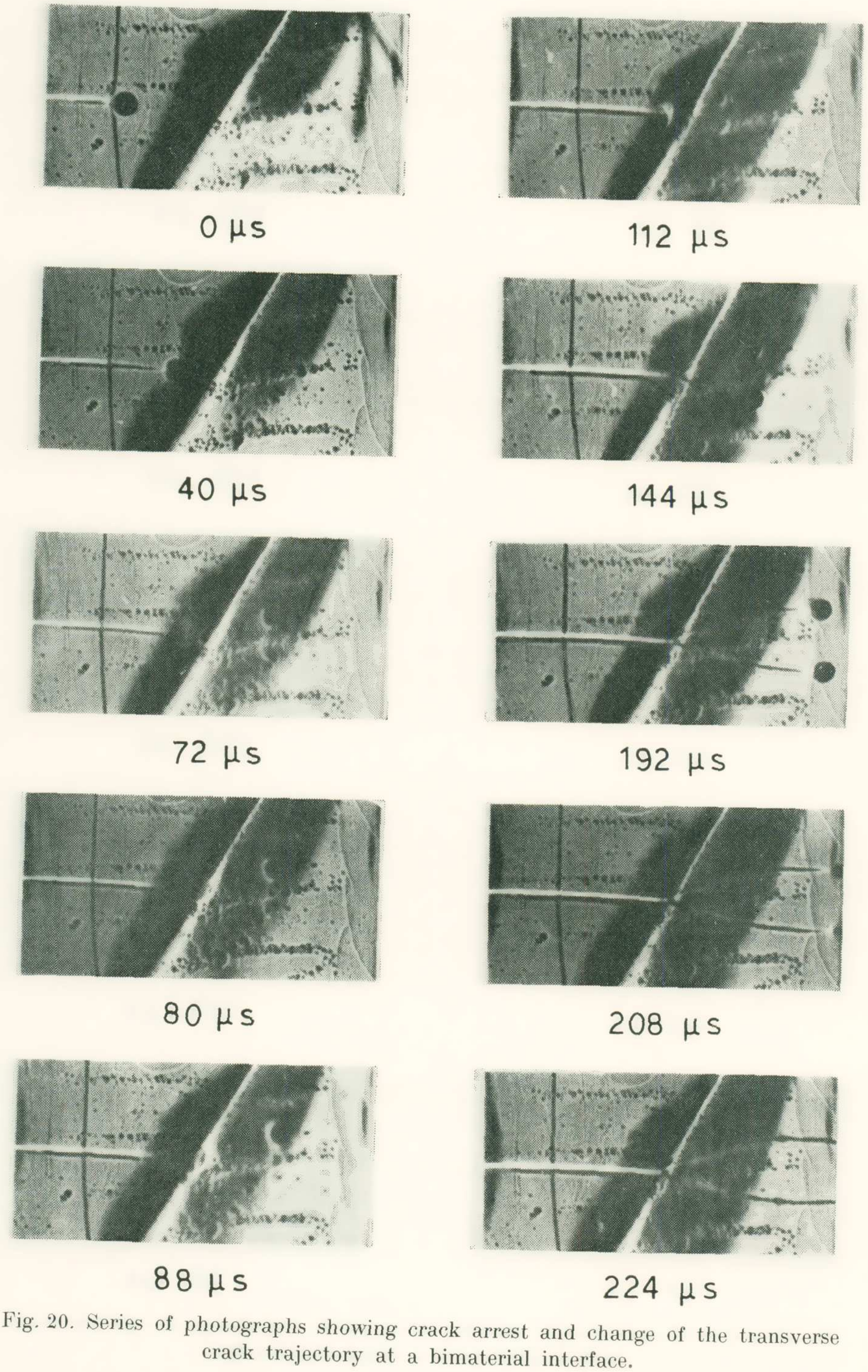

Fig. 20. Series of photographs showing crack arrest and change of the transverse crack trajectory at a bimaterial interface.

