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ΕΦΗΡΜΟΣΜΕΝΑ ΜΑΘΗΜΑΤΙΚΑ.— **Stability concepts of dynamical systems. Applications to flight dynamics and numerical analysis** * by *Demetrios G. Magiros* **. 'Ανεκοινώθη ὑπὸ τοῦ 'Ακαδημαϊκοῦ κ. 'Ιωάν. Ξανθάκη.

Abstract

I will draw attention to groups of new problems which are very important both in theory and practice.

Our problems are essentially problems of basic and fundamental research of physical and engineering sciences, and, as we know, in such problems one does not ask for a necessarily direct and specific commercial objective. But our problems have the advantage that they are immediately applicable to practice and can be directed towards definite goals or applications of the current interest.

The above groups of problems appear when one uses and applies different concepts of stability coming from physical considerations and their mathematical interpretation.

We first give some remarks on stability concepts which are of basic importance for the formulation of our problems.

* ΔΗΜΗΤΡΙΟΥ Γ. ΜΑΓΕΙΡΟΥ, 'Επὶ τῶν ὁρισμῶν εὐσταθείας δυναμικῶν συστημάτων. Ἐφαρμογαὶ εἰς τὴν Μηχανικὴν τῶν τροχιῶν καὶ τὴν ἀριθμητικὴν ἀνάλυσιν.

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By using these stability remarks, one can formulate many new problems in a variety of fields, in mathematics and physics, in engineering and technology, in astrodynamics and meteorology, in chemistry and biology, in medicine and econometrics.

We will confine ourselves here to formulate some stability problems in some «*Precessional Phenomena*» of mechanics and flight dynamics, and in the «*Stability of Numerical Processes*».

PART A. STABILITY CONCEPTS OF DYNAMICAL SYSTEMS (Ref. 1 a, b)

Many problems in many fields of physical, technological and life sciences can be formulated and solved as problems of stability of dynamical systems.

During recent years the concepts of stability of dynamical systems have been advanced, either by modifying old ideas or by creating new ones, and these advances permit a deeper penetration into the more profound problems of stability.

I. One may have different kinds of stability concepts depending upon the nature of the system, the manner in which the system approaches a state or deviates from it, the properties of perturbations, etc.

Physically speaking, a state of a system is «*stable*», if small disturbances to the state produce small changes to the state. If these changes are considerable, the state is «*unstable*».

These physical stabilities are of «*qualitative nature*», then as such they cannot help to solve stability problems.

One needs a «*quantitative discussion*» of the stability concepts, that is an analytical or mathematical description of these concepts, and for this one needs to know relationships between the magnitudes of the disturbances of the state of the system and the magnitudes of their effects on the state.

II. The disturbances are due to *perturbations*, which can be considered as minor disturbing forces acting to the system either momentarily or permanently.

«*The momentarily acting perturbations*» give disturbances to the initial conditions of the system and they do not appear in the formulation

of the equations of the motion of the system. In this case we can get the equations of the motion of the system in the form :

$$\left. \begin{aligned} \dot{x}_i(t) &= X_i(t, x_1, \dots, x_n); \quad i = 1, \dots, n \\ x_i(t_0) &= x_{i0}, \quad X_i(t_1, 0, \dots, 0) = 0; \quad t_0 \leq t \end{aligned} \right\} \quad (1)$$

«The permanently acting perturbations» give disturbances to the system itself, and the equations of the motion of the system must contain the persistent perturbations, then the equations are :

$$\dot{x}_i(t) = X_i(t, x_1, \dots, x_n) + p_i(t, x_1, \dots, x_n) \quad (2)$$

where p_i are the persistent perturbations.

The functions X_i and p_i in (1) and (2) must satisfy appropriate conditions for a unique solution of the above equations.

The distinction of the perturbations into sudden and persistent, leads to the classification of the stability concepts into two big categories.

III. *The effect of perturbations* is a change of the original (unperturbed) motion with orbit L into the perturbed motion with orbit \bar{L} .

