

ΠΡΑΓΜΑΤΕΙΑΙ ΤΗΣ ΑΚΑΔΗΜΙΑΣ ΑΘΗΝΩΝ, ΤΟΜΟΣ 40, ΑΡΙΘ. 1 - 2, 1977















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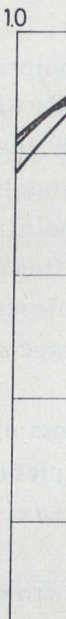
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