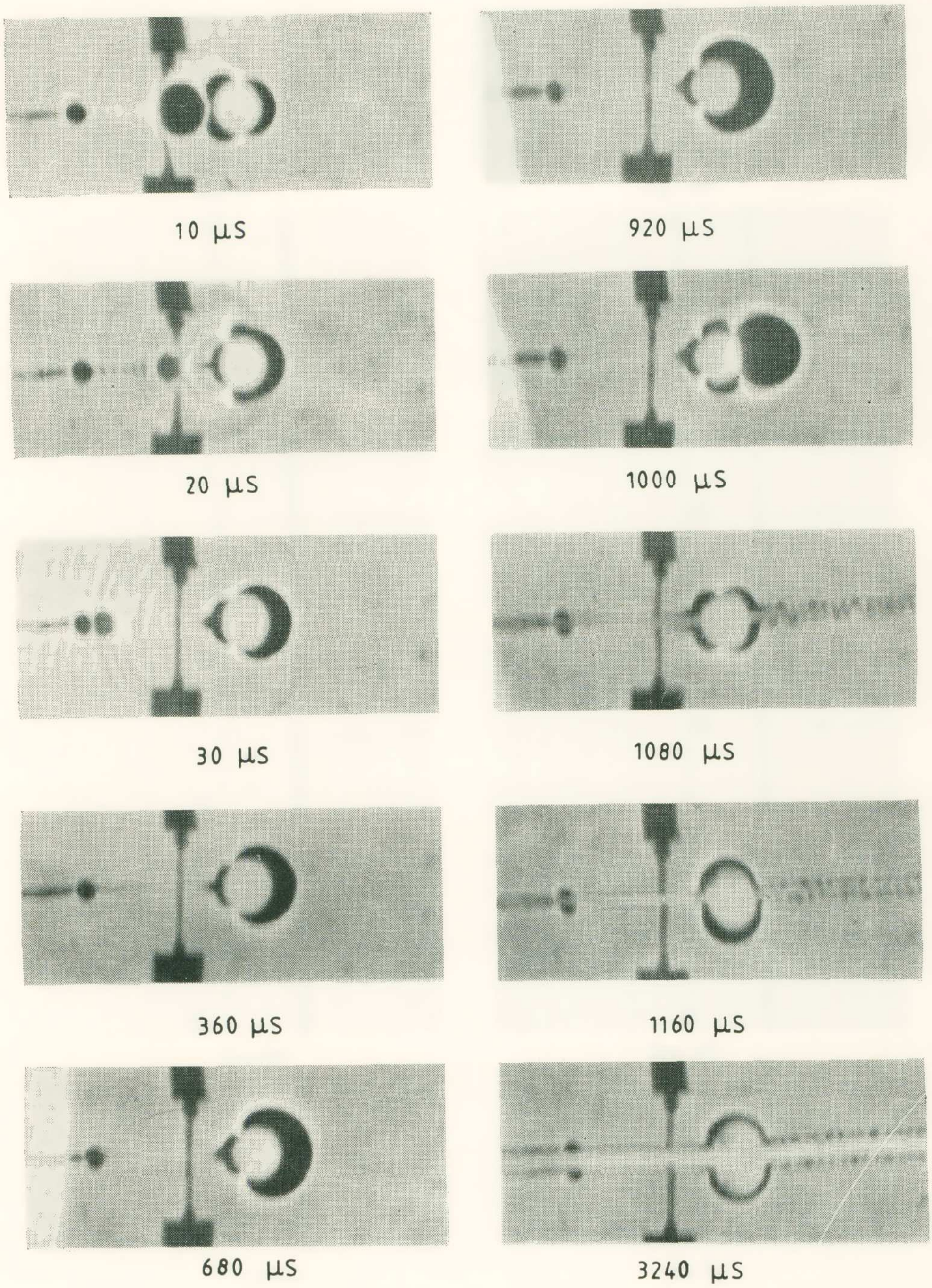

Fig. 22. Series of photographs showing crack arrest at a hole in a PMMA-plate.

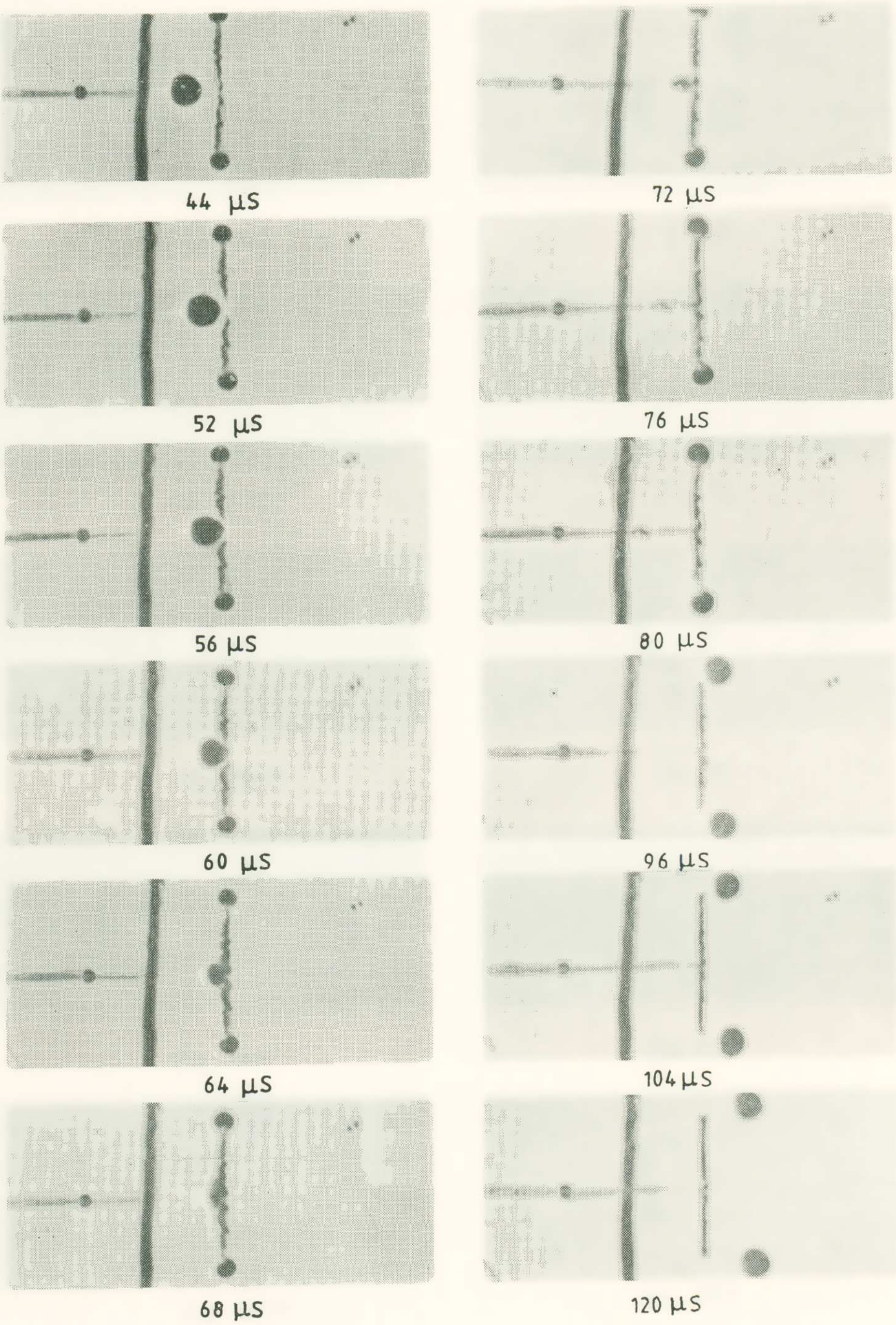

Fig. 23. Series of photographs showing crack arrest at a longitudinal crack in a PMMA-plate.