The states of a system at any time can be designated by points of the orbit, and if the unperturbed state P on L corresponds to the perturbed state \bar{P} on \bar{L} , their distance $q = P\bar{P}$ (Fig. 1), can be taken as

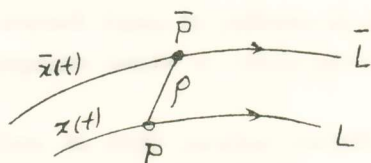


Fig. 1.

the magnitude of the effect of the perturbations on the state P .

The above distance q , called «*stability distance*», of which the end points are, by definition, correspondent points, plays a decisive role in the discussion of the stability problems.

Now the following question arises: «If the original orbit L and its perturbed \bar{L} are given, and if the point P on L is given, which is the point \bar{P} on \bar{L} , that corresponds to p ?

To answer this question, one must make assumptions on the correspondence, and for each assumption one has a unique correspondence, which characterizes a special stability concept of the trajectory L .

One can distinguish two different correspondences between the points P and \bar{P} , which give two different stability distances, when one has two different stability concepts very important physically.

The first is the case where P and \bar{P} correspond to each other at the «same time», when the distance $q = P\bar{P}$ is time dependent, called «Liapunov distance» (Fig. 2).

The second is the case where the distance $q = P\bar{P}$ is the «minimum» of the distances of P from points of \bar{L} (Fig. 3).

IV. By using the Liapunov and Poincaré distances, one has two distinct stability concepts, namely the «Liapunov stability» and the «Poincaré stability», respectively.

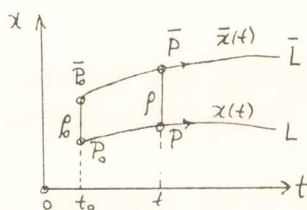


Fig. 2.

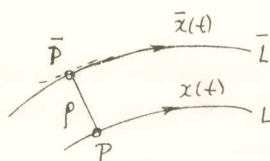


Fig. 3.

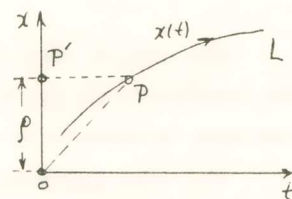


Fig. 4.

caré stability», respectively. The first is appropriate for the stability of motions, and the second for the stability of orbits (orbital stability).

A third kind of stability, connected with the concept of boundedness, called «Lagrange stability», comes by using the distance $q = OP'$ (Fig. 4), the distance of the state P from the origin in the state variables system, the «distance in Lagrange sense».

V. The above three kinds of stability concepts can be expressed quantitatively by the same «stability conditions»:

$$q_0 < \delta, \quad q < \varepsilon, \quad \lim_{t \rightarrow \infty} q = 0, \quad \|p\| < \eta \quad (3)$$

where $\delta, \varepsilon, \eta$ are positive constants, p the persistent perturbations, and q_0, q distances in the sense of Liapunov, or Poincaré, or Lagrange, when these conditions express stability definitions in the sense of Liapunov, or Poincaré, or Lagrange, respectively.

In case of «sudden perturbations», the first two conditions (3) define the «stability» of the trajectory L , and the first three of these conditions its «asymptotic stability».

In case of «persistent perturbations» all four conditions (3) express the stability, and one can get any kind of stability by an appropriate inter-

pretation of the distance ϱ and the kind of the norm used for the perturbations p . So *the conditions (3) unify the stability concepts*.

If the norm of p is: $\|p\| = \max_{x_i} |p_i|$, one has *total stability*,

If $\|p\| = \int_0^{\infty} \max_{x_i} |p_i| dt$, *integral stability*, and

If $\|p\| = \int_0^T \max_{x_i} |p_i| dt$, *mean stability*.

We remark that the knowledge of the regions of δ , ε , η , for which the conditions (3) are valid, will make the above mathematical stability concepts *practically accepted*.