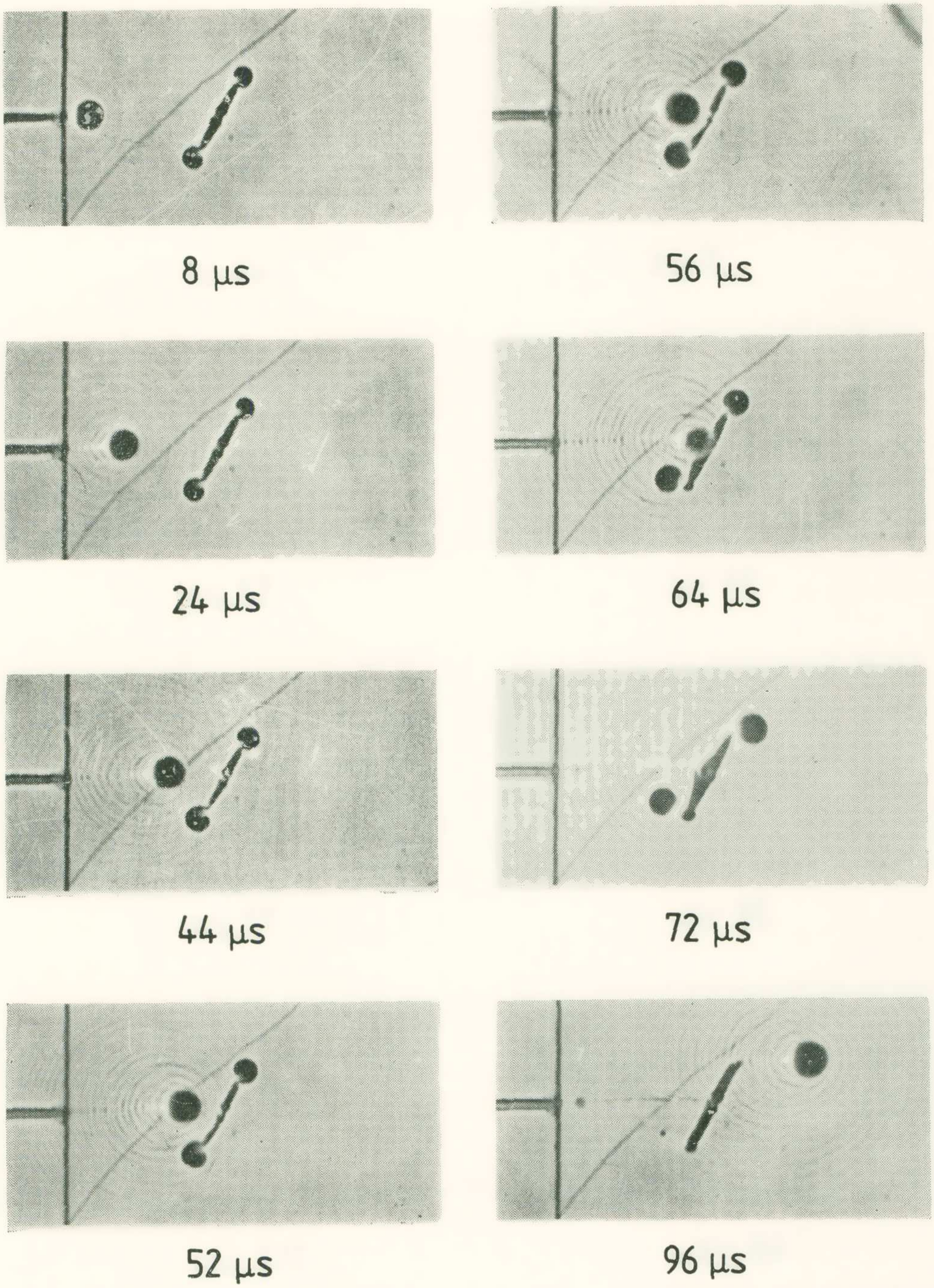

Fig. 24. Series of photographs showing a transverse edge crack, moving to an internal crack, inclined at angle of $\beta = 30$ deg. with the axis of loading.

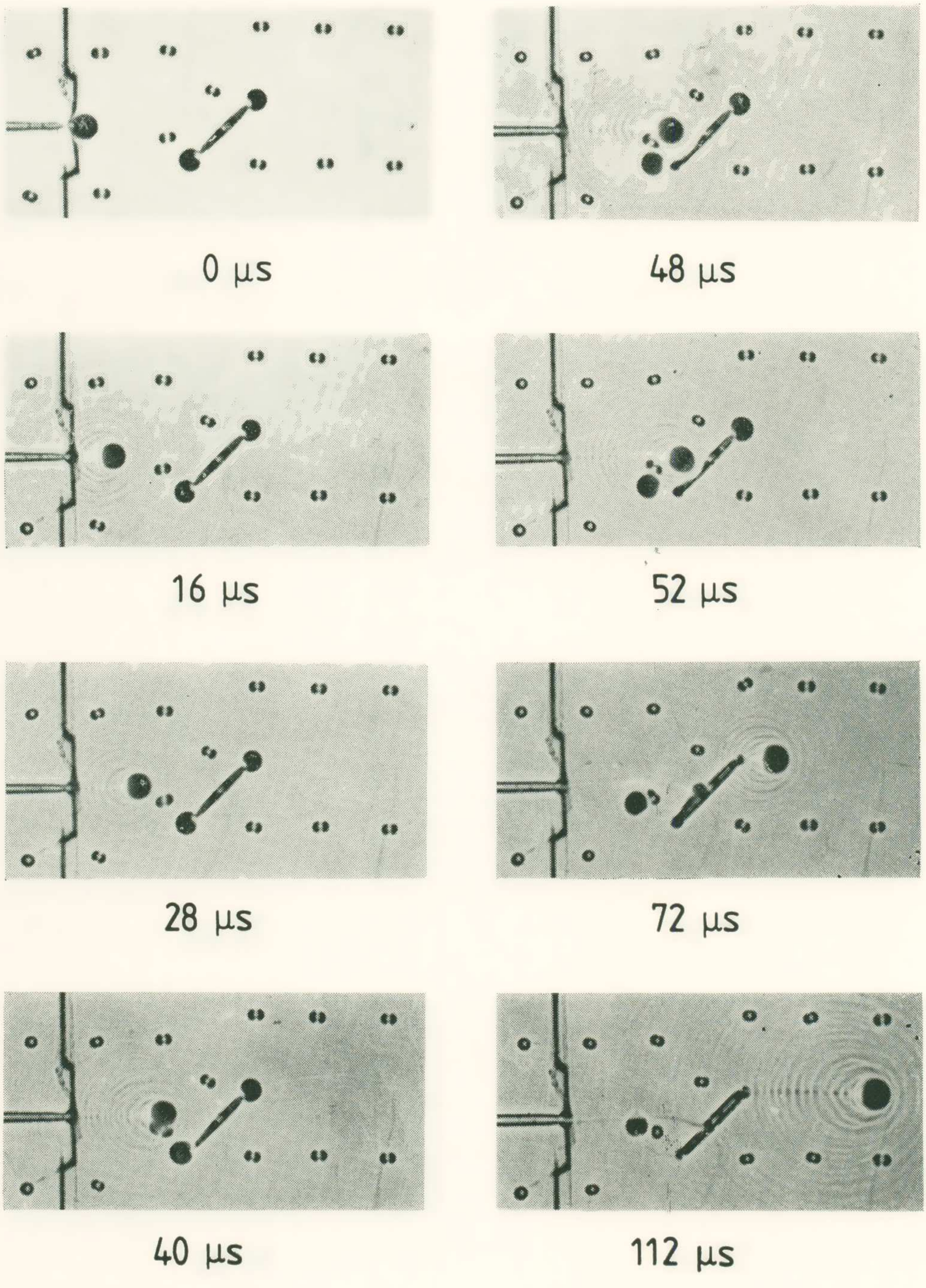

Fig. 25. Series of photographs showing the orientation of the principal stresses when the main crack approaches the internal fault in the case of $\beta = 45$ deg.

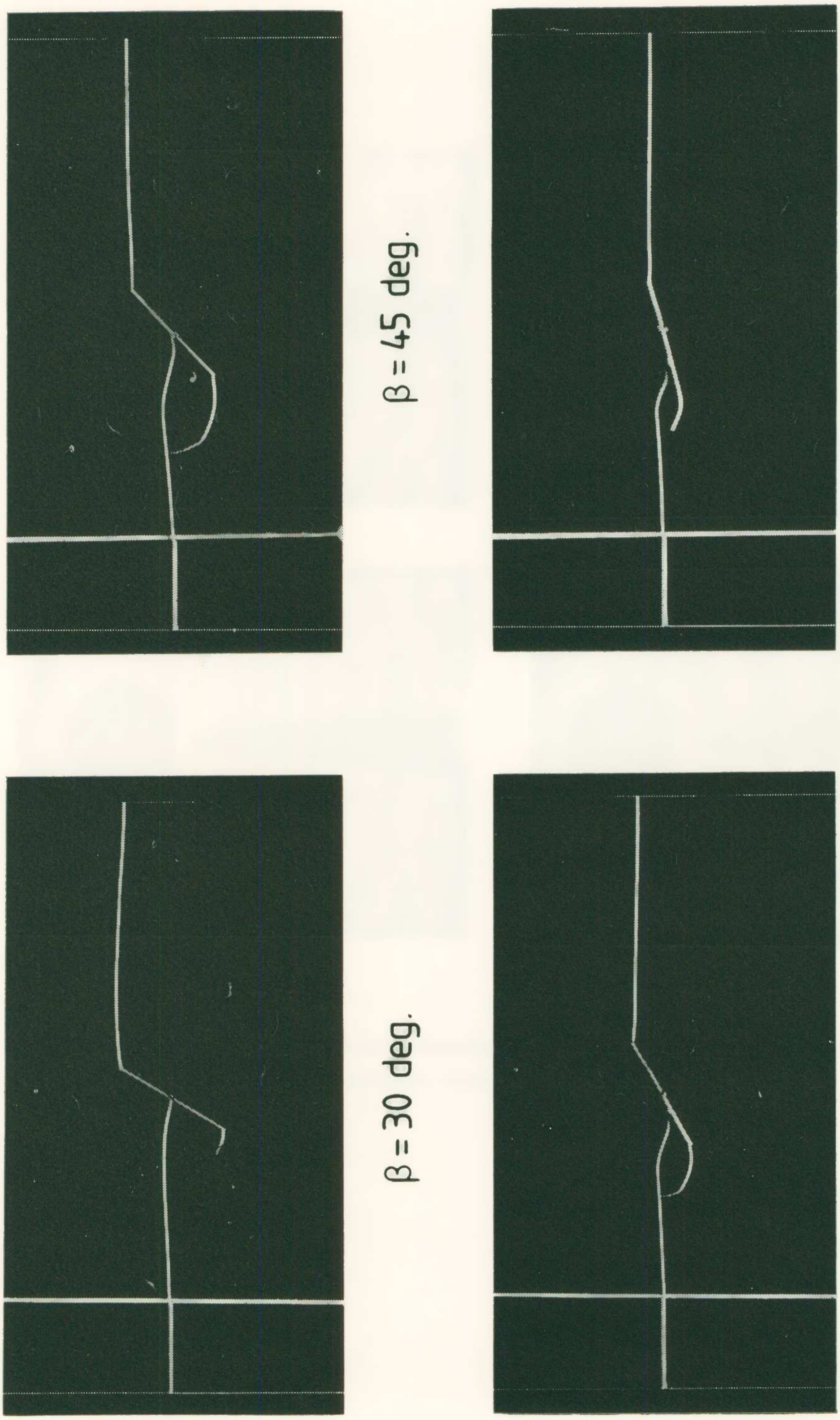

Fig. 26. Photographs of specimens showing the crack propagation paths for angles $\beta = 30$, 45, 60 and 75 deg.