VI. The unification of the stability concepts in the conditions (3) leads to important results, some of which, needed in the present paper, are listed in the following:

1. For the equilibrium states of a system, the stability concepts in the sense of Liapunov and Poincaré are equivalent.

For periodic states, the Liapunov stability concept is a narrow one compared to the Poincaré stability concept (orbital).

The isochronism, that is the constancy of the frequency, characterizes and suggests the use of the Liapunov concept in periodic states.

2. A state stable in Liapunov sense is stable in Poincaré sense, and a state unstable in Poincaré sense is unstable in Liapunov sense, but a state stable in Poincaré sense may be either stable or unstable in Liapunov sense.
3. The Lagrange stability concept, applied either to equilibrium states or to periodic motions in finite distances, classifies these special motions as stable, so the Lagrange stability concept is useless in these motions.
4. The stability concept of a motion in the Lagrange sense is exactly the boundedness property of the motion, but this is not so in the stabilities in the sense of Liapunov and Poincaré.

A motion $x(t)$, which, as $t \rightarrow \infty$, becomes infinite is Lagrange

unstable; but in this motion the Liapunov and Poincaré distances, being of the type $|\bar{x}(t) - x(t)|$, of which the limiting form may be $(\infty - \infty)$, it is possible to be finite and satisfies the conditions (3), when $x(t)$ may be stable in Liapunov or Poincaré sense.

5. One and the same physical phenomenon may be, mathematically speaking, stable or unstable depending on the stability concept used for the discussion of the phenomenon.

It is necessary that for any stability problem one must select beforehand the stability concept on which the stability discussion will be based, must know which stability concept is the appropriate one for the problem in hand, also to know the sizes of the $\delta, \varepsilon, \eta$ regions.

PART B. APPLICATIONS

FIRST APPLICATION: STABILITY OF A CLASS OF PRECESSIONAL PHENOMENA (Ref. 1 c, d)

I. The precessions

If a rigid body rotates around its axis of symmetry, precessional phenomena occur depending on the nature of the external torque acting on the body. The stability of an important class of precessions will be discussed in the following as an application to the preceding stability remarks.

The rotational motion of the body is given by the Euler's equations:

$$\left. \begin{aligned} \dot{\omega}_1 &= L_1 / I_1 \\ \dot{\omega}_2 &= L_2 / I - \omega_1 \omega_3 (I_1 - I) / I \\ \dot{\omega}_3 &= L_3 / I + \omega_1 \omega_2 (I_1 - I) / I \end{aligned} \right\} \quad (4)$$

$\omega_1, \omega_2, \omega_3$ are components of the angular velocity $\underline{\omega}$, which characterize a precessional motion of the body, and L_1, L_2, L_3 components of the resultant torque \underline{L} along the axes of the coordinate system $O_1 x_1 x_2 x_3$, which is fixed in the moving body. I_1 and $I_2 = I_3 = I$ are moments of inertia about the x_1 -axis and a perpendicular to it, respectively.

The vector $\underline{\omega}$ has as starting point the mass center O_1 of the body.
To the torque

$$L_1 = L_1(t), \quad L_2 = L_3 = 0 \quad (5)$$

the following solution of the equations (4) corresponds :

$$\left. \begin{aligned} \omega_1 &= (I/I_1) \int L_1(t) dt + c \\ \omega_2 &= A \cos Q(t) \\ \omega_3 &= A \sin Q(t) \\ Q(t) &= [(I_1 - I)/I] \int \omega_1(t) dt \end{aligned} \right\} \quad (6)$$

where c, A are constants depending on the initial conditions $\omega_{10}, \omega_{20}, \omega_{30}$.

The solution (6) gives a class of precessions corresponding to the torque (5), and by elimination of the time t in (6) one has the corresponding «precessional curve», which plays an important role for the stability of the precessions (6).