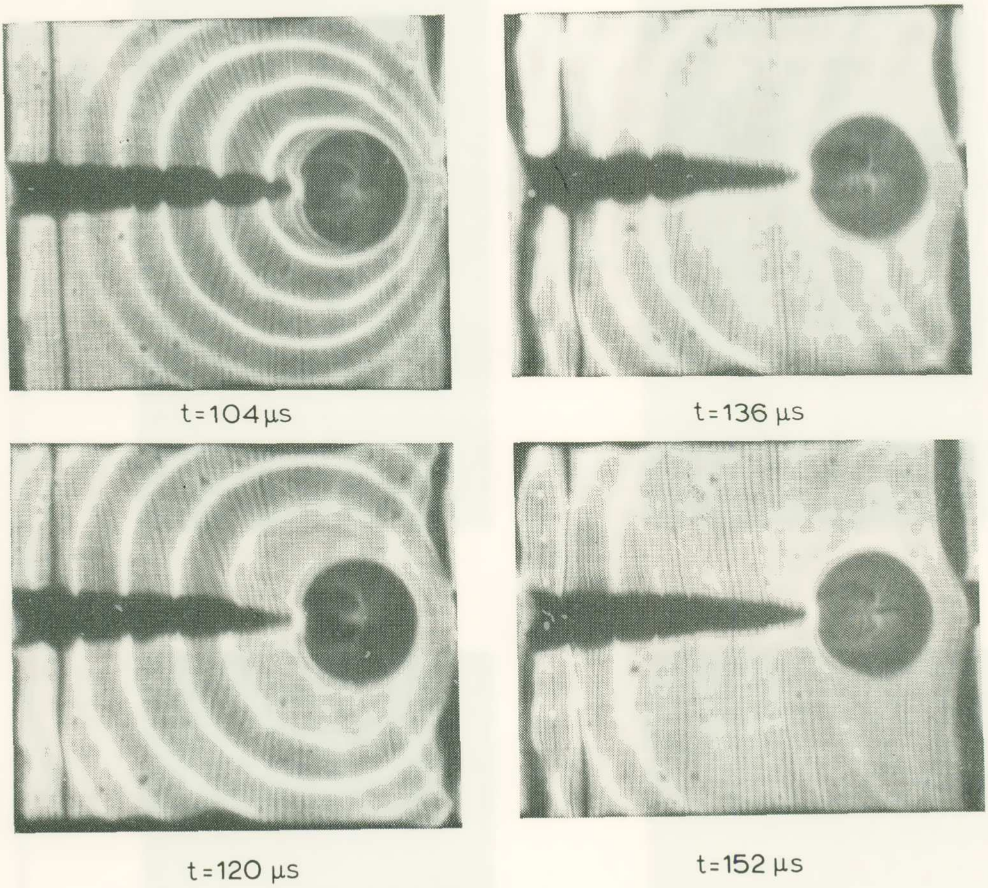

Fig. 27. Series of photographs showing succesive crack arrests by the formation of plastic zones and Rayleigh waves emission in a PCBA plate.

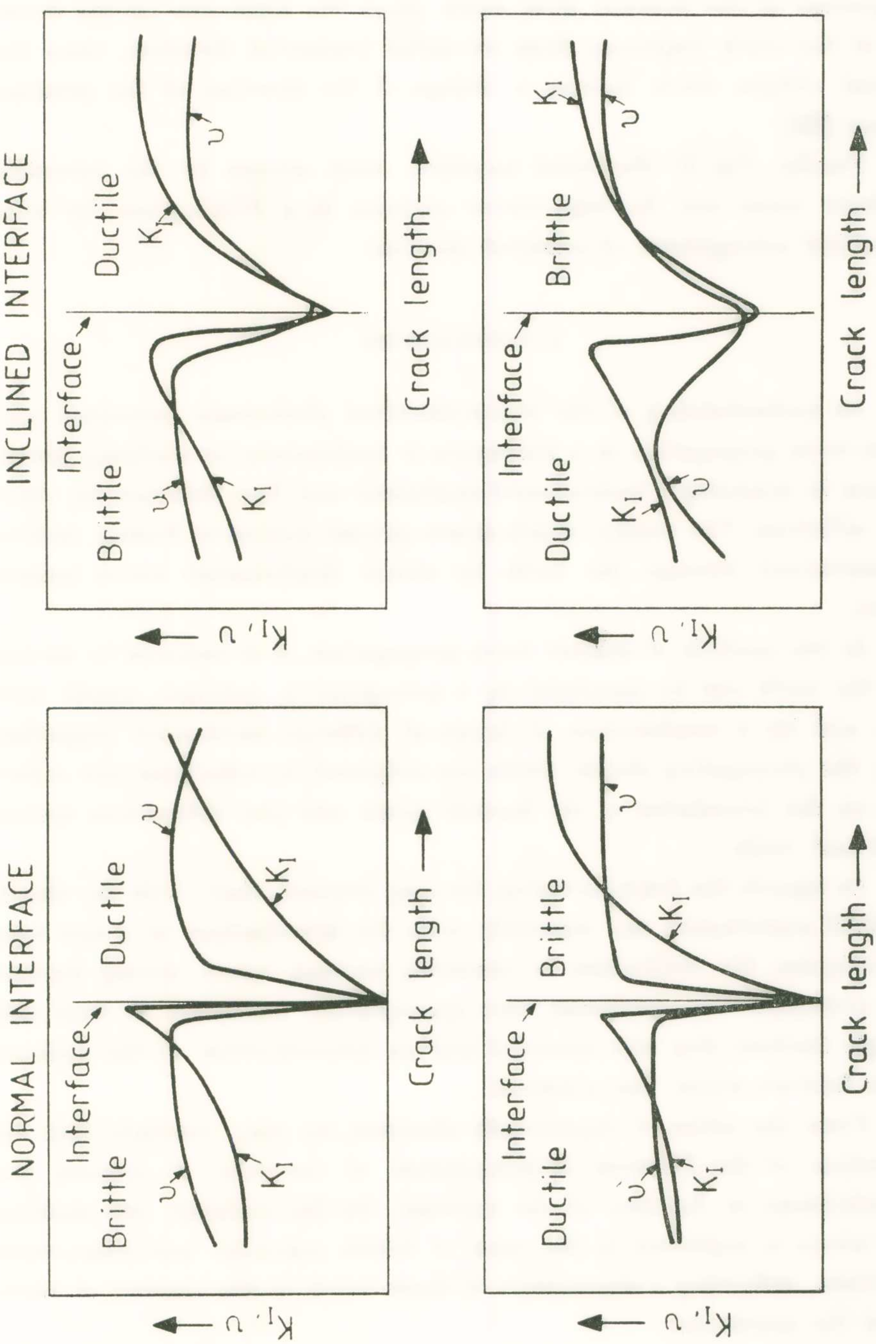

Fig. 21. Variation of the crack-velocity and dynamic stress intensity factor K_I^d, versus crack length in bimaterial plates.

obliqueness of the internal slant crack plays the main role on the deviation of the crack trajectory from its initial transerval direction, since this internal oblique crack induces a change of the direction of the principal stresses [**25**].