By specifying the torque as :

$$L_1 = \begin{cases} 0 \\ \bar{L}_1 = \text{const.}, \\ \sin t \end{cases} \quad L_2 = L_3 = 0 \quad (7)$$

the following special precessions occur :

$$\left. \begin{aligned} \omega_1 &= c = \text{const.}, \quad \omega_2 = A \cos Q_1(t), \quad \omega_3 = A \sin Q_1(t), \\ Q_1(t) &= \frac{I_1 - I}{I} ct \end{aligned} \right\} \quad (8.1)$$

$$\left. \begin{aligned} \omega_1 &= \frac{\bar{L}_1}{I_1} t + c, \quad \omega_2 = A \cos Q_2(t), \quad \omega_3 = A \sin Q_2(t), \\ Q_2(t) &= \frac{I_1 - I}{I} ct + \frac{(I_1 - I) \bar{L}_1}{2I_1 I} t^2 \end{aligned} \right\} \quad (8.2)$$

$$\left. \begin{aligned} \omega_1 &= -\frac{1}{I_1} \cos t + c, \quad \omega_2 = A \cos Q_3(t), \quad \omega_3 = A \sin Q_3(t), \\ Q_3(t) &= \frac{I_1 - I}{I} ct - \frac{I_1 - I}{I_1 I} \sin t \end{aligned} \right\} \quad (8.3)$$

(8.1) is the «regular precession», of which the precessional curve is a circumference PMNP in Fig. 5.

(8.2) the «*helicoid precession*», with precessional curve an helicoid curve, $P_0 P M N \dots$, (Fig. 6), and

(8.3) the «*non-regular periodic precession*», with precessional curve a non-planar curve, $P_0 P_1 P_2 P_3 P_0$ in Fig. 7.

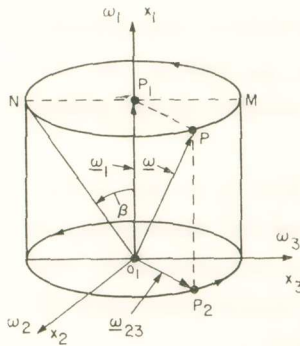


Fig. 5.

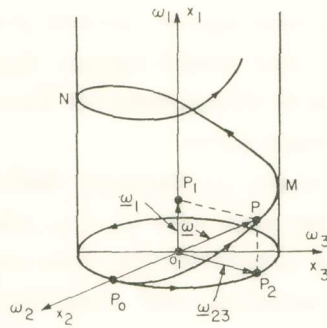


Fig. 6.

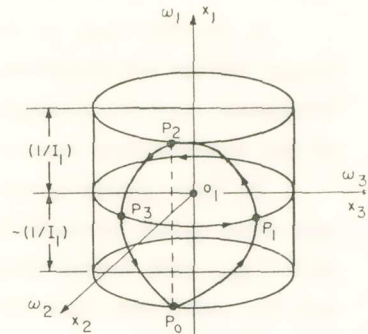


Fig. 7.

II. The stability of the precessions (8)

1. Lagrange Stability.

Since the precessional curves of the «*regular*» and «*non-regular periodic precessions*» have all their points at finite distance, the corresponding «*precessions*» are «*Lagrange-stable*». And since, as $t \rightarrow \infty$, the helicoid precessional curve tends to ∞ , the «*helicoid precession*» is «*Lagrange-unstable*».

2. Liapunov Stability.

In the «*helicoid*» and the «*non-regular periodic precessions*», the frequency of the motion of ω_{23} , projection of $\underline{\omega}$ on the x_2, x_3 — plane, is time-dependent, then the corresponding precessions are «*Liapunov-unstable*». In the «*regular-precession*», the frequency is given by: $f = (I_1 - I) \omega_{10} / I$, then, if the perturbations do not affect ω_{10} , this precession is «*Liapunov-stable*», but if ω_{10} is affected by the perturbations, this precession is «*Liapunov-unstable*».

3. Poincaré (Orbital) Stability.

a. In the case of the «*regular-precession*», the unperturbed and perturbed precessional curves are circumferences on planes perpendicular

to ω_1 — axis which contains their centers, then the Poincaré distance q at any point P of the unperturbed precessional curve is a constant, $q = q_0$.

If, given ε , one selects $\delta = \varepsilon$, the first two conditions (6) are satisfied, but not the third one, then the «regular precession» is «orbitally stable», but not «orbitally asymptotically stable».

b. In the case of the «non-regular periodic precession», the precessional curves are nonplanar and closed curves. Let q_M and q_m be the maximum and the minimum of all the Poincaré distances of all the points of the unperturbed precessional curve.

If, now, given ε , one wants all Poincaré distances q to be: $q < \varepsilon$, one must select perturbations such that $q_M < \varepsilon$, when an appropriate δ is $\delta < q_m$, and under these conditions the first two conditions (3), but not the third one, can be satisfied, and then the above precession is «orbitally stable», but not «orbitally asymptotically stable».

c. The discussion of the Poincaré stability of the «helicoid precession» can be based on the following definitions and theorems.

A generator of the cylinder surface intersects any two particular helicoid precessional curves L and \bar{L} into two infinite sets of points, $s: P_0, P_1, P_2, \dots, P_n, \dots$ on L , and $\bar{s}: P_0, P_1, P_2, \dots, P_n, \dots$ on \bar{L} , (Fig. 8(a)).

We consider three kinds of distances at a point P_n of L , shown in Fig. 8(b). The distance $D_n = P_n P_{n+1}$, between two consecutive points P_n and P_{n+1} of L , the distance $d_n = P_n \bar{P}_n$ between the point P_n of L and its consecutive \bar{P}_n on \bar{L} , and the Poincaré distance $q_n = P_n \bar{P}'_n$, where \bar{P}'_n is the intersection point of \bar{L} by the plane through P_n of L perpendicular to L at \bar{P}'_n .

The following theorem can be proved:

«The distances D_n , d_n , q_n tend to zero as the point P_n goes to infinity».

Based on this theorem, we can prove that the «helicoid precession» is «orbitally asymptotically stable».

We remark that for any particular helicoid precession, corresponding to given initial conditions, one can determine the appropriate « δ and ε — regions», when the orbital asymptotic stability of the helicoid precession is practically useful.

4. We refer here an important application of the concept of helicoid precession in flight dynamics, (Ref. 2).

bations the stability situation of the above precessions is not known.

5. The above results are summarized in the following table :

TABLE OF RESULTS

<i>Kinds of Precession</i> <i>Kinds of Stability</i>	<i>Regular Precession</i> (Eq. 8. 1)	<i>Helicoid Precession</i> (Eq. 8. 2)	<i>Non - Regular Periodic Precession</i> (Eq. 8. 3)
<i>Liapunov Stability</i>	<i>Stable</i> — if the perturbations do not change ω_1 <i>Unstable</i> — if the perturbations do change ω_1	<i>Unstable</i>	<i>Unstable</i>
<i>Poincaré Stability (Orbital)</i>	<i>Stable</i>	<i>Asymptotically Stable</i>	<i>Stable</i>
<i>Lagrangé Stability (Boundedness)</i>	<i>Stable</i>	<i>Unstable</i>	<i>Stable</i>

SECOND APPLICATION: STABILITY OF NUMERICAL PROCESSES,
(Ref. 3)

We formulate here and propose for investigation stability problems of numerical processes as applications of the stability remarks of the first part of this paper.

I. The modern numerical processes by using high speed computers brought in recent years new important problems as, i. e., the problem of the «numerical stability» of the computation processes. Previously, calculations were so short that a process could be identified with its model. Today the computers cannot, in general, realize the ideal process from which the actual process of the computer differs. This is due to the «round-off-errors», which disturb the numerical process, during its realization by the computer, in the same way as the noise disturbs the transmissions in telephone circuits; and as in the transmissions we study their capacities in order to preclude absorption by noise, in the same way we

must study the correspondent phenomenon during the realization of a numerical process, a phenomenon which corresponds to the stability of the process.

The problem of the stability of a numerical process is an investigation of the distance of the ideal process in the field of exact real numbers from the realization of this process by the computer and we can recognize immediately that this distance is of the same nature as the stability distance q of the previous part of the paper, namely, the distance in the sense of Liapunov, Poincaré, Lagrange. Furthermore, we can trust a formula used as a model for calculation, if the numerical process, realized by the computer, is stable.