Finally, Fig. 27 illustrates succesive crack arrests by the formation of plastic zones and Rayleigh-waves emission in a PCBA-plate, by using the optical arrangement of reflected caustics.

5. CONCLUSIONS

An understanding of the above-described phenomena associated with elastic-wave propagation in a half-space is fundamental in studying ground motions in seismology, underground explosions and flaw detection by ultrasonic reflection. The energy, which causes ground motion or footing motion, is transmitted through the earth by elastic displacement waves (seismic waves).

In the analysis of seismic wave propagation, it is common to assume that the earth can be simulated by a homogeneous, isotropic, elastic half-space and by a combination of layers of different mechanical properties. Thus, the propagating elastic waves are subjected to reflections and refractions on the boundaries of the layered media and also diffractions around faults and voids.

As regards the fracture-waves, we may conclude that, with the above-described experiments and especially with the introduction of a new type of specimens, the difficulties in observing emitted waves during fracture were overcome. The specimens were appropriately machined, so that only a single fracture step was observed and an intensification of the emission of the fracture waves was achieved.

From the series of experiments executed we may conclude that the generation of the P-waves is independent of the state of stresses and the brittleness or ductility of the material. On the contrary, the emission of R-waves is supressed in the cases of brittle materials and plane-strain conditions, indicating a dependence of these waves to the amount of ductility of the specimens.

SP-waves appear as trailing waves from the incidence of the primary P-waves at glancing angles, on the lips of the propagating crack.

For the explanation of the non-circular scheme of Rayleigh waves, emanating from the tip of a propagating crack in rate-dependent elastic media, a satisfactory model was introduced.

By using the method of caustics and the model, it was possible to evaluate the transient dynamic stress- and strain-fields at every point of the plate, when the moving crack was propagating under mode-I deformation. An approximate stepwise procedure was introduced to evaluate the variation of the dynamic elastic modulus of the material, as the crack propagated, which was used for defining the velocities of the Rayleigh wavefronts allover the cracked medium.

Finally, many experiments with cracked plates containing discontinuities, in which running cracks propagate against these discontinuities, show that the interface between dissimilar phases or internal stable cracks act as «barriers», which cause crack deceleration and arrest phenomena.

ΠΕΡΙΛΗΨΙΣ

Ἡ παροῦσα μελέτη ἀφορᾶ εἰς τὴν θεωρητικὴν καὶ πειραματικὴν διερεύνησιν τῶν φαινομένων, τὰ ὁποῖα συνδέονται μὲ τὴν ἐκπομπὴν καὶ τὴν διάδοσιν ἐλαστικῶν κυμάτων εἰς συνεχῆ μέσα. Καίτοι τὰ φαινόμενα αὐτὰ ἔτυχον μέχρι τοῦδε τῆς προσοχῆς ἱκανοῦ ἀριθμοῦ ἐρευνητῶν, καὶ εἰς τὴν βιβλιογραφίαν ὑπάρχει πλῆθος θεωρητικῶν καὶ πειραματικῶν ἐργασιῶν, ὡρισμένα ἐνδιαφέροντα προβλήματα χρήζουν εὐρυτέρας ἢ γενικῆς ἐρεύνης.

Ἡ ἐλαστικὴ θεωρία τοῦ ἀπείρου ἡμιχώρου ὁρίζει δύο βασικοὺς τύπους τασικῶν κυμάτων, τὰ κύματα ὄγκου καὶ τὰ κύματα ἐπιφανείας. Τὰ κύματα ὄγκου διαχωρίζονται ἐκ νέου εἰς διαμήκη, προκαλοῦντα τὴν μεταβολὴν τοῦ ὄγκου, καὶ εἰς ἐγκάρσια, προκαλοῦντα τὴν μεταβολὴν τοῦ σχήματος τοῦ στοιχείου. Τὰ ἐπιφανειακὰ κύματα ἀποτελοῦν συνδυασμὸν τῶν ἀνωτέρω κυμάτων καὶ διαδίδονται μόνον εἰς τὰ ὅρια τοῦ σώματος, εἴτε αὐτὰ εἶναι ἐλεύθερα, εἴτε εἶναι συνδεδεμένα μεθ' ἑτέρων σωμάτων.

Ἐλαστικὰ κύματα δύνανται νὰ δημιουργηθοῦν εἰς τὴν ἐδαφικὴν μᾶζαν μὲ πλῆθος τρόπων λόγῳ σεισμῶν, ἐκρήξεων, κατὰ τὴν ἔμπηξιν πασσάλων, ἢ ταλαντουμένων θεμελίων. Ἡ πηγὴ αὐτῶν ἠμπορεῖ νὰ εὑρίσκεται ἐντὸς τοῦ ὑπὸ ἐξέτασιν στερεοῦ ἢ καὶ εἰς τὴν ἐπιφάνειάν του.