Let us go more deeply to the problem.

Although a numerical process is a succession of elementary arithmetic operations on numbers, the numerical stability concept can be given in a generalized way by using vectors, matrices, normed spaces, an approach which permits the same characterization of numerical stability for all types of computers.

When one asks for numerical solutions of differential equations, one uses formulae which in general are numerical processes, of the form :

$$x_{i+1} = A_i (x_1, x_2, \dots, x_i) \quad (9)$$

where x_i are elements of normed vector spaces X_i , which usually are n -dimensional Euclidian spaces. A_i are continuous arithmetic operators, defined in the cartesian product $X_1 \times \dots \times X_i$ and mapping this product into the space X_{i+1} .

The sequence of elements x_1, x_2, \dots , which satisfy the model (9), is the solution of (9).

In practice, the model (9) cannot be realized, that is the real numerical process carried out by the computer is different than the ideal process (9).

If in the i^{th} step, $\bar{x}_1, \bar{x}_2, \dots, \bar{x}_i$ are the inexact values of x_1, x_2, \dots, x_i , and \bar{A}_i the inexact values of the operations A_i , then \bar{x}_{i+1} is the inexact value really computed, then :

$$\bar{x}_{i+1} = \bar{A}_i (\bar{x}_1, \bar{x}_2, \dots, \bar{x}_i) \quad (10)$$

If the operators A_i and \bar{A}_i operate on $(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_i)$, the results are in general different, and we can write:

$$\bar{A}_i(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_i) = A_i(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_i) + p_i \quad (11)$$

when formula (10) becomes:

$$\bar{x}_{i+1} = A_i(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_i) + p_i \quad (12)$$

p_i are errors with small norm.

II. Formula (12) is the right formula for the discussion of the stability of x_i of the model equations (9), then *«the problem of stability of a numerical process for the solution of differential equations is a problem of stability of dynamical systems under persistent perturbations»*, since the errors p_i act permanently during the computation, and the step of computation can be considered as unit-time. There are methods for determining the errors p_i , say by the actual machine program or by the manner of round-off, etc.

The distance q of the previous part is the norm of the difference $\bar{x}_i - x_i$, $q = \|\bar{x}_i - x_i\|$.

According to the stability conditions (3) of the previous part, we can formulate now the stability of a numerical process (9) as follows:

Given a number $\epsilon > 0$, if it is possible to find two positive numbers δ_1 and δ_2 such that starting from an initial absolute error of the process with norm smaller than δ_1 , $q_0 = \|\bar{x}_1 - x_1\| < \delta_1$, and if the norm of the error p_i is smaller than δ_2 , $\|p_i\| < \delta_2$, and, as a result of these assumptions, the norm of the absolute error is, for all steps of the calculation, smaller than ϵ , $q = \|\bar{x}_i - x_i\| < \epsilon$, $i = 1, 2, \dots$, then the numerical process is «stable».

If, in addition, the absolute error tends to zero, as the number of steps is increasing to infinity, $\lim_{i \rightarrow \infty} \|\bar{x}_i - x_i\| = 0$, then the numerical process is «asymptotically stable».

We remark that the above stability concepts of numerical processes are dependent on the kind of norm selected and independent of the hypothesis of boundedness of the values of the processes.

An investigation of the stability of numerical processes, according to the above concepts, will lead to new results concerning stability of the numerical processes, and these results may affect the whole field of numerical analysis.

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Π Ε Ρ Ι Λ Η Ψ Ι Σ

Εἰς τὴν παροῦσαν ἐργασίαν ἐκτίθενται παρατηρήσεις ἐπὶ τῶν ὁρισμῶν εὐσταθείας δυναμικῶν συστημάτων, βάσει τῶν ὁποίων καθίσταται δυνατόν νὰ μελετηθοῦν ζητήματα εὐσταθείας διαφορῶν κλάδων ἐρεύνης εἰς τὰ μαθηματικά, τὴν Φυσικὴν, Μηχανικὴν, Χημείαν, Βιολογίαν, Οἰκονομετρικὴν κλπ.

Λί ἄνωτέρω παρατηρήσεις ἐφαρμόζονται ἀποτελεσματικῶς διὰ τὴν λύσιν προβλημάτων εὐσταθείας διαφορῶν ομάδων μεταπτώσεων (precessions) ὑπὸ «στιγμιαίως ἐφαρμοζομένας ἐπιδράσεις». Ἡ λύσις τοῦ προβλήματος τῆς σπουδῆς τοῦ λάθους τοῦ προσανατολισμοῦ διαστημικοῦ ὀχήματος — ἐν σπουδαῖον σύγχρονον πρόβλημα — ἐπιτυγχάνεται δι' εἰσαγωγῆς τῆς ἐννοίας τῆς ἐλικοειδοῦς precession.

Τὸ πρόβλημα τῆς εὐσταθείας τῶν ἀριθμητικῶν λύσεων διαφορικῶν ἐξισώσεων ἐκφράζεται, βάσει τῶν ἄνωτέρω παρατηρήσεων, ὡς πρόβλημα εὐσταθείας δυναμικῶν συστημάτων ὑπὸ «συνεχῶς ἐφαρμοζομένας ἐπιδράσεις».



Ὁ Ἀκαδημαϊκὸς κ. Ἰωάν. Ξανθάκης ἀνακοινῶν τὴν ὡς ἄνω ἐργασίαν εἶπε τὰ ἑξῆς :

Ἔχω τὴν τιμὴν νὰ παρουσιάσω εἰς τὴν Ἀκαδημίαν ἐργασίαν τοῦ κ. Δημ. Μαγείρου, μαθηματικοῦ συμβούλου τῆς General Electric τῶν Ἡν. Πολιτειῶν, ὑπὸ τὸν τίτλον :

«Ἐπὶ τῶν ὁρισμῶν εὐσταθείας δυναμικῶν συστημάτων. Ἐφαρμογαὶ εἰς τὴν Μηχανικὴν τῶν τροχιῶν καὶ τὴν ἀριθμητικὴν ἀνάλυσιν».