Τὰ ἐλαστικὰ κύματα σώματος, τὰ ὁποῖα ἀκολουθοῦν κάποιαν ἔμπηξιν ἢ κροῦσιν στερεοῦ, διαδίδονται ἀκτινικῶς, ἀπομακρυνόμενα ἐκ τῆς πηγῆς ἐπὶ ἡμισφαιρικῶν μετώπων, ἐνῷ τὰ ἐπιφανειακὰ κύματα ἐπὶ κυλινδρικῶν μετώπων. Ὁ νόμος τῆς γεωμετρικῆς ἀποσβέσεως τοῦ εὔρους τῶν κυμάτων εἶναι διὰ μὲν τὰ διαμήκη καὶ ἐγκάρσια ἀντιστρόφως ἀνάλογος τῆς ἀκτινικῆς ἀποστάσεως ἀπὸ τὴν πηγὴν γεννέσεώς των, διὰ δὲ τὰ ἐπιφανειακὰ ἀντιστρόφως ἀνάλογος τῆς τετραγωνικῆς ρίζης αὐτῆς. Διὰ ταλαντουμένην πηγὴν ἐπὶ τῆς ἐπιφανείας ὁμογενοῦς, ἰσοτρόπου, ἐλαστικοῦ ἡμιχώρου, ἡ διανομὴ τῆς συνολικῆς διαδιδομένης ἐνεργείας ἔχει ὡς ἑξῆς: 67 % ἐπιφανειακὰ κύματα, 26 % ἐγκάρσια κύματα, καὶ 7 % διαμήκη κύματα.

Ἐκ τῶν ἀνωτέρω καθίσταται σαφὴς ἡ σημασία τῶν ἐπιφανειακῶν κυμάτων, τόσον εἰς προβλήματα Ἐδαφοδυναμικῆς καὶ Σεισμῶν, ὅσον καὶ διερευνήσεως τῶν ὑλικῶν μὲ τὰς μεθόδους τῆς Ἀκουστικῆς.

Αἱ ταχύτητες διαδόσεως τῶν ἐλαστικῶν κυμάτων διαφέρουν μεταξύ των καὶ δίδουν ταυτοχρόνως τὴν δυνατότητα ἀνιχνεύσεως καὶ διαχωρισμοῦ αὐτῶν. Αἱ ταχύτητες αὐταὶ ἐξαρτῶνται ἀποκλειστικῶς ἐκ τῶν ἐλαστικῶν σταθερῶν καὶ τῆς πυκνότητος τῆς μάζης τοῦ μέσου διαδόσεως.

Ὅταν ἀρχικὸς διαμήκης ἢ ἐγκάρσιος παλμὸς προσκρούει ἐπὶ λείας ἐπιφανείας ἢ ἐπὶ ἐπιπέδου ὁρίου πλακός, εἰς τὴν ὁποίαν διαδίδεται, ἐμφανίζεται ἀνάκλασις καὶ δημιουργία νέων δευτερευόντων διαμήκων καὶ ἐγκαρσίων παλμῶν. Τὸ αὐτὸ φαινόμενον ἐπαναλαμβάνεται καὶ εἰς τὴν ὁριακὴν περίπτωσιν τῆς ὑπὸ ὀρθὴν γωνίαν προσπτώσεως τοῦ ἀρχικοῦ παλμοῦ. Εἰς τὴν περίπτωσιν αὐτὴν δημιουργεῖται κυματοδηγὸς κατὰ τὴν ἔννοιαν τοῦ πάχους τῆς λεπτῆς πλακός. Ὁ νόμος ἀνακλάσεως εἶναι ὁ γνωστὸς νόμος τοῦ Snellius, ὁ ὁποῖος ἰσχύει εἰς τὴν ὀπτικήν.

Ὅσον ἀφορᾷ εἰς τὴν ἐκπομπὴν ἐλαστικῶν κυμάτων κατὰ τὴν διάρκειαν τοῦ φαινομένου τῆς θραύσεως, ἀναφέρεται ὅτι, λόγῳ τῆς ἀποτόμου θραύσεως τῆς πλακός, δημιουργοῦνται αἱ ἑξῆς μορφαὶ κυμάτων: Διαμήκη καὶ ἐγκάρσια κύματα, ὡς ἀποτέλεσμα τῆς αἰφνιδίας μειώσεως τῶν ὀρθῶν καὶ διατμητικῶν τάσεων ἀντιστοίχως εἰς τὴν θραυομένην διατομήν. Ἐπιφανειακὰ κύματα διαδιδόμενα ἐπὶ τῶν χειλέων τῆς ρωγμῆς καὶ ἐπὶ τῆς ἐπιφανείας τοῦ δοκιμίου, ὡς ἀποτέλεσμα τῆς δυναμικῆς ἐφαρμογῆς τάσεων καὶ μετατοπίσεων ἀντιστοίχως, καθέτων πρὸς αὐτὰς τὰς ἐπιφανείας.

Εἰς τὸ πειραματικὸν μέρος τῆς παρούσης μελέτης, ὅπου ἐχρησιμοποιήθη ἡ μέθοδος τῶν ἐξ ἀνακλάσεως καὶ διελεύσεως καυστικῶν ἐν συνδυασμῷ μὲ διατάξεις ταχείας φωτογραφίσεως διὰ καταλλήλων φωτογραφικῶν μηχανῶν, ἐγένετο πλήρης καὶ συστηματικὴ διερεύνησις, ἀφορῶσα τὴν ψαθυρότητα ἢ τὴν ὀλκιμότητα τῶν χρησιμοποιηθέντων ὑλικῶν τῶν δοκιμίων καὶ τὴν ἐπίπεδον παραμορφωσιακὴν ἢ ἐπίπεδον ἐντατικὴν κατάστασιν αὐτῶν. Διὰ τὸν σαφῆ διαχωρισμὸν καὶ τὴν φωτογράφισιν ἑνὸς ἑκάστου τῶν ἐκπεμπομένων κυμάτων ἐπενοήθη νέος τύπος δοκιμίων, τὰ ὁποῖα ἔφερον μόνον μικρὸν σύνδεσμον ὑλικοῦ μεταξὺ τῶν δύο τμημάτων των, τὰ ὁποῖα συνεδέοντο μὲ τὰς ἁρπάγας τῆς μηχανῆς φορτίσεως.

Ἡ μελέτη ἔδειξεν ὅτι ἡ δημιουργία διαμήκων παλμῶν λόγῳ τῆς θραύσεως εἶναι ἀνεξάρτητος τῆς ψαθυρότητος ἢ ὀλκιμότητος τοῦ δοκιμίου, καθὼς καὶ τοῦ εἴδους τῆς ἐντατικῆς καταστάσεως αὐτοῦ. Ἀντιθέτως, ἐπιφανειακὰ κύματα εἰς τὰ ψαθυρὰ ὑλικὰ δημιουργοῦνται μόνον ὑπὸ συνθήκας ἐπιπέδου ἐντατικῆς καταστάσεως, ὅπου ἡ βύθισις εἰς τὸ ἄκρον τῆς ρωγμῆς εἶναι ἀξιοσημείωτος, λόγῳ τῆς πλευρικῆς συστολῆς.