Εἰς τὴν ἐργασίαν ταύτην ὁ κ. Μάγειρος ἐπισύρει κατ' ἀρχὰς τὴν προσοχὴν ἐπὶ μιᾷ ὁμάδῳ νέων προβλημάτων, τὰ ὅποια κέκτηνται ἰδιαιτέραν σημασίαν τόσον ἀπὸ θεωρητικῆς ὅσον καὶ ἀπὸ πρακτικῆς ἐπόψεως. Ἡ ὁμὰς δὲ αὕτη τῶν προβλημάτων ἔλκει τὴν ἀρχὴν τῆς ἐκ τῆς χρήσεως καὶ τῶν ἐφαρμογῶν τῶν διαφόρων ὁρισμῶν καὶ ἀντιλήψεων ἐπὶ τῆς εὐσταθείας, ποὺ πηγάζουν τόσον ἀπὸ φυσικὰς θεωρήσεις ὅσον καὶ ἀπὸ μαθηματικὰς ἐπεξηγήσεις. Πολλὰ δὲ προβλήματα διαφόρων πεδίων τῶν φυσικῶν ἐπιστημῶν καὶ τῆς τεχνολογίας δύνανται νὰ διατυπωθῶσι καὶ νὰ ἐπιλυθῶσι ὡς προβλήματα εὐσταθείας δυναμικῶν συστημάτων. Δέον δὲ νὰ σημειωθῇ ὅτι αἱ ἀπόψεις μας καὶ ἀντιλήψεις μας ἐπὶ τῆς εὐσταθείας δυναμικῶν συστημάτων ὑπέστησαν κατὰ τὰ τελευταῖα ἔτη σημαντικὰς ἐξελίξεις εἴτε διὰ τῆς τροποποιήσεως παλαιῶν ἰδεῶν εἴτε διὰ τῆς προτάσεως νέων τοιούτων. Δύνανται δέ τις νὰ ἔξη πολλὰ εἴδη εὐσταθείας ἐξαρκώμενα ἀπὸ τὴν φύσιν τοῦ συστήματος, ἀπὸ τὸν τρόπον καθ' ὃν τὸ σύστημα προσεγγίζει πρὸς μίαν κατάστασιν ἢ ἀπομακρύνεται ταύτης, ἀπὸ τὰς ιδιότητας τῶν ἐξασκουμένων παρελκτικῶν ἐνεργειῶν ἢ διαταραχῶν κλπ. Ἀπὸ φυσικῆς δὲ ἐπόψεως ἡ κατάστασις ἑνὸς συστήματος θεωρεῖται ὡς σταθερά, ὅταν μικραὶ διαταραχαὶ προκαλοῦν μικρὰς μεταβολὰς τῆς καταστάσεώς του, ἐὰν αἱ μεταβολαὶ εἶναι σημαντικαί, ἡ κατάστασις τοῦ συστήματος θεωρεῖται ἀσταθής. Ἀλλὰ αἱ οὕτω ὀριζόμεναι φυσικαὶ εὐστάθειαι εἶναι μόνον «ποιοτικῆς φύσεως» καὶ δὲν μᾶς ὑποβοηθοῦν εἰς τὴν λύσιν τῶν συναφῶν προβλημάτων εὐσταθείας. Διὰ τὴν ποσοτικὴν διερεῦνησιν εἶναι ἀναγκαῖον νὰ ἔχωμεν μίαν ἀναλυτικὴν ἢ μαθηματικὴν περιγραφὴν τῶν περὶ εὐσταθείας ἀντιλήψεων μας. Πρὸς τοῦτο δὲν εἶναι ἀναγκαῖον νὰ γνωρίζωμεν τὰς σχέσεις ποὺ συνδέουν τὰ μεγέθη τῶν ἐξασκουμένων διαταραχῶν μὲ τὰ μεγέθη τῶν ἀποτελεσμάτων αὐτῶν ἐπὶ τῆς καταστάσεως ἑνὸς συστήματος.

Εἰς τὴν παροῦσαν ἐργασίαν ὁ κ. Μάγειρος διατυπώνει ὁρισμένας παρατηρήσεις καὶ ἀπόψεις ἐπὶ τῶν ὁρισμῶν εὐσταθείας κατὰ Liapunov, Poincaré καὶ Lagrange μὲ ἀντικειμενικὸν σκοπὸν τὴν ἐνοποίησιν τῶν ὁρισμῶν τούτων ὑπὸ ὁρισμένας ἀρχικὰς συνθήκας. Τὰς ἀπόψεις του ταύτας ὁ συγγραφεὺς ἐφαρμόζει εἰς τὴν λύσιν προβλημάτων εὐσταθείας διαφόρων ὁμάδων μεταπτώσεων (precessions) ὑπὸ «στιγμιαίως ἐνεργουσῶν παρελκτικῶν ἐνεργειῶν ἢ ἐπιδράσεων». Οὕτω, ἡ σπουδὴ καὶ ἡ λύσις τοῦ προβλήματος τῶν παρεκκλίσεων εἰς τὸν προσανατολισμὸν διαστημικοῦ ὀχήματος, — ἐπὶ λίαν ἐνδιαφέροντος συγχρόνου προβλήματος — ἐπιτυγχάνεται διὰ τῆς εἰσαγωγῆς τῆς ἐννοίας τῆς ἐλικοειδοῦς μεταπτώσεως. Ὅμοιως, τὸ πρόβλημα τῆς εὐσταθείας τῶν ἀριθμητικῶν λύσεων διαφορικῶν ἐξισώσεων διατυποῦται, βάσει τῶν ἀπόψεων τοῦ συγγραφέως, ὡς πρόβλημα εὐσταθείας δυναμικῶν συστημάτων «ὑπὸ συνεχῶς ἐξασκουμένων ἐπιδράσεων».