Κατὰ τὴν θραῦσιν δοκιμίων, μορφῆς λεπτῆς πλακός, φερόντων μικρὰν μόνον ἀκραίαν ἀρχικὴν ρωγμήν, παρετηρήθη ὅτι τὰ ἐκπεμπόμενα συνεχῶς ἐπιφανειακὰ κύματα δὲν σχηματίζουν κυκλικῆς μορφῆς μέτωπα ἐπὶ τῆς πλακὸς καὶ ὁμοιάζουν κατὰ τὸ μᾶλλον καὶ ἧττον μὲ ἐλλείψεις. Διὰ τὴν διερεύνησιν τοῦ φαινομένου προτείνεται ὑπόδειγμα στηριζόμενον εἰς τὴν δημιουργίαν, κατὰ τὴν διάδοσιν τῆς ρωγμῆς, περιοχῶν μὲ ριζικῶς διάφορα μέτρα ἐλαστικότητος λόγῳ τῆς διαφόρου ἐντάσεως τοῦ πεδίου εἰς αὐτὰς τὰς περιοχάς.

Ἐχρησιμοποιήθησαν ἐπὶ πλέον τὰ ἀποτελέσματα ἑτέρων πειραμάτων μὲ δυναμικῶς φορτιζόμενα δοκίμια μορφῆς ράβδου, τὰ ὁποῖα παρέσχον τὴν καμπύλην τῆς μεταβολῆς τῶν μέτρων ἐλαστικότητος συναρτήσει τοῦ ρυθμοῦ παραμορφώσεως τοῦ δοκιμίου. Ἐν συνεχείᾳ ἐγένετο χρῆσις τῶν ἐκφράσεων διὰ τὸν προσδιορισμὸν τοῦ δυναμικοῦ ἐντατικοῦ πεδίου πλησίον τῆς διαδιδομένης ψαθυρᾶς ρωγμῆς, καὶ κατέστη δυνατὴ ἡ εὕρεσις τῶν μεταβαλλομένων μηχανικῶν ἰδιοτήτων τοῦ ὑλικοῦ τῆς πλακὸς κατὰ τὴν διάρκειαν ἐξελίξεως τοῦ φαινομένου. Αἱ εὑρεθεῖσαι τιμαὶ τοῦ δυναμικοῦ μέτρου ἐλαστικότητος ὁδηγοῦν εἰς διαφόρους ταχύτητας τῶν ἐπιφανειακῶν κυμάτων κατὰ διαφόρους κατευθύνσεις ἐπὶ τῆς ἐπιφανείας τοῦ δοκιμίου.

Τελικῶς, πλῆθος πειραμάτων, τὰ ὁποῖα ἐξετελέσθησαν εἰς δοκίμια μορφῆς πλακός, τὰ ὁποῖα περιεῖχον ἐσωτερικὰς σταθερὰς ρωγμάς, ἢ διαχωριστικὰς ἐπιφανείας, δεικνύουν τὴν ἰσχυρὰν ἐπιβράδυνσιν, ἡ ὁποία προκαλεῖται εἰς τὰς διαδιδομένας ρωγμὰς λόγῳ τῶν ἐμποδίων αὐτῶν. Κατὰ τὴν ἐπιβράδυνσιν καὶ τὴν ἐκ νέου ἐπιτάχυνσιν τῆς κινουμένης ρωγμῆς ἐλαστικὰ κύματα ἐκπέμπονται εἰς ὅλα τὰ πειράματα, λόγῳ τῆς ἀποτόμου μεταβολῆς τῶν τάσεων εἰς τὸ ἄκρον τῆς ἐπιβραδυνομένης ἢ ἐπιταχυνομένης ρωγμῆς.

Ἐκ τῆς ὅλης αὐτῆς μελέτης συνάγεται τὸ συμπέρασμα ὅτι τὴν διάδοσιν τῶν διαφόρων τύπων κυμάτων, τῶν δημιουργουμένων εἴτε ἐκ κρουστικῶν φορτίων, εἴτε ἐκ τῆς διαδόσεως ρωγμῶν καὶ ἄλλων ἀσυνεχειῶν εἰς τὰ σώματα, ἐπηρεάζουν αἱ μηχανικαὶ ἰδιότητες τῶν σωμάτων, ἡ μορφή των, καὶ αἱ τυχὸν ὑπάρχουσαι γεωμετρικαὶ ἀσυνέχειαί των.

Ἰδιαιτέρως τονίζεται ὅτι σημαντικὰ ἐμπόδια εἰς τὴν ὁμαλὴν διάδοσιν τῶν κυμάτων ἀποτελοῦν ὅλαι αἱ ἀσυνέχειαι τῶν σωμάτων καὶ ἰδιαιτέρως κάθε ἀλλαγὴ φάσεως, αἱ τυχὸν ὑπάρχουσαι στατικαὶ ρωγμαί, ὀπαί, ἐγκλείσματα, κ.λπ.

Δεδομένου ὅτι ἐξ ὅλων τῶν τύπων κυμάτων τὰ κύματα Rayleigh εἶναι πλέον ἐνδιαφέροντα, ὡς δυσκόλως ἀποσβεννύμενα, εἰδικαὶ πειραματικαὶ μελέται ἀπέδειξαν τοὺς νόμους διαδόσεως τῶν κυμάτων αὐτῶν. Ἡ σπουδαιότης τῶν κυμάτων αὐτῶν καταφαίνεται καὶ ἐκ τῆς συγγενείας των μὲ τὰ κύματα τὰ δημιουργούμενα κατὰ τοὺς σεισμοὺς καὶ ὅλας τὰς συναφεῖς διαταραχὰς τῶν κατασκευῶν.

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